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**A Test for Autocorrelation
in Dynamic Panel Data Models**

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A Test for Autocorrelation in Dynamic Panel Data Models *

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Abstract

This paper presents an autocorrelation test that is applicable to dynamic panel data models with serially correlated errors. Our residual-based GMM t -test (hereafter: t -test) differs from the m_2 and Sargan's over-identifying restriction (hereafter: Sargan test) in Arellano and Bond (1991), both of which are based on residuals from the first-difference equation. It is a significance test which is applied after estimating a dynamic model by the instrumental variable (IV) method and is directly applicable to any other consistently estimated residual. Two interesting points are found: the test depends only on the consistency of the first-step estimation, not on its efficiency; and the test is applicable to both forms of serial correlation (i.e., AR(1) or MA(1)).

Monte Carlo simulations are also performed to study the practical performance of these three tests, the m_2 , the Sargan and the t -test for models with first-order auto-regressive AR(1) and first-order moving-average MA(1) serial correlation. The m_2 and Sargan test statistics appear to accept too often in small samples even when the autocorrelation coefficient approaches unity in the AR(1) disturbance. Overall, our residual based t -test has considerably more power than the m_2 test or the Sargan test.

Keywords: Dynamic panel data; Residual based GMM t -test; m_2 and Sargan tests

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1 Introduction

The phenomenon of serial correlation, i.e., cases where successive residuals appear to be correlated with each other, is very often encountered in econometric models. For example, adaptive expectations, stock adjustments, and price adjustments frequently call for estimating a model in which lagged endogenous variables and autocorrelated disturbances coexist. However, while a vast number of methods have been proposed to deal with these problems in time-series data (e.g., Taylor and Wilson, 1964; Wallis, 1967) few have addressed them in the context of panel data (Ahn and Schmidt, 1997).

The main purpose of this paper is to propose a test of serial correlation in dynamic panel models that is applicable after estimating a dynamic model from panel data by the IV method, and to compare this with the m_2 and the Sargan test proposed by Arellano and Bond (1991)(hereafter: AB). While these two tests are the most commonly used methods to detect serial correlation of the error term in a dynamic model based on panel data, their application is limited to uncorrelated disturbances under the null and moving-average errors under the alternative. In other words, they do not consider the case of autoregressive errors by representing this kind of model as a dynamic regression with non-linear common factor restrictions and uncorrelated errors (e.g., Sargan, 1980).

However, if there are *a priori* reasons to expect autoregressive errors in a panel regression model, or if the dynamics of the model have been incorrectly specified, there is a strong possibility of autocorrelation being present in the residuals. Hence, it is natural that we may consider a test of uncorrelated errors as a null against an AR(1) error as an alternative. If the disturbance has an AR(1) structure, the usual instruments of lagged values of the dependent variables in the differenced equations as used by, for example, Anderson and Hsiao (1981, 1982) and AB (1991) are no longer valid. Furthermore, an estimator that uses lags as instruments under the assumption of white noise errors loses its consistency if in fact the disturbances are autocorrelated.

Thus, AB's m_2 and Sargan tests are no longer applicable because they use inconsistently estimated residuals based on one-step consistent estimation (hereafter GMM1) or optimal two-step estimation (hereafter: GMM2) which also use invalid instruments. In order to remedy this problem, the t -test utilizes consistently estimated residuals based on IV estimation which uses the lags of exogenous variable as instruments for the lagged dependent variables. The assumption of strict exogeneity of an explanatory variable is rather strong. However, it is safer to assume this than to restrict the serial correlation structure of the errors where it is suspected that the error has an autoregressive structure.

The remainder of this paper is organized as follows. The next section presents the model and the performance of the m_2 and the Sargan test where the disturbances follow an AR(1) process. In Section 3, we propose a t -test for zero first-order serial correlation. Section 4 shows that the t -test is applicable to both forms of serial correlation (i.e., AR(1) or MA(1)). Section 5 reports the simulation results using generated data, while Section 6 concludes. The notation is fairly standard and self-explanatory. ' \rightarrow ' denotes convergence in probability while ' \sim ' or ' \Rightarrow ' are used for convergence in distribution. The non-stochastic limit of a sequence is also denoted by ' \rightarrow ' when the context makes the usage clear.

2 Models and Two Autocorrelation Tests by AB (1991)

An IV type estimator that uses lags of dependent variables as instruments under the assumption of white noise errors becomes inconsistent if in fact the errors are serially correlated. It is therefore essential to confirm that the errors are not serially correlated. We may use the m_2 test or the Sargan test for models with MA(1) errors where the lagged values of the dependent variable itself become valid instruments in the differenced equations corresponding to later periods. For the AR(1) error process, however, we cannot directly use these tests because they use the residuals from standard GMM estimations that may not be consistent in this case.

