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**Cointegration, Integration, and  
Long-Term Forecasting**

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# COINTEGRATION, INTEGRATION, AND LONG-TERM FORECASTING

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## ABSTRACT

It is widely believed that taking cointegration and integration into consideration is useful in constructing long-term forecasts for cointegrated processes. This paper shows that imposing neither cointegration nor integration leads to superior long-term forecasts.

Key words: Forecasting; Cointegration; Integration.

JEL classifications : C12, C32

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# 1 INTRODUCTION

A question that has received considerable research interest is whether imposing cointegration improves long-term forecasts. The reason is that while the variance of forecast errors from a unit root process diverges as the forecast horizon goes to infinity, the variance of the cointegrating combination of forecast errors is finite even in the long-run. Therefore, intuitively, imposing cointegration would seem to improve the accuracy of long-term forecasts. A number of studies have provided support for this intuition. Engle and Yoo (1987), for example, have shown that their long-term forecasts, which impose cointegration constraint, are superior to those from an unrestricted vector autoregressive model. Along similar lines, Lin and Tsay (1996) found in their Monte Carlo simulations that the specification of the cointegration relationship plays an important role in long-term forecasts, and that correct specification leads to better forecasts. The experimental results of Reinsel and Ahn (1992) also indicated that imposing the cointegration constraint helps to improve long-term forecasts. These studies have spread the belief that as long as the specification of the cointegrating relationship is correct, imposing cointegration helps long-term forecasts.

Rejecting this notion, Christoffersen and Diebold (1998) point out that in order to evaluate the effect of imposing cointegration on forecast accuracy, the above-mentioned studies compared the accuracy of forecasts which impose both integration and cointegration to that of forecasts which impose neither. Christoffersen and Diebold go on to argue that such a comparison does not necessarily show the effect of imposing cointegration and that a comparison between forecasts which impose both integration and cointegration and forecasts which impose only integration would be superior. They analytically show that in the long-run, Engle and Yoo's (1987) cointegrated system forecast (which imposes both integration and cointegration) and a forecast based on the univariate ARIMA model of an individual series (which imposes only integration) are equally accurate in terms of the mean square error (MSE) measure of accuracy. This means that imposing cointegration does not improve long-term forecasts.<sup>1</sup> The authors argue that in long-term forecasts

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<sup>1</sup>Christoffersen and Diebold (1998) note that whether imposing cointegration improves the accuracy of long-term forecasts depends on the adopted measure of accuracy. They propose an alternative accuracy

the effect of imposing cointegration on forecast accuracy is dominated by the effect of imposing integration. This is because the cointegrated system forecast performs as well as the univariate ARIMA forecast. In other words, Christoffersen and Diebold (1998) argue that only integration, but not cointegration, matters in long-term forecasts.

In this paper, we extend Christoffersen and Diebold's (1998) result and argue that when using the MSE criterion, *neither* integration nor cointegration matters in long-term forecasts where the prediction horizon is infinite. In particular, we analytically show that as the forecast horizon goes to infinity, the accuracy of the cointegrated system forecast decreases to that of forecasts which impose neither integration nor cointegration. Because our analytical result is an extension of the analytical result of Christoffersen and Diebold (1998), the derivation of our result can be given by a straightforward generalization of the derivation of their result.

The remainder of this paper is organized as follows. Section 2 provides a brief description of the model and the accuracy measure of forecasts. Section 3 shows that neither integration nor cointegration matters in long-term forecasts. Section 4 provides a Monte Carlo simulation corroborating the result in Section 3. Section 5 concludes.

## 2 THE MODEL AND THE ACCURACY MEASURE

Consider an  $m$ -variate vector cointegrated process which is given by a vector moving average of infinite order:

$$(1 - L)y_t = \mu + C(L)\varepsilon_t = \mu + \sum_{i=0}^{\infty} C_i \varepsilon_{t-i} \quad (1)$$

where  $\mu$  is a vector of constants,  $C_i$  is an  $m \times m$  matrix with absolute summability of  $\{sC_s\}_{s=0}^{\infty}$  and  $\varepsilon_t$  is an  $m \times 1$  i.i.d.  $(0, \Omega)$  process. Under regularity conditions, a necessary and sufficient condition for the existence of an  $m \times r$  matrix of cointegration  $\beta$  such that  $\beta'y_t \sim I(0)$ , is given by  $\beta'C(1) = 0$  with  $\text{rank}(C(1)) = m - r$ .

