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# Bootstrap Inference for Impulse Response Functions in Factor-Augmented Vector Autoregressions

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# Bootstrap Inference for Impulse Response Functions in Factor-Augmented Vector Autoregressions

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**Abstract**

This paper investigates structural identification and residual-based bootstrap inference schemes for impulse response functions (IRFs) in factor-augmented vector autoregressions (FAVARs). I first discuss general conditions for structural identification, which also resolve the random rotation of the principal components estimates. I also provide empirically popular three such identification schemes: short-run, long-run and contemporaneous restrictions with sign restrictions. Second, two bootstrap procedures for the identified structural IRFs are compared: A) bootstrap with factor estimation and B) bootstrap without factor estimation. Although both procedures are asymptotically valid in the first-order under $\sqrt{T}/N \to 0$ ($T$ and $N$ are the time and the cross sectional dimensions), the errors in the factor estimation produce higher-order discrepancies. The asymptotic normal intervals also tend to provide smaller coverage ratios and are quite erratic. Monte Carlo simulations and an empirical example confirm the theoretical findings.

**JEL Classification Number:** C14, C22

**Keywords:** structural identification, principal components, factor rotation, coverage ratios, factor estimation errors

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## 1 Introduction

Factor-augmented vector autoregressions (FAVARs), initiated by Bernanke et al. (2005) and further explored by Stock and Watson (2005, 2010), have at least two attractive features for empirical researchers. First, the dynamic factor model essentially reduces the data dimension hence it enables the conventional small-scaled VAR framework to accommodate vast amount of information contained in a large panel data set. Second, macroeconomists have long been considering that certain concepts in economic models, say "productivity" or "inflation", are often better captured by latent factors measured by multiple indicators rather than by a single specific series. (Sargent and Sims, 1977, for example[1]) Empirical applications of FAVARs are rapidly growing across various topics and a current incomprehensive list includes Ang and Piazzesi (2003), Giannone et al. (2005), Boivin et al. (2007), Acconcia and Simonelli (2008), Moench (2008), Ludvigson and Ng (2009ab), Gilchrist et al. (2009) and Boivin et al. (2010) among others.

This paper considers bootstrap inference methods for the impulse response functions (IRFs) in FAVARs when the latent factors are extracted using the method of principal components. Recently Bai and Ng (2006b) and their seminal works have developed a benchmark asymptotic normal inference for the coefficients in factor-augmented regression models. They show that under certain conditions, including $\sqrt{T}/N \to 0$ as $N, T \to \infty$, one can replace the latent factors with their principal components estimates in FAVAR models and still rely on the same asymptotic distribution. However, there are two concerns in applying this method to impulse response analysis in FAVARs. First, the errors in the latent factor estimation can be relevant in finite samples, especially in cases where $N$ is much smaller than $T$, i.e. $\sqrt{T}/N \to 0$ is not appropriate. This caveat is substantiated by Ludvigson and Ng (2009b) and Gonçalves and Perron (2011). From this perspective, bootstrap methods are a potential alternative to the normal approximation. Indeed, Gonçalves and Perron (2011) study theoretical properties of residual-based bootstrap inference in factor-augmented regressions under more general framework $\sqrt{T}/N \to c$ $(0 \leq c < \infty)$ and show that the bootstrap method involving a bias correcting procedure works well. The second concern pertains to the structural identification schemes in FAVARs. In general, structural identification methods are more involved in FAVARs than in conventional small-scaled VARs in which all the variables are observed. This is because, as Bai and Ng (2010) recently pointed out, the individual

[1]See Bai and Ng 2006a, Armah and Swanson 2008, for examples of comparison between observed and latent factor models.

parameters are not statistically identified in factor models and the factors and the attached coefficients estimates are randomly rotated from their true counterparts. Hence, in order to estimate the individual parameters and IRFs, identifying restrictions and estimations must explicitly account for this random factor rotation problem. Bai and Ng (2010) propose three sets of parameter restrictions to achieve the statistical identification, however, these restrictions are not necessarily compatible with what are placed in the structural VAR literature. Hence I revisit the principle of identification schemes in a structural manner and provide empirically popular examples, i.e. short-run, long-run, and contemporaneous identification schemes involving sign restrictions. Importantly, these identification schemes do not only achieve statistical identification in the original sample space, but also do in the bootstrap space so that the IRFs are identified in each bootstrap replication as well.

In order to justify the bootstrap procedures provided in this paper, I exploit most recent papers Gonçalves and Perron (2011) and Shintani and Guo (2011), who show the asymptotic validity of bootstrap inference in factor-augmented models. My suggestion is in the same line as theirs and this paper particularly compares two bootstrap algorithms: namely, A) bootstrapping with factor estimation and B) bootstrapping without factor estimation. I show that although these procedures are both asymptotically valid in the first-order, the errors in the factor estimation produce higher-order discrepancies. Monte Carlo simulations also indicate that Procedure A performs well overall and is of more practical use. Conversely, although Procedure B is considered as a straightforward extension of the methods conducted in many small-scaled VAR excercises under the assumption that the estimated factors are factual, it is unable to capture the effects of higher-order factor estimation errors and may produce a smaller coverage ratio than the nominal level in finite samples. Indeed, our simulation results confirm this finding, especially when $N$ is relatively small when compared with $T$. The asymptotic normal intervals also tend to provide smaller coverage ratios and are quite erratic.

The rest of the paper is structured as follows. Section 2 introduces the models and regularity conditions. Section 3 discusses the identification and estimation methods for the IRFs. In Sections 4 and 5, I propose bootstrap inference procedures and discuss their asymptotic validity. Section 6 assesses the finite sample properties of the suggested and alternative procedures via Monte Carlo simulations using artificial data along with calibrated models of US macroeconomic data. Section 7 provides some concluding remarks. Finally, the appendices include technical derivations and validity of conditions in the main text.