A simple dynamic panel model with strictly exogenous variables is an autoregressive specification of the form (e.g Hsiao, 1986; Nerlove, 1971a; Baltagi and Li, 1995)

$$\begin{aligned} y_{it} &= \delta y_{i,t-1} + x'_{it}\beta + u_{it}, & |\delta| < 1. \\ u_{it} &= \mu_i + v_{it} & \mu_i \sim \text{NID}(0, \sigma_\mu^2) \end{aligned} \quad (1)$$

with the one-way error component u_{it} where for $i = 1, \dots, N$ and $t = 2, \dots, T$. To begin with, we assume that μ_i and v_{it} have the familiar error component structure in which

$$E(\mu_i) = E(v_{it}) = E(\mu_i v_{it}) = 0 \quad \forall \quad i, t \quad (2)$$

and

$$E(v_{it} v_{is}) = 0 \quad \forall \quad i, t \neq s \quad (3)$$

Let us assume that a random sample of N individual time series (y_{i1}, \dots, y_{iT}) is available where T is small and N is large. To focus only on the impact of serial correlation in the error process, we consider the simplest model without exogenous variables for the time being.¹ Adopting the standard assumptions concerning the error component, i.e., a white noise error v_{it} , AB (1991) noted the validity of the following $m = (T-1)(T-2)/2$ linear moment restrictions for the dynamic model (1).

$$E[(\Delta y_{it} - \delta \Delta y_{i,t-1}) y_{i,t-j}] = 0 \quad \text{for } (j = 2, \dots, t-1; t = 3, \dots, T) \quad (4)$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$. For convenience, the moment restrictions in (4) can be expressed more compactly as $E(W'_{yi} \Delta v_i) = \mathbf{0}$ where $\Delta v_i = (\Delta v_{i3}, \dots, \Delta v_{iT})'$ and W_{yi} is a $(T-2) \times m$ block diagonal matrix given by

$$W_{yi} = \begin{bmatrix} [y_{i1}] & & & 0 \\ & [y_{i1}, y_{i2}] & & \\ & & \ddots & \\ 0 & & & [y_{i1}, \dots, y_{iT-2}] \end{bmatrix} \quad (5)$$

However if the standard assumption of a white noise error in (3) for v_{it} is violated, the above orthogonality conditions would no longer hold so that the use of the values of y lagged two periods or more as instruments for $\Delta y_{i,t-1}$ would be impossible.

¹The existence of exogenous variables is crucial for our GMM t -test in order to obtain the estimator of the serial correlation coefficient in the first-step IV estimation.

Let us consider two alternative cases of the serially correlated disturbances. First, the case of an AR(1) stationary disturbances in the classical error term v_{it} :

$$v_{it} = \rho v_{i,t-1} + \epsilon_{it} \quad 0 < \rho < 1 \quad (6)$$

And second, the case of an invertible MA(1) disturbance:

$$v_{it} = e_{it} + \theta e_{i,t-1} \quad 0 < \theta < 1 \quad (7)$$

In addition, we make the standard assumption that

$$E[\epsilon_{i,t-s} y_{i,t-j}] = E[e_{i,t-s} y_{i,t-j}] = 0 \quad s < j \quad (8)$$

In either of these two cases, values of y lagged two periods are no longer valid as instruments for the later periods in the equations in first differences since

$$E[(\Delta y_{it} - \delta \Delta y_{i,t-1}) y_{i,t-j}] \neq 0, \quad j \geq 2, \quad \forall i, t \quad (9)$$

For the AR(1) error, if $j = 2$

$$\begin{aligned} E[(\Delta y_{it} - \delta \Delta y_{i,t-1}) y_{i,t-2}] &= E[\Delta u_{it} y_{i,t-2}] \\ &= E[\Delta v_{it} y_{i,t-2}] \\ &= E[(\rho - 1)v_{i,t-1} + \epsilon_t] y_{i,t-2}] \\ &= E[(\rho - 1)(\rho v_{i,t-2} + \epsilon_{t-1}) + \epsilon_t] y_{i,t-2}] \\ &= E[\rho(\rho - 1)v_{i,t-2} y_{i,t-2}] \neq 0 \end{aligned}$$

Hence, the linear moment restriction in vector form is

$$E[W'_{yi} \Delta v_i] = \begin{bmatrix} \rho(\rho - 1)E[v_{i1} y_{i1}] \\ \rho^2(\rho - 1)E[v_{i1} y_{i1}] \\ \rho(\rho - 1)E[v_{i2} y_{i2}] \\ \vdots \\ \rho^{T-2}(\rho - 1)E[v_{i1} y_{i1}] \\ \vdots \\ \rho(\rho - 1)E[v_{i,t-2} y_{i,t-2}] \end{bmatrix} \neq \mathbf{0} \quad \forall i \quad (10)$$

for the AR(1) serial correlation.² On the other hand, for the MA(1) error

$$\begin{aligned} E[(\Delta y_{it} - \delta \Delta y_{i,t-1}) y_{i,t-2}] &= E[\Delta u_{it} y_{i,t-2}] \\ &= E[\Delta v_{it} y_{i,t-2}] \\ &= E[(e_{it} + (\theta - 1)e_{i,t-1} - \theta e_{i,t-2}) y_{i,t-2}] \\ &= -\theta E[e_{i,t-2} y_{i,t-2}] \neq 0 \end{aligned}$$

For $j \geq 3$, all the orthogonality conditions remain valid so that the $(m \times 1)$ vector of the moment restrictions is

²If we focus on the inapplicability of the moment restrictions condition in AB, we can assume that $E(y_{i1} v_{i1}) = E(y_{i2} v_{i2}) = \dots = E(y_{it} v_{it})$ and omit these terms from equation (10) for convenience.