We now turn to the accuracy measure used in this paper. The MSE is one of the most commonly used measures of forecast accuracy in scalar processes. When we consider an 

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measure under which the cointegrated system forecast outperforms the univariate ARIMA forecast.

$m$ -variate process, the MSE is the  $m \times m$  matrix  $E(e_{t+h}e'_{t+h})$ , where  $e_{t+h}$  is the error vector of  $h$ -step-ahead forecast. In multivariate processes, a commonly used accuracy measure is  $\text{trace}(E(e_{t+h}e'_{t+h}))$  (see, e.g., Lin and Tsay (1996)). Following Christoffersen and Diebold (1998), we call this the “trace MSE.” They argue that in order to examine the relative accuracy of two forecasts, say 1 to 2, it is standard to use the ratio

$$\frac{\text{trace MSE}(e_{1,t+h})}{\text{trace MSE}(e_{2,t+h})}. \quad (2)$$

Following Christoffersen and Diebold (1998), we refer to this ratio as the “trace MSE ratio” and use this as the measure of accuracy.

We briefly review those results of Engle and Yoo (1987) and Christoffersen and Diebold (1998) which are required for our study. Engle and Yoo (1987) consider model (1) and derive an  $h$ -step ahead forecast:

$$\hat{y}_{t+h} = (t+h)\mu + \sum_{i=1}^t \sum_{j=0}^{t+h-i} C_j \varepsilon_i \quad (3)$$

They also derive its forecast error:

$$\hat{e}_{t+h} = y_{t+h} - \hat{y}_{t+h} = \sum_{i=1}^h \sum_{j=0}^{h-i} C_j \varepsilon_{t+i}. \quad (4)$$

They show that the trace MSE of  $\hat{e}_{t+h}$  diverges as  $h$  goes to  $\infty$ :

$$\text{trace MSE}(\hat{e}_{t+h}) \equiv \text{trace}(E(\hat{e}_{t+h}\hat{e}'_{t+h})) = O(h). \quad (5)$$

In contrast, because of the stationarity of  $\beta'\hat{e}_{t+h}$ , the trace MSE of  $\beta'\hat{e}_{t+h}$  does not diverge even in the long-run:

$$\lim_{h \rightarrow \infty} \text{trace MSE}(\beta'\hat{e}_{t+h}) = \beta'Q\beta < \infty,$$

where  $Q$  is a finite-valued constant matrix. Engle and Yoo (1987) also show that due to the equation

$$\lim_{h \rightarrow \infty} \sum_{j=0}^{t+h-i} C_j = C(1), \quad (6)$$

the cointegration constraint, that is,

$$\lim_{h \rightarrow \infty} \beta'\hat{y}_{t+h} = 0, \quad (7)$$

is satisfied in forecast (3).

Christoffersen and Diebold (1998) argue that a comparison between the accuracy of cointegrated system forecast (3) and of the univariate ARIMA forecast of an individual series allows us to evaluate the effect of imposing cointegration on forecast accuracy. This is because cointegrated system forecast (3) imposes both integration and cointegration, and their univariate ARIMA forecast imposes only integration. They show that

$$\lim_{h \rightarrow \infty} \frac{\text{trace MSE}(\tilde{e}_{t+h})}{\text{trace MSE}(\hat{e}_{t+h})} = 1, \quad (8)$$

where  $\tilde{e}_{t+h} = y_{t+h} - \tilde{y}_{t+h}$  and  $\tilde{y}_{t+h}$  denotes the univariate ARIMA forecast. Equation (8) means that in the long-run, imposing cointegration does not improve the cointegrated system forecast as long as the univariate ARIMA forecast imposes integration. Based on this result, they argue that only integration, but not cointegration, matters in long-term forecasts.

### 3 THE RELATIVE ACCURACY OF LONG-TERM FORECASTS

In this section, we show that a forecast which imposes neither cointegration nor integration performs as well as cointegrated system forecast (3), when the forecast horizon goes to infinity.