Throughout the paper, the following notations are used. The Euclidean norm of vector

$x$ is denoted by $\|x\|$. For matrices, the vector-induced norm is used. The symbols "$\xrightarrow{p}$" and "$\xrightarrow{d}$" represent convergence in probability under the probability measure $P$ and convergence in distribution. $O_p(\cdot)$ and $o_p(\cdot)$ are the order of convergence in probability under $P$. I define $P^*$ as the bootstrap probability measure, conditional on the original sample. For any bootstrap statistic $T^*$, I write $T^* \xrightarrow{p*} 0$, in probability, or $T^* = o_{p*}(1)$, in probability, when for all $\epsilon > 0$, $P^*(|T^*| > \epsilon) = o_p(1)$. I write $T^* = O_{p*}(\cdot)$, in probability, when for all $\epsilon > 0$ there exists $M(\epsilon) < \infty$ such that $\lim_{N,T\to\infty} P[P^*(|T^*| > M(\epsilon)) > \epsilon] = 0$. I also write $T^* \xrightarrow{d*} D$, in probability, if conditional on a sample with probability that converges to one, $T^*$ converges in distribution to $D$ under $P^*$. Let $\delta = \min\left\{\sqrt{N}, \sqrt{T}\right\}$ and $L$ be the standard lag operator. $Chol(X)$ denotes the Cholesky factorization of a positive definite matrix $X$ returning a lower triangular matrix $W$ such that $W'W = X$. The operator $vec(X)$ transforms an $m \times n$ matrix $X$ into an $mn \times 1$ vector by stacking the columns.

## 2 Models and Assumptions

### 2.1 Reduced-form models

Consider the following factor model:

$$X_t = \mu + \Lambda F_t + u_t, \quad t = 1, ..., T, \tag{1}$$

where $X_t$ is an $N \times 1$ vector of observations and $N$ is the (typically large) number of equations. I assume that $X_t$ is driven by much lower dimensional unobservable factors $F_t$ $(r \times 1)$ with time-invariant unobservable factor loadings $\Lambda = [\lambda_1, ..., \lambda_N]'$ $(N \times r)$. $u_t = [u_{1t}, ..., u_{Nt}]'$ is an $N \times 1$ idiosyncratic shock. $\mu$ is an $N \times 1$ vector of constant.

In addition, the factors $F_t$ form a VAR of order $p$ with $r \times r$ coefficient parameters $\Phi_j$ $(j = 1, ..., p)$, an $r \times 1$ constant vector $\upsilon$, and an error term $e_t$ $(r \times 1)$ so that:

$$F_t = \upsilon + \textstyle\sum_{j=1}^{p} \Phi_j F_{t-j} + e_t, \tag{2}$$

If I write variables without their associated $t$ subscript, then they denote the entire matrix of observations, for example, $X = [X_1, ..., X_T]'$ is a $T \times N$ matrix and $F = [F_1, ..., F_T]'$ is a $T \times r$ matrix. Define $Z = [\iota,\ F_{(-1)},\ F_{(-2)}, \dots\ ,F_{(-p)}]$ $(T \times (rp+1))$ with $\iota$ being a $T \times 1$ vector of ones, $F_{(-j)} = [F_{1-j}, ..., F_{T-j}]'$ and $\Phi = [\upsilon,\ \Phi_1, ..., \Phi_p]'$ $((rp+1) \times r)$ so that (2) can equivalently be written as:

$$F = Z\Phi + e.$$

Note that the constant terms in the models can be omitted in case the data are demeaned[2].

### 2.2 Structural models

Structural VAR framework can be straightforwardly employed to identify the contemporaneous relationships among variables of interest in practice in FAVARs given a particular interpretation for the factors. Stock and Watson (2005) detail a comprehensive modeling strategy, hence I follow their lead. Using an $r \times r$ invertible matrix $S$, let the structural factor model be defined as:

$$X_t = \mu^s + \Lambda^s F_t^s + u_t, \tag{3}$$

$$F_t^s = \upsilon^s + \sum_{j=1}^{p} \Phi_j^s F_{t-j}^s + \zeta_t, \tag{4}$$

where $\Lambda^s = \Lambda S$, $F_t^s = S^{-1}F_t$, $\Phi_j^s = S^{-1}\Phi_j S$ and $\zeta_t = S^{-1}e_t$ is a structural innovation. $\mu^s$ and $\upsilon^s$ are vectors of constant. Note that the models in this paper are simpler than ones in Stock and Watson (2005) in order to focus on the essence of the problem.

### 2.3 Assumptions

I require standard regularity conditions for the remainder of the analysis. First, let the data generating processes above be defined on a probability space $(\Omega, \mathcal{F}, P)$ and the following assumptions hold. Note that $M < \infty$ is a generic constant.

**Assumption 1.**

a. The common factors $F_t$ in (1) and (2) satisfy $E\left\|F_t\right\|^4 < M$, and $T^{-1}\sum_{t=1}^{T} F_t F_t' \xrightarrow{p} \Sigma_F$ as $T \to \infty$ for a nonrandom $r \times r$ positive definite matrix $\Sigma_F$;

b. The factor loadings $\lambda_i$ in (1) are either deterministic such that $\|\lambda_i\| < M$, or stochastic such that $E\left\|\lambda_i\right\|^4 < M$. In either case, $\Lambda'\Lambda/N \xrightarrow{p} \Sigma_\Lambda$ as $N \to \infty$ for a nonrandom $r \times r$ matrix $\Sigma_\Lambda$;

c. The eigenvalues of the $r \times r$ matrix $\Sigma_\Lambda \Sigma_F$ are distinct.