$$E[W'_{yi}\Delta v_i] = \begin{bmatrix} (-\theta) \\ (0, -\theta)' \\ (0, 0, -\theta)' \\ \vdots \\ (0, 0, \dots, -\theta)' \end{bmatrix} \neq \mathbf{0} \quad \forall i \quad (11)$$

where $E[e_{i,t-j}y_{i,t-j}]$ in all the elements is omitted. As we expect that the breaks of the orthogonality conditions will affect the m2 and Sargan tests, it is interesting to show how these statistics work in the AR(1) error process. The consistency of the GMM estimator relies upon the fact that $E[\Delta u_{it}\Delta u_{i,t-2}] = 0$. Therefore, a test for the hypothesis that there is no second-order serial correlation for the disturbances of the first-differenced equation, based on the average covariances of $\Delta \hat{v}'_{-2}\Delta \hat{v}_*$ takes the form

$$m_2 = \frac{\Delta \hat{v}'_{-2}\Delta \hat{v}_*}{\tilde{v}^{1/2}} \sim N(0, 1) \quad (12)$$

where $\Delta \hat{v}_{-2}$ is the vector of residuals lagged twice, of order $q = N(T - 4)$, and $\Delta \hat{v}_*$ is a $q \times 1$ vector of trimmed $\Delta \hat{v}$ to match $\Delta \hat{v}_{-2}$.³ To focus on the effect of the AR(1) serial correlation on the numerator in (12) under $H_1 : 0 < \rho < 1$, we obtain

$$\begin{aligned} E[\Delta u_{it}\Delta u_{i,t-2}] &= E[(v_{it} - v_{i,t-1})(v_{i,t-2} - v_{i,t-3})] \\ &= 2\gamma_2 - \gamma_1 - \gamma_3 \\ &= \frac{\sigma_\epsilon^2}{\rho^2 - 1} [\rho^2 - 2\rho + 1] \rho \\ &= \frac{\rho(\rho - 1)}{\rho + 1} \sigma_\epsilon^2 \neq 0 \end{aligned} \quad (13)$$

where γ_h is an autocovariance function of v_{it} for a fixed i .⁴ Equation (13) reveals that not only the usual standard normal asymptotic in (12) is unavailable but also the power of the test depends on ρ if the error follows an AR(1) process.

On the other hand, the invalidity of the orthogonality condition will also affect the power of the Sargan test

$$S = \Delta \hat{v}'Z \left(\sum_{i=1}^N Z'_i \Delta \hat{v}_i \Delta \hat{v}'_i Z_i \right)^{-1} Z' \Delta \hat{v} \sim \chi^2_{p-k} \quad (14)$$

where the valid instrument matrix Z is chosen appropriately. The impact of the autocorrelated errors on $\Delta \hat{v}'Z$ in S is briefly expressed as

$$E[\Delta \hat{v}'_{it}y_{i,t-s}]^2 \cong \rho^2(\rho - 1)^2, \rho^4(\rho - 1)^4, \rho^6(\rho - 1)^6 \dots \quad \text{for } s \geq 2 \quad (15)$$

Overall, we expect that the power of the two tests decreases as ρ approaches to unity under the AR(1) alternative.

³If we drop 'Δ' from all variables in first differences to follow the notation in AB(1991), $\tilde{v} = \sum_{i=1}^N \hat{v}'_{i,-2}\hat{v}_{i*}\hat{v}'_{i,-2} - 2\hat{v}'_{i,-2} X_*(X'Z A_N Z' X)^{-1} X'Z A_N (\sum_{i=1}^N Z'_i \hat{v}_i \hat{v}'_{i*} \hat{v}_{i,-2}) + \hat{v}'_{i,-2} X_* \widehat{\text{avar}}(\hat{\delta}) X'_* \hat{v}_{i,-2}$, where Z, A_N are chosen appropriately. This implies that the m_2 statistics depends on the efficiency of the first step estimation through the $\widehat{\text{avar}}(\hat{\delta})$.

⁴The equation will be 0 if the errors in the model in levels are not autocorrelated or follow a random-walk process.

Proposition 2.1 *The power of the m_2 and Sargan tests have their maximum values at $\rho = 0.5$ and around $\rho = 0.7$ respectively, under the AR(1) alternative.⁵ Consequently, the probabilities of Type II error increase as ρ approaches unity in these tests, signalling possible misspecifications.*

3 A Residual-based t -test in the AR(1) Case

The poor performance of the two standard tests, the m_2 and the Sargan test, in the presence of AR(1) disturbances provides the motivation for the discussion in this section. The idea is to consider a standard first-difference GMM estimation for the previously estimated residual \hat{u}_{it} to test whether the coefficient ρ is significantly different from 0. Because the test is performed for any consistently estimated residuals, it can be easily extended to models with time-invariant covariates.