Consider a forecast  $\hat{y}_{t+h}$  that can be expressed as

$$\hat{y}_{t+h} = \begin{cases} v_{t+h} & \text{if } \mu = 0 \\ (t+h)\mu + v_{t+h} & \text{if } \mu \neq 0 \end{cases}, \quad (9)$$

where  $v_{t+h}$  is an  $m \times 1$  random vector, which possibly depends on a set of variables observed at date  $t$ , with trace  $E(v_{t+h}v'_{t+h}) = O(1)$ . We find that many forecasts, including the univariate ARIMA forecast, can be written in the form of (9). Further, a forecast  $\hat{y}_{t+h} = (t+h)\mu$ , which does not impose integration and cointegration, is given by (9) with  $v_{t+h} = 0$ . We therefore find that (9) forms a wide class of forecasts. Even a forecast which fails to impose both integration and cointegration can be in this class.

Upon comparing the accuracy of cointegrated system forecast (9) to forecast (3) in terms of the trace MSE ratio, we obtain the following proposition:

*Proposition:*

$$\lim_{h \rightarrow \infty} \frac{\text{trace MSE}(\hat{\hat{e}}_{t+h})}{\text{trace MSE}(\hat{e}_{t+h})} = 1, \quad (10)$$

where  $\hat{\hat{e}}_{t+h} = y_{t+h} - \hat{\hat{y}}_{t+h}$ .

*Proof:* We prove the above proposition in roughly the same way as Proposition 1 of Christoffersen and Diebold (1998). We express  $\hat{\hat{e}}_{t+h}$  as follows:

$$\begin{aligned} \hat{\hat{e}}_{t+h} &= y_{t+h} - \hat{\hat{y}}_{t+h} \\ &= (y_{t+h} - \hat{y}_{t+h}) + (\hat{y}_{t+h} - \hat{\hat{y}}_{t+h}) \\ &\approx \hat{e}_{t+h} + ((t+h)\mu + C(1)\xi_t - (t+h)\mu - v_{t+h}) \end{aligned} \quad (11)$$

$$= \hat{e}_{t+h} + C(1)\xi_t - v_{t+h}, \quad (12)$$

where  $\xi_t = \sum_{i=1}^t \varepsilon_i$ . The approximation in line (11) holds for large  $h$  because we obtain

$$\hat{y}_{t+h} = (t+h)\mu + \sum_{i=1}^t \sum_{j=0}^{t+h-i} C_j \varepsilon_i \approx (t+h)\mu + C(1)\xi_t$$

as  $h$  becomes large. Using (12), we derive the following equation:

$$\begin{aligned} \text{trace MSE}(\hat{\hat{e}}_{t+h}) &= \text{trace } E[\hat{\hat{e}}_{t+h} \hat{\hat{e}}'_{t+h}] \\ &= \text{trace } E[(\hat{e}_{t+h} + C(1)\xi_t - v_{t+h})(\hat{e}_{t+h} + C(1)\xi_t - v_{t+h})'] \\ &= \text{trace } E(\hat{e}_{t+h} \hat{e}'_{t+h}) + \text{trace } E(C(1)\xi_t (C(1)\xi_t)') + \text{trace } E(v_{t+h} v'_{t+h}) \\ &\quad + 2\text{trace } E(\hat{e}_{t+h} (C(1)\xi_t)') - 2\text{trace } E(\hat{e}_{t+h} v'_{t+h}) - 2\text{trace } E(C(1)\xi_t v'_{t+h}) \\ &= \text{trace MSE}(\hat{e}_{t+h}) + V, \end{aligned} \quad (13)$$

where  $V = \text{trace } E(C(1)\xi_t (C(1)\xi_t)') + \text{trace } E(v_{t+h} v'_{t+h}) - 2\text{trace } E(C(1)\xi_t v'_{t+h})$ , since  $\text{trace } E(\hat{e}_{t+h} (C(1)\xi_t)') = \text{trace } E(\hat{e}_{t+h} v'_{t+h}) = 0$  due to the serial independence of  $\varepsilon_t$ . We find that  $V = O(1)$  because  $\text{trace } E(C(1)\xi_t (C(1)\xi_t)') = O(1)$ ,  $\text{trace } E(v_{t+h} v'_{t+h}) = O(1)$  by assumption, and Schwarz's inequality and the above results ensure that  $\text{trace } E(C(1)\xi_t v'_{t+h}) = O(1)$ . On the other hand, the first term of equation (13) is  $O(h)$ , as shown in (5). Thus, the dominating term in  $\text{trace MSE}(\hat{\hat{e}}_{t+h})$  is just  $\text{trace MSE}(\hat{e}_{t+h})$ . Hence, the trace MSE ratio,  $(\text{trace MSE}(\hat{\hat{e}}_{t+h})) / (\text{trace MSE}(\hat{e}_{t+h}))$ , converges to unity as  $h$  becomes large and the proof is complete.