**Assumption 2.**

[2]For simplicity, theoretical derivations in this paper do not include the constant term, assuming that the data is demeaned. In practice, when the model does not include a constant term and demeaned data is used, it is important to make sure that the residuals will be demeaned in the bootstrap procedures. See Section 4.

a. $E(u_{it}) = 0$ and $E\left|u_{it}\right|^8 \leq M$ for all $(i, t)$;

b. $E(u_{it}u_{js}) = \sigma_{ij,ts}$ and $|\sigma_{ij,ts}| \leq \bar{\sigma}_{ij}$ for all $(t, s)$ and $|\sigma_{ij,ts}| \leq \tau_{ts}$ for all $(i, j)$ such that $\frac{1}{N}\sum_{i,j=1}^{N} \bar{\sigma}_{ij} \leq M$, and $\frac{1}{T}\sum_{t,s=1}^{T} \tau_{ts} \leq M$ and $\frac{1}{NT}\sum_{i,j,ts=1}^{T} |\sigma_{ij,ts}| \leq M$;

c. For every $(t, s)$, $E\left|N^{-1/2}\sum_{i=1}^{N}(u_{is}u_{it} - E(u_{is}u_{it}))\right|^4 \leq M$;

d. $E\left(\frac{1}{N}\sum_{i=1}^{N}\left\|\frac{1}{\sqrt{T}}\sum_{t=1}^{T} F_t u_{it}\right\|^2\right) \leq M$, where $E(F_t e_{it}) = 0$ for all $(i, t)$;

e. For each $t$, $E\left\|\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N} F_s\left[u_{is}u_{it} - E(u_{is}u_{it})\right]\right\|^2 \leq M$;

f. $E\left\|\frac{1}{\sqrt{NT}}\sum_{t=1}^{T}\sum_{i=1}^{N} F_t u_t' \Lambda\right\|^2 \leq M$, where $E(F_t \lambda_i' u_{it}) = 0$ for all $(i, t)$;

g. $E\left(\frac{1}{T}\sum_{t=1}^{T}\left\|\frac{1}{\sqrt{N}}\sum_{i=1}^{N} \lambda_i u_{it}\right\|^2\right) \leq M$, where $E(\lambda_i u_{it}) = 0$ for all $(i, t)$;

h. As $N, T \to \infty$, $\frac{1}{TN}\sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_i \lambda_j' u_{it} u_{jt} - \Gamma \xrightarrow{p} 0$, where $\Gamma \equiv \lim_{N,T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \Gamma_t > 0$ and $\Gamma_t \equiv Var\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N} \lambda_i u_{it}\right)$;

i. For each $i$, $T^{-1/2}\sum_{t=1}^{T} F_t u_{it} \xrightarrow{d} N(0, \Theta_i)$.

**Assumption 3.**

a. $E(e_t) = 0$, $E(e_t e_t') = \Sigma_e$ an $r \times r$ positive definite matrix, and $e_t$ and $e_s$ are independent for $s \neq t$;

b. $E\left|e_{it}e_{jt}e_{kt}e_{lt}\right| \leq c$ for $i, j, k, l = 1, ..., r$, and all $t$;

c. $e_t$ are independent of $u_{is}$ for all $i$, $t$ and $s$;

d. For $h = 1, 2, ...,$ $T^{-1/2}\sum_{t=1}^{T} vec\left(Z_t e_t'\right) \xrightarrow{d} N(0, \Sigma)$ with $\Sigma = \Sigma_Z \otimes \Sigma_e$ and $\Sigma_Z \equiv p\lim_{T\to\infty} Z'Z/T$;

e. Roots of $\det(I_r - \Phi_1\eta - \Phi_2\eta^2 - \cdots - \Phi_p\eta^p) = 0$ lie outside the unit circle;

f. The $r \times r$ matrix $S$ has full rank.

Most of these assumptions are based on the usual regularity conditions discussed in the seminal work on factor models by Bai (2003) and Bai and Ng (2006b) and in the standard VAR literature including Lütkepohl (2005). Assumptions 1(a) and 1(b) allow general second moments for the factors and loadings. Assumption 1c guarantees the uniqueness of the limit of $\hat{F}'F/T$, which is important when discussing the behavior of the factor rotation. Assumption 2 is fairly standard and allows for weak dependence in cross sections and allows for general time dependence in $u_{it}$. Assumption 3 is standard in the VAR literature to enforce a stable system that is estimable by least squares. Assumption 3(a) imposes a white noise property on $\{e_t\}$ since a stable covariance matrix $\Sigma_e$ is needed to obtain structural identifications using up to the second moments of the residuals. In Assumption 3(c), $\{u_t\}$ and $\{e_t\}$ (thus $\{u_t\}$ and $\{F_t\}$) are assumed to be independent at all leads and lags.