Let us assume that the AR(1) dynamic panel model (1) is given with the standard assumption for the error component, but no information on the classical error term v_{it} is available at this stage. There are three possible correlation structures between the x_{it} and the v_{it} error process. The first possibility is that x_{it} is strictly exogenous. The second is that x_{it} is weakly exogenous or predetermined. The third, finally, is that x_{it} is endogenously determined. Depending on the type of correlation, we can determine the suitable instruments that can be used to estimate δ and β consistently. For the sake of simplicity, however, we will assume that x'_{it} is strictly exogenous (possibly autocorrelated) and is independent of μ_i and v_{it} .⁶

The *a priori* expectation that autoregressive errors may be present in the regression model rules out the use of lags of dependent variables as instruments. As an alternative, we therefore use $x_{i,t-j}$ as instruments corresponding to $y_{i,t-j}$ where we have no confidence that the disturbances fulfil the i.i.d. requirement.⁷ Then, the IV estimators of δ, β are given by

$$\begin{pmatrix} \hat{\delta}_{IV} \\ \hat{\beta}_{IV} \end{pmatrix} = ([x_{-1}, x]'[y_{-1}, x])^{-1}([x_{-1}, x]'y) \quad (16)$$

where x is the stacked $N(T-2) \times (k-1)$ matrix of observations on x_{it} , and x_{-1} or y_{-1} are vectors of the one-period lagged values of their counterparts x and y , respectively. In order to keep the notation simple we set $\hat{\delta} = (\hat{\delta}_{IV}, \hat{\beta}_{IV})'$ and $X_{it} = (y_{i,t-1}, x_{it})$ so that the consistently estimated residual

$$\hat{u}_{it} = y_{it} - X_{it}\hat{\delta} = u_{it} - X_{it}(\hat{\delta} - \delta) \quad (17)$$

Because the residual \hat{u}_{it} includes the individual specific effects μ_i , i.e. $\widehat{\mu_i + v_{it}}$, we are not able to test directly for serial correlation using a significance test for ρ based on the

⁵It is not essential to find the exact value of ρ that give the maxima of the two tests. The essential point is that the two tests are biased and inconsistent under H_1 .

⁶If there is a correlation between x_{it} and the unobserved individual specific effects μ_i , the Hausman and Taylor (1981) estimation procedure should be adopted. The only concern is whether the estimation is consistently estimated.

⁷The lagged exogenous variables are weak instruments if the true disturbances are not serially correlated.

simple least square regression ⁸

$$\hat{u}_{it} = \rho \hat{u}_{i,t-1} + \epsilon_{it} \quad (18)$$

As an alternative, we suggest using the usual t -statistic of the first-order serial correlation coefficient of the previously estimated residuals which is obtained after the standard GMM1 estimation. The first differenced disturbances in levels are

$$\Delta u_{it} = u_{it} - u_{i,t-1} = v_{it} - v_{i,t-1} = \Delta v_{it}$$

It does not matter whether the unobserved disturbance v_{it} follows an AR(1) or MA(1) process. This is a very simple but powerful relationship between u_{it} and v_{it} in first differences that is useful for deriving our t -test. In the case of an AR(1) disturbance as in (6), the first-differenced equation is

$$\Delta v_{it} = \rho \Delta v_{i,t-1} + \Delta \epsilon_{it} \quad (19)$$

where ϵ_{it} is independent and homoskedastic both across individuals and over time. If we replace the Δv_{it} in the above equation with the first differenced Δu_{it} , equation (19) exactly matches the AR(1) dynamic random effects specification in AB (1991). ⁹ Consequently, we wish to test $H_0 : \rho = 0$ after obtaining the GMM estimator $\hat{\rho}$ and its t -value, $t_{\hat{\rho}}$. The significance test for ρ in (19) is, therefore, an autocorrelation test for the classical error term in (1).

To operationalize this estimation, Δu_{it} is replaced with the estimated differenced residual $\Delta \hat{u}_{it}$ which is obtained from the first step IV estimation in (16). If we use $\Delta \hat{u}_{it} = \Delta u_{it} - \Delta X_{it}(\hat{\delta} - \delta)$, we obtain

$$\begin{aligned} \Delta \hat{u}_{it} &= \rho \Delta \hat{u}_{i,t-1} + \Delta \epsilon_{it} - (\Delta X_{it} - \rho \Delta X_{i,t-1})(\hat{\delta} - \delta) \\ &= \rho \Delta \hat{u}_{i,t-1} + \Delta \eta_{it} \quad \text{say} \end{aligned} \quad (20)$$

For $T \geq 3$, the newly derived AR(1) dynamic model (20) also implies the following linear moment restrictions

$$E[(\Delta \hat{u}_{it} - \rho \Delta \hat{u}_{i,t-1}) \hat{u}_{i,t-j}] = 0 \quad (j = 2, \dots, (t-1); \quad t = 3, \dots, T) \quad (21)$$

The moment equations is conveniently written in vector form as $E[W'_{ui} \Delta \eta_i] = 0$ where $\eta_i = (\eta_{i3} \dots \eta_{iT})'$ and W_{ui} is a block diagonal matrix whose s th block is given by $(\hat{u}_{i1} \dots \hat{u}_{is})$. The GMM estimator $\hat{\rho}$ is based on the sample moments $N^{-1} \sum_{i=1}^N W'_i \Delta \eta_i$ and is given by

$$\hat{\rho} = \operatorname{argmin}_{\rho} (\Delta \eta' W_u) V_N (W'_u \Delta \eta) \quad (22)$$

where $\Delta \eta = (\Delta \eta'_1, \dots, \Delta \eta'_N)$ and $W_u = (W'_{u1}, \dots, W'_{uN})$. The one-step GMM estimator $\hat{\rho}$ is obtained by setting $V_N = (N^{-1} \sum_{i=1}^N W'_{ui} G W_{ui})^{-1}$ where G is a $(T-2)$ square

⁸The significance test of ρ , $H_0 : \rho = -1/(T-1)$ is possible in a fixed effect dynamic panel model, although the test depends on T . (e.g., Wooldledge, 2002; Bhargava et al., 1982).