The above proposition indicates that as the forecast horizon goes to infinity, a forecast which does not impose integration and cointegration constraints performs as well as the cointegrated system forecast which correctly imposes integration and cointegration constraints. This is an extension of Christoffersen and Diebold's (1998) result (8). Christoffersen and Diebold (1998) argue that integration rather than cointegration matters in long-term forecasts. The above proposition straightforwardly extends their result and shows that neither cointegration nor integration matters in the long-run. Based on the above proposition, we argue that only the drift term  $(t + h)\mu$  matters in the long-run, and if there is no drift, i.e.  $\mu = 0$ , there is no information that helps in long-term forecasts.

## 4 MONTE CARLO EXPERIMENTS

In this section, we present the results of our Monte Carlo experiments. In order to ensure that the Proposition holds, we compare the accuracy of cointegrated system forecast (3) and other forecasts (introduced below) in terms of trace MSE ratio.

### *The Monte Carlo Design*

In our experiment, we consider the following bivariate cointegrated model:

$$x_t = \mu + x_{t-1} + \varepsilon_t$$

$$y_t = \lambda x_t + u_t,$$

where the disturbances, i.e.  $\varepsilon_t$  and  $u_t$ , are distributed  $i.i.d.N(0, \sigma_\varepsilon^2)$  and  $i.i.d.N(0, \sigma_u^2)$ , respectively, and  $\varepsilon_t$  and  $u_t$  are independent of each other. This is the same model as that employed by Christoffersen and Diebold (1998). Throughout our experiment, we set the sample size to 100, and the number of replications is 4000. We consider two data generating processes (DGPs).

#### Case 1

Following Christoffersen and Diebold (1998), we set  $\lambda = 1$ ,  $\mu = 0$ ,  $\sigma_\varepsilon^2 = 1$  and  $\sigma_u^2 = 1$ .



Case 2

We set  $\lambda = 1$ ,  $\mu = 1$ ,  $\sigma_\varepsilon^2 = 1$  and  $\sigma_u^2 = 1$ .

The important point in designing the DGPs is the value of  $\mu$ . There is no drift in Case 1. In contrast, the DGP of Case 2 contains the drift.

For these two cases, we compare the accuracy of the following four forecasts:

- (i) The cointegrated system forecast: 
$$\begin{bmatrix} \hat{x}_{t+h} \\ \hat{y}_{t+h} \end{bmatrix} = \begin{bmatrix} \mu h + x_t \\ \lambda \mu h + \lambda x_t \end{bmatrix}.$$
- (ii) The univariate ARIMA forecast:<sup>2</sup> 
$$\begin{bmatrix} \tilde{x}_{t+h} \\ \tilde{y}_{t+h} \end{bmatrix} = \begin{bmatrix} \mu h + x_t \\ \lambda \mu h + y_t + \theta \nu_t \end{bmatrix},$$
 where  $\theta$  is a parameter and  $\nu_t$  is a disturbance such that  $(1 - L)u_t + \lambda \varepsilon_t = \nu_t + \theta \nu_{t-1}$ .
- (iii) A misspecified forecast: 
$$\begin{bmatrix} \hat{\hat{x}}_{t+h} \\ \hat{\hat{y}}_{t+h} \end{bmatrix} = \begin{bmatrix} \mu(t+h) + \sum_{i=1}^3 x_i/3 \\ \lambda \mu(t+h) + \sum_{i=1}^3 y_i/3 \end{bmatrix}.$$
- (iv) An idle forecast: 
$$\begin{bmatrix} \bar{x}_{t+h} \\ \bar{y}_{t+h} \end{bmatrix} = \begin{cases} 0 & \text{for Case 1} \\ 100 & \text{for Case 2} \end{cases}.$$