### 2.4 Impulse response functions

I consider the standard form of IRFs defined for the observable variable $X_i$ to the VAR innovations in both the reduced-form and the structural models. For the reduced-form models, (1) and (2) can be rewritten in vector moving-average form under Assumption 3(e) such that:

$$X_{it} = \mu + \lambda_i' \Psi(L) e_t + u_t,$$

where $\Psi(L) \equiv \sum_{j=0}^{\infty} \Psi_j L^j$ with $\Psi_0 = I_r$ and $\Psi(L) = [\ I_r - \sum_{j=1}^{p} \Phi_j L^j]^{-1}$. Let the reduced-form IRF of observable $X_i$ at time horizon $h$ ($h = 0, 1, 2, ...$) be $\psi_{ih}$. Then,

$$\psi_{ih} \equiv \frac{\partial X_{it+h}}{\partial e_t} = \lambda_i' \Psi_h.$$

The structural IRFs $\varphi_{ih}$ will be similarly defined based on the models of (3) and (4). It can be straightforwardly shown that:

$$\varphi_{ih} \equiv \frac{\partial X_{it+h}}{\partial \zeta_t} = \lambda_i^{s\prime} \Psi_h^s,$$

where the moving average parameters are such that $\Psi^s(L) \equiv \sum_{j=0}^{\infty} \Psi_j^s L^j = \left[ I_r - \sum_{j=1}^{p} \Phi_j^s L^j \right]^{-1} = S^{-1}\Psi(L)S$ with $\Psi_0^s = I_r$ and $\Phi_j^s$ is defined in (4). On the one hand, I note that the structural IRF can take a form that involves only structural parameters and no reduced-form parameters. This fact suggests that the identification of all the structural parameters guarantees the identification of any structural IRFs. On the other hand, the structural IRFs can equivalently be written as:

$$\varphi_{ih} = \lambda_i' \Psi_h S, \tag{5}$$

by using the reduced-form parameters $\lambda_i$ and $\Psi_h$. I also use this reduced-form representation to derive the asymptotic distribution in the next section.

# 3 Identification

## 3.1 Statistical identification

It is well known that in the standard factor model (1) the individual factors and loadings are not statistically identified. In fact, only the space spanned by the factors is identified. As Bai and Ng (2010) point out, this fact is not problematic per se as long as the researcher's interest lies only in the conditional mean or the values of the dependent variables. However, if the analysis involves coefficient values separated from the attached variables, then the identification of the individual factors must be achieved. As the IRF is nothing but a function of individual coefficients, the identification of the individual parameters is then necessary.

I briefly review the consequence of this statistically nonidentification problem in the FAVAR setting. Throughout the paper, I assume that the reduced-form models (1) and (2) are estimated by the following two-step PC procedure. In the first step, I extract the factors using the popular static PC method. This is implemented by finding the solution of:

$$(\hat{F}, \hat{\Lambda}) = \arg\min_{\Lambda, F} \textstyle\sum_{i=1}^{N} \sum_{t=1}^{T} (X_{it} - \lambda_i' F_t)^2. \tag{6}$$

In the second step, the VAR equation for $\hat{F}_t$ is estimated using standard least squares.

However, the problem (6) is not uniquely solvable since, for any $r \times r$ invertible matrix $H$, $\lambda_i' H^{-1}$ and $HF_t$ are also solutions for (6). Also $HF_t$ can be generated through (2) by a combination of $\{H\Phi_j H^{-1}, He_t\}$ instead of $\{\Phi_j, e_t\}$. To overcome this observational equivalence problem between two sets $\{\lambda_i', F_t, u_t, \Phi_j, e_t\}$ and $\{\lambda_i' H^{-1}, HF_t, u_t, H\Phi_j H^{-1}, He_t\}$ embedded with the system (1) and (2), the PC method uses an arbitrary normalization $F'F/T = I_r$ to somehow fix $r^2$ parameters. This yields an estimate $\hat{F}$ of $F$, which is the eigenvectors of $XX'/(TN)$ corresponding to the $r$ largest eigenvalues (multiplied by $\sqrt{T}$). As Bai (2003) shows, the particular $H$ obtained through the above PC estimation is:

$$H_{NT} = V_{NT}^{-1} (\hat{F}' F/T)(\Lambda' \Lambda / N), \tag{7}$$

where $V_{NT}$ is a diagonal matrix with its diagonal elements being the $r$ largest eigenvalues of $XX'/(TN)$ in descending order. The actual value of $H_{NT}$ depends on the realized unobservable process $F$, an estimate $\hat{F}$, and unknown parameters $\Lambda$. What makes the situation unique is the fact that the researchers neither observe nor are able to consistently estimate the realization of $H_{NT}$.

Bai and Ng (2010) further investigate this statistical nonidentification problem in the factor model (1) and provide three sets of parameter restrictions with which PC estimation yields $H_{NT}$, which converges to the identity matrix as $N, T \to \infty$ up to sign normalization. In other words, if one of their restrictions holds, then the estimated factors and parameters are individually identified up to sign.

### 3.2 Structural identification

Because of this statistical nonidentification problem in the factor models, the conventional structural VAR identification schemes do not simply go through with FAVARs under the standard regularity conditions. Also, the identifying assumptions proposed by Bai and Ng (2010) may not be fully justified by any underlying economic interpretations[3]. Recent structural VAR literature emphasize the importance of structural parameter restrictions. See Rubio-Ramirez et al. (2010) for a comprehensive review. Therefore, I propose different identification schemes from Bai and Ng (2010) in the sense of imposing identifying restrictions on the structural parameters rather than on the parameters in models (1) and (2) and still accounting for the factor rotation. Indeed, these identification schemes are technically distinct from, but conceptually common in, many existing structural VAR studies. It is also seen that through these identifying restrictions, although the reduced-form parameters are not identified, so are the structural ones (A similar result is obtained in Komunjer and Ng, 2011 in DSGE settings). To this effect, I introduce the following identifying assumptions:

**Assumption 4.** The lag order $p$ and the number of factors $r$ are known.

**Assumption 5.** $E(\zeta'\zeta) = I_r$.

**Assumption 6.** I have either:

a. (short-run restriction) The short-run IRFs $\varphi_0 \equiv \Lambda^s = \begin{bmatrix} \Lambda^s_{1:r} \\ \Lambda^s_{r+1:N} \end{bmatrix}$ have $\Lambda^s_{1:r} = [\lambda^s_1, ..., \lambda^s_r]'$ an $r \times r$ (upper or) lower triangular matrix with positive diagonal elements; or