⁹The individual effects that may exist in the estimated residuals in the level equation are successively eliminated by first-differencing.

matrix which has twos in the main diagonal, minus ones in the first sub-diagonals and zeros otherwise.¹⁰ For consistency, however, we need to confirm the convergence of the sample moments to the population moments.

Lemma 3.1 *The difference between the average sample and population moments converges in probability to zero.*

$$\lim_{N \rightarrow \infty} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T [(\Delta \hat{u}_{it} - \rho \Delta \hat{u}_{i,t-1}) \hat{u}_{i,t-j}] - E[(\Delta \hat{u}_{it} - \rho \Delta \hat{u}_{i,t-1}) \hat{u}_{i,t-j}] \rightarrow 0 \quad (23)$$

For $j = 2$, for example, the average sample moment

$$\begin{aligned} & \lim_{N \rightarrow \infty} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T [\Delta \epsilon_{it} - (\Delta X_{it} - \rho \Delta X_{i,t-1})(\hat{\delta} - \delta)]' [u_{i,t-2} - X_{i,t-2}(\hat{\delta} - \delta)] \\ = & \lim_{N \rightarrow \infty} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T [\Delta \epsilon'_{it} u_{i,t-2}] + \lim_{N \rightarrow \infty} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T [\Delta \epsilon'_{it} X_{i,t-2}(\hat{\delta} - \delta)] \\ + & \lim_{N \rightarrow \infty} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T [(\hat{\delta} - \delta)' (\Delta X_{it} - \rho \Delta X_{i,t-1})' u_{i,t-2}] \\ + & \lim_{N \rightarrow \infty} \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T [(\hat{\delta} - \delta)' (\Delta X_{it} - \rho \Delta X_{i,t-1})' X_{i,t-2}(\hat{\delta} - \delta)] \end{aligned}$$

Since $E[\epsilon'_{i,t-s} u_{i,t-j}] = 0$ for $s < j$, the first part converges to 0. The remaining three parts also vanish respectively, as $\sqrt{N}(\hat{\delta} - \delta) = O_p(1)$. For $j > 2$, the same convergence holds.

We also make an assumption about the convergence of the weighting matrix V_N .

Assumption 1 *There exists a non-random sequence of positive definite matrices \bar{V}_N such that*

$$V_N - \bar{V}_N \rightarrow 0 \quad (24)$$

With this assumption, the one-step GMM estimator $\hat{\rho}$

$$\hat{\rho} = ([\Delta \hat{u}_{-1}]' W_u \hat{V}_N^{-1} W_u' [\Delta \hat{u}_{-1}])^{-1} ([\Delta \hat{u}_{-1}]' W_u \hat{V}_N^{-1} W_u' \Delta \hat{u}) \quad (25)$$

is consistent where \hat{u}_{-1} is an $N(T-2) \times 1$ vector of $\hat{u}_{i,t-1}$.

Lemma 3.2 *A consistent estimate of the asymptotic variance is given by*

$$\text{Avar } \hat{\rho} = \hat{\sigma}_\eta^2 \left([\Delta \hat{u}_{-1}]' W_u \hat{V}_N^{-1} W_u' [\Delta \hat{u}_{-1}] \right)^{-1} \quad (26)$$

where

$$\hat{\sigma}_\eta^2 = \sum_{i=1}^N \sum_{t=3}^T \Delta \hat{\eta}_{it}^2 / 2N(T-2) \quad (27)$$

¹⁰ $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [\Delta \eta_i \Delta \eta_i'] \rightarrow G \quad \forall i.$

Proposition 3.1 *Under the null of $H_0 : \rho = 0$,*

$$t_{\hat{\rho}} = \hat{\sigma}_\eta([\Delta\hat{u}_{-1}]'W_u\hat{V}_N^{-1}W_u'[\Delta\hat{u}_{-1}])^{-\frac{1}{2}}([\Delta\hat{u}_{-1}]'W_u\hat{V}_N^{-1}W_u'\Delta\hat{u}) \sim N(0, 1) \quad (28)$$

A proof of asymptotic normality is quite straightforward and therefore is not presented. However, an interesting point worth mentioning is that unlike in the \tilde{v} in the m_2 statistics, the term ' $\widehat{\text{avar}}(\hat{\delta} - \delta)$ ' does not appear in the above asymptotic convergency so that the t -test does not rely on the efficiency of the first-step estimator, i.e., $\hat{\delta}$. The ' $\widehat{\text{avar}}(\hat{\delta} - \delta)$ ' which possibly appears in the estimation of σ_η in (28) vanishes as $N \rightarrow \infty$, since $\sqrt{N}(\hat{\delta} - \delta) = O_p(1)$.