Forecasts (i) and (ii) are given by Christoffersen and Diebold (1998) and (iii) is a forecast which can be expressed as (9). Forecast (iii) may appear unusual, but it is given by (9) with  $v_{t+h} = [\sum_{i=1}^3 x_i/3, \sum_{i=1}^3 y_i/3]'$ . In contrast, (iv) is not expressed as (9) in Case 2. In Case 1, (iv) is given by (9) with  $v_{t+h} = 0$ . We consider the following three trace MSE ratios:

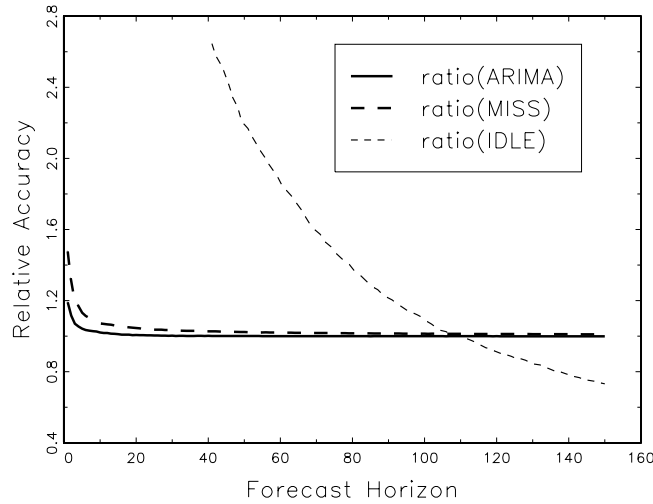
$$\begin{aligned} \text{ratio(ARIMA)} &= \frac{\text{trace MSE}(\tilde{e}_{t+h})}{\text{trace MSE}(\hat{e}_{t+h})}, \\ \text{ratio(MISS)} &= \frac{\text{trace MSE}(\hat{\hat{e}}_{t+h})}{\text{trace MSE}(\hat{e}_{t+h})} \text{ and} \\ \text{ratio(IDLE)} &= \frac{\text{trace MSE}(\bar{e}_{t+h})}{\text{trace MSE}(\hat{e}_{t+h})}, \end{aligned}$$

where  $\hat{e}_{t+h}$ ,  $\tilde{e}_{t+h}$ ,  $\hat{\hat{e}}_{t+h}$  and  $\bar{e}_{t+h}$  are the forecast errors of forecasts (i), (ii), (iii) and (iv), respectively. If the Proposition holds, as  $h$  becomes large, ratio(ARIMA) and ratio(MISS) converge to unity both in Case 1 and in Case 2. On the other hand, ratio(IDLE) converges to unity in Case 1 while it does not converge to unity in Case 2. Computing these trace

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<sup>2</sup>Due to space limitations, we omit the derivation of the univariate ARIMA forecast. Interested readers may refer to Christoffersen and Diebold (1998).

Figure 1: Trace MSE ratios in Case 1



Note: “ratio(ARIMA)” shows the trace MSE ratio of the univariate ARIMA forecast to the cointegrated system forecast. “ratio(MISS)” shows the trace MSE ratio of the misspecified forecast to the cointegrated system forecast. “ratio(IDLE)” shows the trace MSE ratio of the idle forecast to the cointegrated system forecast.

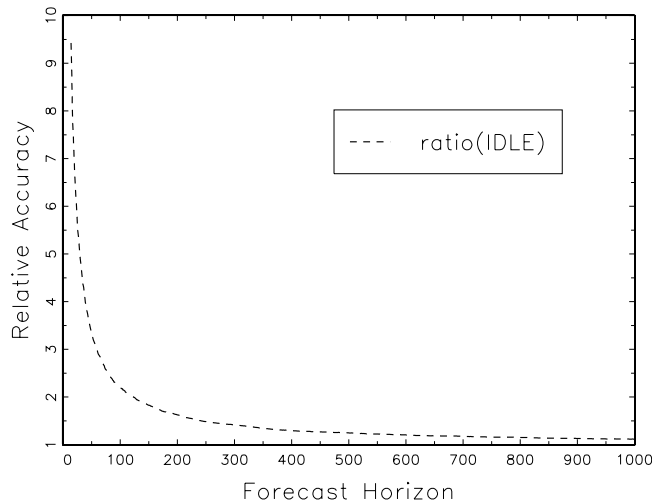
MSE ratios, we replace population parameters  $\mu$ ,  $\lambda$  and  $\theta$  with their estimates, because in practice population parameters must be estimated.