[3] For example, one of the Bai and Ng (2010)'s assumptions requires the orthogonality of the latent factors (a restriction on the second moment of the latent factors). However, if we give a particular interpretation to the factors in structural VARs, such an assumption can be too restrictive.

b. (long-run restriction) The long-run IRFs $\varphi_{1:r,\infty} \equiv \Lambda^s_{1:r}\left(\sum_{h=0}^{\infty}\Psi^s_h\right)$ from the 1st to the $r$th observations are an $r \times r$ (upper or) lower triangular matrix with positive diagonal elements; or

c. (recursive restriction) $Q^{-1\prime}S$ is an (upper or) lower triangular matrix and the signs of its diagonal elements are known where $Q^{-1} = p\lim_{N,T\to\infty} H'_{NT}$ with $H_{NT}$ defined in (7).

Assumption 4 excludes model uncertainty from the analysis and simplifies the identification and inference problems. Any attempts to relax this assumption should be practically relevant and of great interest, however, this problem is beyond our scope[4]. Assumption 5 imposes the orthogonality of the structural shocks, which is common in structural VAR literature. Note that Assumption 4 fixes the total number of parameters in the model and Assumption 5 imposes restrictions on $\frac{r^2+r}{2}$ parameters as the covariance matrix is symmetric by definition.

One of the three conditions in Assumption 6 imposes $\frac{r^2-r}{2}$ zeros on the structural parameters, respectively. Hence, any of the restrictions in Assumption 6 together with Assumption 5 achieves the necessary order condition of $r^2$ parameter restrictions on the structural models. Importantly, Assumption 6 also leads a sufficient condition for structural identification and this plays an essential role in the current analysis. Assumption 6(a) provides a set of restrictions on the short-run structural IRFs. This requires researchers to find at least $r-1$ observable variables where the $k$th ($k \leq r-1$) is contemporaneously affected only by the first $k$ factors. Assumption 6(b) works similarly, but restricts the long-run IRFs instead of the short-run IRFs. The implication of the long-run IRF restriction follows from, for example, Blanchard and Quah (1989). Note that these two assumptions formalize the exact identification methods suggested by Stock and Watson (2005).

Assumption 6(c) is similar to the popular recursive restriction in structural VARs as it imposes zeros on $\frac{r^2-r}{2}$ parameters in an invertible matrix $Q^{-1\prime}S$. However, in FAVARs I do not restrict the contemporaneous matrix $S$ itself but one needs to consider its rotation $Q^{-1\prime}S$ to achieve the statistical identification. In this sense, this is not a structural parameter restriction and may be of limited use. However, as it involves the popular Cholesky identification procedure, I further break down Assumption 6(c) into the following set of conditions to ascertain its feasibility:

[4]For example, Dufour and Stevanović (2011) discuss that when the factors are a linear combination of observables, their dynamics are represented in general by VARMA processes rather than finite-order VARs.

**Assumption 6(c)':** The following three restrictions imply Assumption 6c:

1. $\Sigma_F$ is diagonal;
2. $\Sigma_\Lambda$ is diagonal;
3. $S$ is an (upper or) lower triangular matrix and the signs of diagonal elements of $Q^{-1\prime}S$ are known.

The first two parts of Assumption 6(c)' imply that the model involves orthogonal factors and loadings in its reduced-form and they are rather statistical assumptions. Given these two statistical restrictions, I am now able to impose the recursive structure not on $Q^{-1\prime}S$ but on $S$ as in conventional structural VARs. The signs of the diagonal elements of $Q^{-1\prime}S$ are barely known in practice as the matrix $Q$ does not have any structural interpretations. However, one can deduce them by using the signs of structural IRFs, as in Uhlig (2005). Appendix B provides a proof that Assumption 6(c)' implies Assumption 6(c).

Given the above assumptions and the two-step PC estimation, I can proceed by introducing a sufficient identifying condition for the structural parameters and IRFs.

**Condition 1. (Sufficient condition for structural identification)** *Under Assumptions 1-6 one obtains an $r \times r$ matrix $\hat{S}$ such that:*

$$\hat{S} - H_{NT}S \xrightarrow{p} 0,$$

*as $N, T \to \infty$ where $H_{NT}$ is defined in (7).*

The next question is how to obtain $\hat{S}$ as in Condition 1. In the following subsection, I discuss examples of how the restrictions in Assumption 6 enable us to consistently estimate the structural parameters and the IRFs.

**Remark 1** *It is also important to ensure that Condition 1 holds in the bootstrap replications. To achieve this, the identifying assumptions must hold in the bootstrap space (for example, assumptions on $\varphi_{0,1:r}$ carry over to the same assumptions on $\hat{\varphi}_{0,1:r}$ in the limit). For ID1 and ID2, this is trivial since restrictions are on structural parameters and all the structural parameters and IRFs are consistently estimated (not up to rotation). For ID3, I will show that the restrictions incidentally hold in the bootstrap space. See Appendix B.*

### 3.3 Estimation of identified structural models

Once the reduced-form models are estimated using the two-step PC method, structural parameter estimates are obtained by the contemporaneous coefficient matrix $\hat{S}$, which satisfies Condition 1. The following three schemes are simple to implement and are often used in empirical applications.

**ID1 (short-run restriction):**

1. Construct a short-run IRF estimate for observations from 1 to $r$ :

$$\hat{\varphi}_{1:r,0} = Chol\left[\hat{\psi}_{1:r,0}(\hat{e}'\hat{e}/T)\hat{\psi}'_{1:r,0}\right],$$

2. Obtain $\hat{S}$ such that:

$$\hat{S} = \hat{\psi}^{-1}_{1:r,0}\hat{\varphi}_{1:r,0},$$

The ID1 scheme achieves Condition 1 under Assumption 6.1.