4 The MA(1) Case

In the previous section, we derived the t -test based on residuals from the IV estimation. In this section, we show that the t -test is valid even if the classical error term in the true disturbances follows MA(1), i.e., $v_{it} = e_{it} + \theta e_{i,t-1}$. Conventionally, we use the m_2 test or the Sargan test to detect any serial correlation in the error term. However, it is possible to consider converting MA(1) error into its AR(1) counterpart to apply our t -test.

$$\begin{aligned} \Delta v_{it} &= \Delta e_{it} + \theta \Delta e_{i,t-1} \\ &= \theta \Delta v_{i,t-1} - \theta^2 \Delta v_{i,t-2}, \dots, + \Delta e_{it} \\ &= \theta \Delta v_{i,t-1} + \sum_{j=2}^{\infty} -(-\theta)^j \Delta v_{i,t-j} + \Delta e_{it} \\ &= \theta \Delta v_{i,t-1} + \Delta \zeta_{it} \end{aligned} \quad (29)$$

$$(30)$$

where $\Delta \zeta_{it} = \sum_{j=2}^{\infty} -(-\theta)^j \Delta v_{i,t-j} + \Delta e_{it}$. This is very similar to the first-difference AR(1) specification in (19)¹¹ and the autocorrelation test in the MA(1) case is again the significance test of θ . Hence, the t -test is readily applicable after estimating (30) by GMM1 to test whether θ is significantly different from 0.

Proposition 4.1 *The residual-based GMM t -test is applicable to both forms of the serial correlation (i.e. AR(1) or MA(1)). Hence, under the null of $H_0 : \theta = 0$, $t_{\hat{\theta}} \sim N(0,1)$.*

Since (29) is of infinite order it is necessary in practice to approximate it by an AR(k) model in order to obtain any estimates.¹² However, if we are concerned about the significance of θ in (30), we need not worry about the lag length k . There are three reasons. Firstly, if θ increases, it is obvious that we should use a longer lag. However, an increase in θ simultaneously raises the significance of θ which makes it easier to detect autocorrelation. Secondly, from a hypothetical point of view

$$H_0 : \theta = 0, \quad H_1 : 0 < \theta \quad (31)$$

$$H_0 : \theta = \theta^2, \dots, \theta^k = 0, \quad H_1 : \text{not } H_0 \quad (32)$$

¹¹The correlation between $v_{i,t-j}$ and ζ_{it} becomes negligible as j grows.

¹²The extra error made by approximating AR(∞) by an AR(k) decreases as k increases, since $|\theta| < 1$.

(32) is redundant to test (30). In order to put these ideas into the GMM framework, let us consider the approximation of the MA(1) process to the AR(2) process. For an AR(2) dynamic panel model, the matrix of instruments would be

$$W_{ui} = \begin{bmatrix} [\hat{u}_{i1}, \hat{u}_{i2}] & & & 0 \\ & [\hat{u}_{i1}, \hat{u}_{i2}, \hat{u}_{i3}] & & \\ & & \ddots & \\ 0 & & & [\hat{u}_{i1}, \dots, \hat{u}_{iT-2}] \end{bmatrix} \quad (33)$$

Hence, no additional linear restrictions are needed for the AR(2) model, given those restrictions which are already exploited from the AR(1) specification. This provides the third reason why the AR(1) approximation is valid. Consequently, to test the significance of θ , it is possible to use the AR(1) approximation even if the true disturbance follows an MA(1) process. Furthermore, the AR(1) approximation is more promising than the AR(k) approximation as long as we are concerned with the significance of θ . Therefore, an attractive feature of this t -test is that it is applicable to both the AR(1) and the MA(1) alternatives.

A shortcoming of the test, however, is that it may not be possible to distinguish an AR(1) from an MA(1) structure if the null hypothesis $\rho = 0$ is rejected. In this particular case, we suggest a different testing strategy. First, apply the t -test to examine whether serial correlation is present. If serial correlation is indeed found to be present, apply either the m_2 or the Sargan test to determine whether the error follows an MA(1) process. If it does not, we can conclude that the error term has an AR(1) structure. This two-step test procedure will be able to detect any serial correlation structure of order one in the error term of a dynamic panel data model.

5 Simulation Study

This section illustrates the performances of the three tests mentioned in a dynamic panel data model. Monte Carlo experiments are carried out to compare the three different tests, the m_2 , the Sargan and the t -test. The data generating process follows Nerlove (1971) and AB (1991).

$$\begin{aligned} y_{it} &= \delta y_{i,t-1} + \beta x_{it} + u_{it} \\ x_{it} &= \alpha x_{i,t-1} + \omega_{it} \quad \omega_{it} \sim U(-1/2, 1/2) \end{aligned} \quad (34)$$

For the random effects specification we generate $u_{it} = \mu_i + v_{it}$ where $\mu_i \sim N(0, 1)$ and the classical error term v_{it} is generated either by the AR(1) process

$$v_{it} = \rho v_{i,t-1} + \epsilon_{it} \quad (35)$$

or by the MA(1) process

$$v_{it} = \epsilon_{it} + \theta \epsilon_{i,t-1} \quad (36)$$

Table 1: Size and Power of the Three Tests (AR(1) error)