### *Monte Carlo Results: Case 1*

Figure 1 shows ratio(ARIMA), ratio(MISS) and ratio(IDLE) plotted against forecast horizon  $h$ . In this figure, we can see that ratio(ARIMA) converges to unity as  $h$  becomes large, which is consistent with equation (8). This result indicates that although the cointegrated system forecast imposes both integration and cointegration, it is not superior to the univariate ARIMA forecast which imposes only integration for large  $h$ . In other words, integration rather than cointegration matters in long-term forecasts. However, it turns out that ratio(MISS) also converges to unity as  $h$  becomes large. That is, the misspecified forecast which imposes neither cointegration nor integration, and the cointegrated system forecast are equally poor for large  $h$ . We therefore find that neither cointegration nor integration matters in long-term forecasts.

Figure 1 also shows that ratio(IDLE) decreases with  $h$  and falls below unity for

Figure 2: ratio(IDLE) with true drift in Case 1



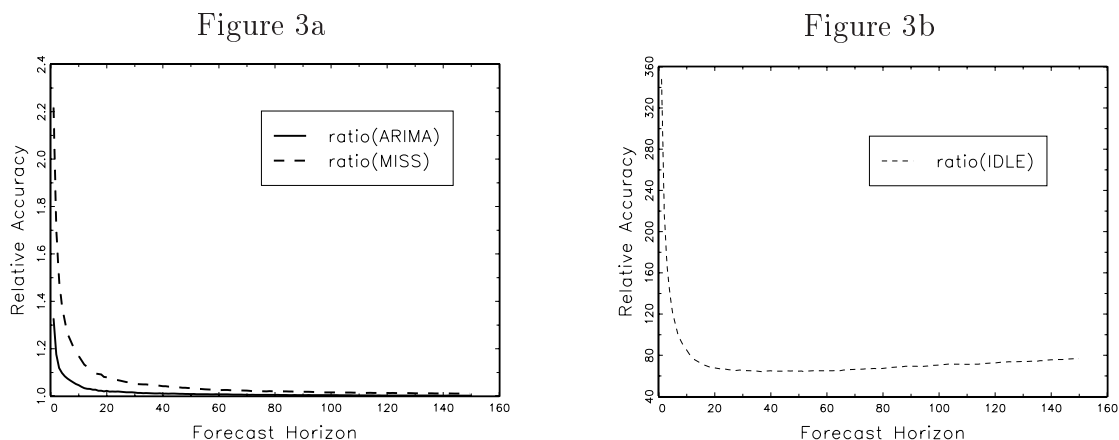
Note: In contrast with Figure 1 which shows trace MSE ratios based on the estimated drift, this figure shows ratio(IDLE) based on the true drift.

$h > 110$ . That is, the idle forecast outperforms the cointegrated system forecast for sufficiently large  $h$ . This result may appear to be inconsistent with our proposition that  $\lim_{h \rightarrow \infty} \text{ratio}(\text{IDLE}) = 1$  but in fact is not. The reason why ratio(IDLE) falls below unity is that the idle forecast happens to impose the true drift, i.e.  $\mu = 0$ , while the cointegrated system forecast contains the estimated drift,  $\hat{\mu}$ , which may not be equal to  $\mu = 0$ . Although the difference between  $\mu$  and  $\hat{\mu}$  is slight,  $\hat{\mu}$  pollutes the cointegrated system forecast with  $\hat{\mu}h$ . This pollution becomes more serious as  $h$  becomes large, resulting in superiority of the idle forecast over the cointegrated system forecast. In fact, Figure 2 shows that ratio(IDLE), which is based on the true  $\mu$  and the other, estimated parameters, converges to unity, which is consistent with our Proposition. This result indicates that when the DGP has no drift, any information on integration and/or cointegration does not produce a superior forecast in the long-run. In Case 1, where the true drift is equal to zero, forecasts (i) - (iv) are equally poor when  $h$  becomes large.