**ID2 (long-run restriction):**

1. Construct a long-run IRF estimate for the observations from 1 to $r$ :

$$\hat{\varphi}_{1:r,\infty} = Chol\left[\hat{\psi}_{1:r,\infty}(\hat{e}'\hat{e}/T)\hat{\psi}'_{1:r,\infty}\right],$$

with $\hat{\psi}_{1:r,\infty} = \hat{\Lambda}_{1:r}\left[I_r - \sum_{h=1}^{p}\hat{\Phi}_h\right]^{-1}$,

2. Obtain $\hat{S}$ such that:

$$\hat{S} = \hat{\psi}^{-1}_{1:r,\infty}\hat{\varphi}_{1:r,\infty},$$

The ID2 scheme achieves Condition 1 under Assumption 6.2.

**ID3 (recursive restriction):**

1. Obtain $\hat{S}$ such that:

$$\hat{S} = Chol\left[\hat{e}'\hat{e}/T\right],$$

2. Adjust the signs of $\hat{\varphi}_{ih}$ $(h = 0, 1, ...)$ if $sign(\hat{\varphi}_{i0})$ is not what was expected.

The ID3 scheme achieves Condition 1 under Assumption 6c or 6c'. Note that the second step is to normalize the signs of $Q^{-1\prime}S$, which are not directly known. However, they are deduced through the sign of the structural IRF $\varphi_{1:r,0}$ in practice for the following reason. Given an estimate for $\Lambda_{1:r}Q$ is available as $\hat{\Lambda}_{1:r}$, and as we know the correct signs of $\varphi_{1:r,0} = \Lambda_{1:r}S$, they imply the signs of $Q^{-1\prime}S$. Hence, the sign restriction in Assumption 6c'.3 has a structural interpretation through the signs of $\varphi_{1:r,0}$.

In Appendix B, we prove that these methods will provide a contemporaneous matrix estimate $\hat{S}$, which satisfies Condition 1. Note that these examples would not be the only ones that lead to the condition, however, the same principle would apply in general.

**Theorem 1** *(Consistency of the structural parameters) Under Assumptions 1–5 and Condition 1,* $\hat{\lambda}_i^s - \lambda_i^s \xrightarrow{p} 0$, $\hat{\Phi}^s - \Phi^s \xrightarrow{p} 0$, *and* $\hat{\varphi}_{ih} - \varphi_{ih} \xrightarrow{p} 0$, *for all* $i$ *uniformly in* $h = 0, 1, 2, ...$ *as* $N, T \to \infty$.

Next I move to the asymptotic distributions of the structural IRFs. First, I require a high-level condition about the limit distribution of $\hat{S}$.

**Condition 2.** *(Asymptotic normality of* $\hat{S}$*) Under Assumptions 1-6, one obtains an estimate* $\hat{S}$ *which satisfies:*

$$\sqrt{T} vec(\hat{S} - H_{NT} S) \xrightarrow{d} N(0, \Sigma_S),$$

*as* $N, T \to \infty$ *and* $\sqrt{T}/N \to 0$ *with* $\Sigma_S$ *an* $r^2 \times r^2$ *fixed positive definite matrix.*

This condition is high-level and enables me to establish the following asymptotic normal results for the IRFs. It can also be interpreted that researchers can find an identification method which satisfies this condition on a case-by-case basis. To make sure the validity for the three suggested schemes in this paper, I break down Condition 2 into more primitive conditions in Appendix C. For example, one needs Condition C1 if ID3 is used and Conditions C1 and C2 if ID1 or ID2 is used. I also note that the finite sample distribution of $\hat{S}$ can be quite contaminated from the normal distribution in practice as I further discuss in Appendix C. The consequence is twofold. First the asymptotic normal inference for the IRFs does not work very well. Second, the distributions of IRF estimates are non-centered so that a certain type of bootstrap confidence interval method is preferred to others. These conjectures are investigated through Monte Carlo simulations in Section 5.

**Theorem 2** *(Asymptotic distribution of structural IRFs) Under Assumptions 1–5 and Conditions 1 and 2,*

$$\sqrt{T}(\hat{\varphi}_{ih} - \varphi_{ih}) \xrightarrow{d} N(0, \Sigma_{\varphi ih}),$$

$\forall i$ *uniformly in* $h = 0, 1, 2, \cdots$ *as* $T, N \to \infty$ *and* $\sqrt{T}/N \to 0$ *provided* $\partial\varphi_{ih}/\partial\theta \neq 0$ *where* $\theta = [\lambda_i,\, vec(\Phi)',\, vec(S)']'$,

$$\Sigma_{\varphi ih} = \frac{\partial\varphi'_{ih}}{\partial\theta'}\Sigma_\theta\frac{\partial\varphi_{ih}}{\partial\theta},$$

*and* $\Sigma_\theta = diag(\Sigma_{\hat{\Lambda}i}, \Sigma_{\hat{\Phi}}, \Sigma_S)$ *with* $\Sigma_{\hat{\Lambda}i} \equiv Q^{-1\prime}\Theta_i Q^{-1}$ *and*

$$\begin{aligned}\Sigma_{\hat{\Phi}} \;\equiv\; & [(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})]^{-1} \\ & \times\left[(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})\right] \otimes \left[Q^{-1\prime}\Sigma_e Q^{-1}\right] \\ & \times[(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})]^{-1\prime}.\end{aligned}$$

There are two comments on this result. First despite the implication that the structural IRFs only involve structural parameters, when I consider the distribution of the IRF estimate I present the expression in terms of the reduced-form parameters. This follows the results of IRFs in the standard structural VARs (See Lütkepohl 1990, 2005). Although the expressions for the covariance matrices of individual parameters are notationally involved because of the factor rotation, their estimates are easily constructed. Second and more importantly, the asymptotic approximation is reasonable when $\sqrt{T}/N \to 0$ is relevant. However, if $N$ is relatively smaller than $T$ and $\sqrt{T}/N \to c$ $(0 < c < \infty)$ is more appropriate, then the parameter estimates suffer from asymptotic bias as studied by Ludvigson and Ng (2009b) and Gonçalves and Perron (2011). This becomes another source of the facts that the normal approximation does not work very well and that the bootstrap distributions which are not centered around the original estimates.