ρ	$\delta = 0.3$			0.5			0.7			0.9		
	m_2	S	t -test	m_2	S	t -test	m_2	S	t -test	m_2	S	t -test
0.0	0.07	0.01	0.06	0.02	0.01	0.05	0.02	0.01	0.06	0.02	0.03	0.04
0.1	0.19	0.05	0.22	0.14	0.05	0.30	0.13	0.05	0.46	0.18	0.03	0.17
0.2	0.31	0.06	0.67	0.28	0.11	0.77	0.28	0.12	0.84	0.25	0.09	0.68
0.3	0.42	0.31	0.95	0.36	0.16	0.95	0.35	0.18	0.93	0.31	0.20	0.84
0.4	0.43	0.31	0.99	0.53	0.36	0.99	0.52	0.36	0.99	0.46	0.39	0.99
0.5	0.52	0.48	1.00	0.59	0.47	1.00	0.58	0.48	1.00	0.61	0.49	1.00
0.6	0.50	0.54	1.00	0.54	0.61	1.00	0.53	0.62	1.00	0.44	0.44	1.00
0.7	0.48	0.65	1.00	0.38	0.60	1.00	0.37	0.60	1.00	0.36	0.32	1.00
0.8	0.39	0.57	1.00	0.34	0.56	1.00	0.35	0.57	1.00	0.21	0.13	1.00
0.9	0.11	0.40	1.00	0.12	0.35	0.99	0.12	0.29	0.99	0.07	0.02	0.98
0.0	0.09	0.00	0.07	0.06	0.00	0.04	0.03	0.00	0.08	0.05	0.02	0.05
0.1	0.19	0.03	0.43	0.19	0.03	0.49	0.20	0.04	0.56	0.21	0.05	0.57
0.2	0.54	0.06	0.95	0.47	0.04	0.96	0.46	0.07	0.97	0.52	0.10	0.98
0.3	0.72	0.14	1.00	0.70	0.08	1.00	0.76	0.09	1.00	0.71	0.14	1.00
0.4	0.87	0.25	1.00	0.83	0.24	1.00	0.85	0.31	1.00	0.83	0.24	1.00
0.5	0.91	0.47	1.00	0.89	0.41	1.00	0.87	0.46	1.00	0.76	0.51	1.00
0.6	0.87	0.56	1.00	0.88	0.61	1.00	0.85	0.58	1.00	0.84	0.49	1.00
0.7	0.80	0.61	1.00	0.77	0.54	1.00	0.75	0.68	1.00	0.50	0.36	1.00
0.8	0.62	0.47	1.00	0.57	0.51	1.00	0.54	0.45	1.00	0.10	0.10	1.00
0.9	0.26	0.20	1.00	0.31	0.21	1.00	0.21	0.22	1.00	0.10	0.03	1.00

Notes.

1. From the top, $T = 7$ and $T = 11$ given $N = 100$, 5000 replications.
2. 'S' refers to the Sargan test.

All the innovations are independent over time and homoskedastic, i.e. $\epsilon_{it} \sim \text{i.i.d}N(0,1)$. For x_{i1} , we used ω_{i1} and for y_{i1} we generate

$$\frac{\beta x_{i1}}{1 - \delta} + \frac{\mu_i}{(1 - \delta)} + \frac{v_{i1}}{\sqrt{1 - \delta^2}} \quad (37)$$

The design of this formulation allows correlation between the initial observations y_{i1} and the individual effects μ_i .¹³ The three testing procedures are repeated five thousand times for each set of parameter values, δ, β, ρ for the AR(1) process, and δ, β, θ for the MA(1) process. The parameter δ is set to have values 0.3, 0.5, 0.7, 0.9 while $\beta = 2, \alpha = 0.4$ is kept fixed. We choose the error process parameters ρ and θ in such a way that $\rho = 0, 0.1, \dots, 1$. In the base design, the sample size is $N = 100$ and $T = 7, T = 11$. The level of significance is set equal to 5 % throughout the experiments.

First, the three tests are applied to the AR(1) case. Table 1 shows the size and power of the three test statistics given an AR(1) error process. The empirical sizes of the m_2 ,

¹³This formulation is useful when there is no information about the initial observation since it imposes almost no restriction on the initial observation (Sevestre and Tronogon, 1985)

Table 2: Size and Power of the Three Tests (MA(1) error)

θ	$\delta = 0.3$			0.5			0.7			0.9		
	m_2	S	t -test	m_2	S	t -test	m_2	S	t -test	m_2	S	t -test
0.0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.1	0.13	0.06	0.26	0.15	0.08	0.29	0.17	0.06	0.35	0.19	0.05	0.39
0.2	0.35	0.11	0.67	0.42	0.16	0.78	0.53	0.14	0.85	0.48	0.05	0.87
0.3	0.73	0.29	0.94	0.83	0.49	0.98	0.88	0.30	0.99	0.76	0.06	0.99
0.4	0.97	0.63	0.99	0.99	0.84	1.00	0.98	0.43	1.00	0.88	0.06	1.00
0.5	1.00	0.91	1.00	1.00	0.99	1.00	0.99	0.52	1.00	0.90	0.07	1.00
0.6	1.00	0.99	1.00	1.00	1.00	1.00	1.00	0.59	1.00	0.87	0.06	1.00
0.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.67	1.00	0.82	0.07	0.99
0.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.00	0.77	0.07	0.99
0.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.78	1.00	0.73	0.07	0.99

Notes.

1. $T = 7$, $N = 100$, 5000 replications.
2. Sizes are corrected previously.

the Sargan, and the residual-based t -test are presented in first row for $\rho = 0$. They have reasonable size properties except that the Sargan test in virtually all of the cases shows a low frequency of rejection. When $T = 11$, the empirical size of the m_2 test and the Sargan test deteriorates while the t -test maintains reasonably good size properties .¹⁴ The result also shows that the t -test is superior to the other two tests in terms of power.