### *Monte Carlo Results: Case 2*

Figure 3 shows ratio(ARIMA), ratio(MISS) and ratio(IDLE) in Case 2. In Figure 3a, ratio(ARIMA) and ratio(MISS) converge to unity, as in Case 1. Forecasts (i) - (iii)

Figure 3: Trace MSE ratios in Case 2

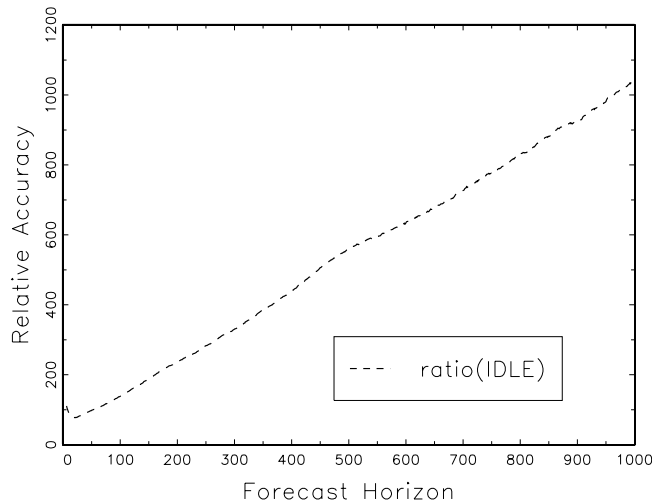


Note: “ratio(ARIMA)” shows the trace MSE ratio of the univariate ARIMA forecast to the cointegrated system forecast. “ratio(MISS)” shows the trace MSE ratio of the misspecified forecast to the cointegrated system forecast. “ratio(IDLE)” shows the trace MSE ratio of the idle forecast to the cointegrated system forecast.

appear to be equally poor for large  $h$ , indicating that neither cointegration nor integration matters in long-term forecasts. In contrast, ratio(IDLE) does not converge to unity in Figure 3b. This is because the idle forecast does not take the drift, which is present in the DGP, into account. Figure 3 thus shows that the drift term matters in long-term forecasts.

Note that we can analytically show that ratio(IDLE) tends to infinity as  $h$  tends to infinity in Case 2, although the divergence of ratio(IDLE) is not readily apparent in Figure 3b (i.e., the value of ratio(IDLE) does not really become large as  $h$  gets large). This is because we compute the cointegrated system forecast based on the estimated drift. That is, not only the idle forecast but also the cointegrated system forecast does not impose the true drift because  $\hat{\mu}$  may not be equal to  $\mu$  in finite samples. Thus, the trace MSE ratio of the idle forecast to the cointegrated system forecast only gradually diverges. When we compute the cointegrated system forecast based on the true drift and the other, estimated parameters, ratio(IDLE) clearly diverges, as shown in Figure 4. We may conclude this section by emphasizing that only the drift term matters in long-term forecasts.

Figure 4: ratio(IDLE) with true drift in Case 2



Note: In contrast with Figure 3b which shows ratio(IDLE) based on the estimated drift, this figure shows ratio(IDLE) based on the true drift.

## 5 CONCLUSION

This paper examined whether imposing integration and cointegration constraints improves long-term forecasts using the MSE ratio as a yardstick. Engle and Yoo (1987) and others, using Monte Carlo experiments, have argued that imposing cointegration produces superior long-term forecasts. However, Christoffersen and Diebold (1998) have noted that these earlier studies have misinterpreted their Monte Carlo results. They analytically showed that the cointegration constraint does not improve long-term forecasts when using the trace MSE ratio as the basis for judgment. They argued that only the integration constraint matters in long-term forecasts. This paper extended their result and showed that in fact neither cointegration nor integration matters in long-term forecasts. As long as a forecast contains a drift term, regardless of whether it imposes integration and/or cointegration, it performs as well as Engle and Yoo’s (1987) cointegrated system forecast. Because the cointegrated system forecast correctly imposes both integration and cointegration, we arrive at the conclusion that only the drift, but not integration and cointegration, matters in long-term forecasts for cointegrated processes. Further, when there is no drift in the DGP, there is no information that would help to improve the forecast.

We should note that our result heavily depends upon our choice of accuracy measure of forecasts. While the trace MSE ratio is one popular measure used in forecast evaluation, there are, as Christoffersen and Diebold (1998) highlighted, alternative accuracy measures. They showed that their argument that only integration matters in long-term forecasts does not hold when these alternative measures are used. Needless to say, our result that only a drift term matters in long-term forecasts may also change when alternative measures are used.

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