# 4 Bootstrap inference

## 4.1 Procedures

This section considers residual-based bootstrap algorithms to construct confidence intervals for the IRFs. I propose to use the i.i.d. bootstrap for the VAR equation in order to make use of the white noise property of $\{e_t\}$, however, I can allow for methods which incorporate more general patterns for $\{u_{it}\}$ as in Assumption 2. For instance, Gonçalves and Perron (2011) propose residual-based wild bootstrap procedures, which are directly applicable to this context if the idiosyncratic errors exhibit heteroskedasticity.

I present two algorithms which are often conducted in empirical studies. The first procedure is A: bootstrapping with factor estimation; and the second procedure is B: bootstrapping without factor estimation. The main feature of Procedure A is that it includes factor estimation within each bootstrap replication so that the confidence intervals can properly account for the uncertainty associated with factor estimation and is in the same line as Gonçalves and Perron (2011) and Shintani and Guo (2011). In contrast, Procedure B does not re-estimate the factors in the bootstrap replications and thus takes the estimated factors as factual. I first outline Procedure A as follows:

**Procedure A: Bootstrapping with factor estimation**

1. Estimate the model using the two-step PC procedure and obtain parameter estimates $\hat{\Lambda}$, $\hat{\Phi}$, $\hat{S}$ and residuals $\hat{u}_t$ and $\hat{e}_t$. Obtain the IRF estimate $\hat{\varphi}_i$.

2. Make sure that the residuals $\hat{u}_t$ and $\hat{e}_t$ are demeaned if the models do not include a constant term. Resample the residuals $\hat{e}_t$ with replacement and label them $e_t^*$. Generate the bootstrapped sample $F_t^*$ by $F_t^* = \hat{\upsilon} + \sum_{j=1}^{p} \hat{\Phi}_j F_{t-j}^* + e_t^*$. Also resample the residuals $\hat{u}_t$ with replacement, and label them $u_t^*$. When $u_t$ is suspected to be heteroskedastic, a wild bootstrap proposed by Gonçalves and Perron (2011) is applied. Generate the bootstrapped observations $X^*$ by $X_t^* = \hat{\mu} + \hat{\Lambda} F_t^* + u_t^*$. At this stage, some type of bias-correction methods can be applied as discussed in Kilian (1998). See Appendix D.

3. Using the bootstrapped observations $X_t^*$, estimate $(\hat{F}^*, \hat{\Lambda}^*)$ using the first step of the PC procedure. Then, estimate the VAR equation of $\hat{F}_t^*$ to obtain the bootstrapped estimates $\hat{\Phi}^*$ and $\hat{S}^*$ using the second step of the PC procedure. This yields the bootstrap IRF estimates $\hat{\varphi}_i^*$.

4. Repeat Steps 2 and 3 $R$ times.

5. Store the recentered statistic $s \equiv \hat{\varphi}_{ih}^* - \hat{\varphi}_{ih}$. Sort the statistics and select the $100 \cdot \alpha^{th}$ and $100 \cdot (1-\alpha)^{th}$ percentiles $(s^{(\alpha)}, s^{(1-\alpha)})$. The resulting $100 \cdot (1-2\alpha)\%$ confidence interval for $\varphi_{ih}$ is $[\hat{\varphi}_{ih} - s^{(1-\alpha)},\, \hat{\varphi}_{ih} - s^{(\alpha)}]$ for $h = 0, 1, ....$

Important features of Procedure A in comparison to Procedure B are as follows. In Step 2, the bootstrap sample $X_t^*$ shares the same data generating process as the original sample $X_t$. In Step 3, the bootstrap IRF estimates involve the same identification and estimation methods, especially factor estimation, as the original so that the dispersions of the bootstrap estimates can mimic those of the original estimates.

**Procedure B: Bootstrapping without factor estimation**

Procedure B requires modification in Steps 3 of Procedure A and is formalized as follows:

3. Using the bootstrapped observations $X_t^*$ and factors $F_t^*$, estimate $\hat{\Lambda}^{**}$, $\hat{\Phi}^{**}$ and $\hat{S}^{**}$. This yields the bootstrap IRF estimates $\hat{\varphi}_i^{**}$.

Procedure B would be a natural and simple extension of the methods conducted in standard structural VAR analyses. However, the generated confidence intervals will not properly account for the factor estimation errors. The theoretical and finite sample properties of both algorithms are further investigated in the following sections. Finally, the bootstrap interval specified in Step 5 is known as Hall's percentile intervals (Hall, 1992). One can alternatively consider what is commonly called Efron's percentile method by storing $s \equiv \hat{\varphi}_{ih}^*$ and constructing $\left[s^{(\alpha)}, s^{(1-\alpha)}\right]$, however, this method is not exact when the interval is not centered even asymptotically (see Efron and Tibshirani, 1994 and Lütkepohl, 2005). As I have seen, the IRFs are likely to produce non-centered distributions in finite samples because of structural identifications and factor estimation errors. Another popular choice is the percentile-$t$ interval. However, Kilian (1999) observed that the percentile-$t$ is not very accurate for IRF estimates especially in long horizons when the sample size is small.