Theoretically, the m_2 test and the Sargan test have their maximum values around $\rho = 0.5$ and $\rho = 0.7$, respectively. This would make the conventional autocorrelation test difficult as ρ approaches one due to the increase in the probability of a Type II error. The biasedness of these two tests implies that the presence of serially correlated errors precludes the use of the past history of y as valid instruments. Consequently, using the past history of y as instruments not only makes the GMM1 and GMM2 estimation biased but also makes these two tests biased.

However, by using the consistently estimated residual from the IV estimation, which does not use the past history of y as instruments, the t -test become unbiased and consistent without being related to the value of ρ . In the case of $\rho = 0.9$, the first difference makes $\Delta\epsilon_{it}$ close to a white noise process and decreases in power even in the t -test. The size-corrected powers in the AR(1) case are also calculated and are available from the author on request. The results remain unchanged.

Next, the same three tests are applied to the MA(1) case where $\theta = 0, 0.1, \dots, 1$. In order to apply the residual-based t -test, the MA(1) error process is approximated by AR(1). Table 2 reports the power of the three test statistics. Even though there is no maximum value of power as in the case of the AR(1) alternative, the m_2 and Sargan tests have lower power than the t -test. Furthermore, the Sargan test completely under-rejects when $\delta = 0.9$ as a result of the weak instruments problem. When we conduct the m_2

¹⁴The under-size problem of the Sargan test is examined in a recent paper by Bowsher (2002). Our simulation experiments confirm it.

Table 3: Size and Power of the Wald Tests (MA(1) error)

$\theta =$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
AR1-1	0.059	0.223	0.703	0.971	0.999	1.000	1.000	1.000	1.000	1.000
AR2-2	0.052	0.162	0.583	0.941	0.998	1.000	1.000	1.000	1.000	1.000
AR1-1	0.050	0.140	0.580	0.927	0.994	1.000	1.000	1.000	1.000	1.000
AR2-2	0.050	0.089	0.443	0.869	0.994	1.000	1.000	1.000	1.000	1.000

Notes.

1. $T = 7$, $N = 100$, 5000 replications. $\delta = 0.5$, fixed.
2. The lower half of the table reports the size-corrected power.

and Sargan tests, we use the number of estimated residuals $T - 4$ and $T - 2$, respectively, while for the t -test the number is $T - 1$. The lower power of the $m2$ and Sargan tests is closely related to this loss of estimated residuals.

On the other hand, when we convert the MA(1) error to the AR(k) error, the choice of lag length seems somewhat problematic. Table 3 reports the size and the power of the Wald test where the lag lengths are $k = 1$ and $k = 2$, respectively. AR1-1 and AR2-2 indicate the autocorrelation coefficients where the MA(1) error is approximated by the AR(1) and AR(2) process, respectively.¹⁵

As stated in Section 4, taking a longer lag length has no advantages in terms of power. The Wald test with the AR(1) approximation is clearly superior to the test with the AR(2) approximation. As long as we are concerned with the significance test of ρ or θ , the AR(1) approximation shows the best performance.

6 Concluding Remarks

The standard first-differenced GMM estimation has become an important tool in the empirical analysis of dynamic panels. However, autocorrelation tests in a dynamic panel model have received little attention, especially in random effects specifications. One of the reasons for this is that when T is fixed and small, the structure of the error component does not seem to be a matter for the estimation itself as long as we are concerned with unbiasedness and consistency. However, the smaller the T , the more accurate the specifications of the structure need to be for better inferences. In a recent paper, Harris and Matyas (2001) showed that the standard first-differenced GMM estimators are severely biased if the errors are autocorrelated and that the bias is an increasing function of ρ .

Next, the robust estimation of the asymptotic standard error in many statistical packages seems to minimize the effects of possible misspecification. However, Monte Carlo studies in a recent paper (Windmeijer, 2004) have shown that estimated asymptotic standard errors of the efficient two-step GMM estimator can be severely biased downward in small samples. Hence, ignoring the structure of errors seems quite problematic when they follows AR(1), which will adversely affect to the estimation of asymptotic standard errors. For these reasons, the validity of autocorrelation tests in a dynamic panel model is quite important together with the parameter estimations.

¹⁵In the case of $k = 2$, the two-step GMM estimation is conducted using the instrument matrix of (33).

In this paper we propose a residual-based GMM t -test that is applicable to dynamic panel data models with serially correlated errors. Two interesting points are found: the t -test depends only on the consistency of the first-step estimation, not on its efficiency; and the test is applicable to both forms of serial correlation (i.e. AR(1) or MA(1)). These points distinguish the t -test from the m_2 and Sargan tests proposed by AB (1991).

To study the practical performance of the three tests, the m_2 , the Sargan and the t -test, Monte Carlo simulations were performed. The m_2 and Sargan tests work reasonably well in the case of MA(1) disturbances, but they perform badly in the case of the AR(1) counterpart. The results also indicate that the t -test shows better performance than these two tests in terms of size and power even under the MA(1) alternative.

We also noticed that the size of the Sargan test is distorted as T grows. The use of too many moment conditions causes the Sargan test to be undersized and to have extremely low power. This result confirms previous work by Bowsher (2002). However, the more T are available, the better the inferences we obtain in the t -test. Consequently, the t -test would be an excellent alternative to the standard m_2 and Sargan tests in terms of size and power as well as performance with large T .

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