## 4.2 Asymptotic validity

This section discusses the asymptotic validity of Procedures A and B. The first-order asymptotic results are given in Theorems 3 and 4, and several remarks on higher-order correctness will follow the statements. To this end, I extensively use some high-level conditions. These conditions can be justified by the following more primitive conditions in the bootstrap DGP.

**Condition BT1.**

a. $E^*(u_{it}^*) = 0$ for all $(i,t)$;

b. $\frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}|\gamma_{st}^*|^2 = O_p(1)$, where $\gamma_{st}^* = E^*\left(\frac{1}{N}\sum_{i=1}^{N}u_{it}^*u_{is}^*\right)$;

c. $\frac{1}{T^2}\sum_{t=1}^{T}\sum_{s=1}^{T}E^*\left|N^{-1/2}\sum_{i=1}^{N}(u_{is}^*u_{it}^* - E(u_{is}^*u_{it}^*))\right|^2 = O_p(1)$;

d. $\frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}F_s^*F_t^{*\prime}\gamma_{st}^* = O_p(1)$;

e. $\frac{1}{T}\sum_{t=1}^{T}E^*\left\|\frac{1}{\sqrt{NT}}\sum_{s=1}^{T}\sum_{i=1}^{N}F_s^*\left[u_{is}^*u_{it}^* - E^*(u_{is}^*u_{it}^*)\right]\right\|^2 = O_p(1)$;

f. $E^*\left\|\frac{1}{\sqrt{NT}}\sum_{t=1}^T\sum_{i=1}^N F_t^*\hat{\lambda}_i' u_{it}^*\right\|^2 = O_p(1)$;

g. $\frac{1}{T}\sum_{t=1}^T E^*\left\|\frac{1}{\sqrt{N}}\sum_{i=1}^N \hat{\lambda}_i u_{it}^*\right\|^2 = O_p(1)$;

h. $\frac{1}{T}\sum_{t=1}^T\left(\frac{\hat{\Lambda}' u_t^*}{\sqrt{N}}\right)\left(\frac{u_t^{*\prime}\hat{\Lambda}}{\sqrt{N}}\right) - \Gamma^* = o_{p*}(1)$, in probability, where $\Gamma^* \equiv \frac{1}{T}\sum_{t=1}^T Var^*\left(\frac{1}{\sqrt{N}}\hat{\Lambda}' u_t^*\right) > 0$;

i. For each $i$, $\Theta_i^{*-1/2}\frac{1}{\sqrt{T}}\sum_{t=1}^T F_t^* u_{it}^* \xrightarrow{d*} N(0, I_r)$, in probability, where $\Theta_i^* \equiv Var^*\left(\frac{1}{\sqrt{T}}\sum_{t=1}^T F_t^* u_{it}^*\right)$;

j. $p\lim \Theta_i^* = Q^{-1\prime}\Theta_i Q^{-1}$.

**Condition BT2.**

a. $E^*(e_t^*) = 0$, $E^*(e_t^* e_t^{*\prime}) = \Sigma_e^*$ an $r \times r$ positive definite matrix, and $e_t^*$ and $e_s^*$ are independent for $s \neq t$;

b. $E^*\left|e_{it}^* e_{jt}^* e_{kt}^* e_{lt}^*\right| = O_p(1)$ for $i, j, k, l = 1, ..., r$, and all $t$;

c. $e_t^*$ are uncorrelated with $u_{is}^*$ for all $i$, $t$ and $s$;

d. $\Sigma^* T^{-1/2}\sum_{t=p+1}^T vec(Z_t^* e_t^*) \xrightarrow{d} N(0, I_{pr^2})$, in probability, where $\Sigma^* \equiv Var^*\left(\frac{1}{\sqrt{T}}\sum_{t=1}^T Z_t^* e_t^{*\prime}\right)$;

e. $p\lim \Sigma^* = \left[\left(I_p \otimes Q^{-1\prime}\right)\Sigma_Z\left(I_p \otimes Q^{-1}\right)\right] \otimes \left(Q^{-1\prime}\Sigma_e Q^{-1}\right)$.

**Condition BT3.**

a. $\sqrt{T}vec(H^{*-1}\hat{S}^* - \hat{S}) \xrightarrow{d} N(0, \Sigma_S)$,in probability, as $N, T \to \infty$ and $\sqrt{T}/N \to 0$.

b. $\sqrt{T}vec(\hat{S}^{**} - \hat{S}) \xrightarrow{d*} N(0, \Sigma_S)$, in probability, as $N, T \to \infty$.

Condition BT1 is a bootstrap analogue of Assumption 2 and follows the high-level conditions considered in Gonçalves and Perron (2011). Condition BT2 is analogous to Assumption 3 where the white noise VAR errors are assumed. Condition BT3 is even higher and can be broken down into more primitive conditions depending on specific structural identification schemes. These more primitive conditions as well as an explicit form of $\Sigma_S$ are discussed in Appendix C. Given these conditions, I obtain the following theorems.

**Theorem 3** *(Asymptotic validity of Procedure A) Under Assumptions 1–6 and Conditions 1, 2, BT1, BT2 and BT3(a),*

$$\sup_{x \in \boldsymbol{R}} \left| P^* \left[ (\hat{\varphi}^*_{ih} - \hat{\varphi}_{ih}) \leq x \right] - P \left[ (\hat{\varphi}_{ih} - \varphi_{ih}) \leq x \right] \right| \xrightarrow{p} 0,$$

*for all $i$ and uniformly in $h = 0, 1, 2, \cdots$ as $N, T \to \infty$, and $\sqrt{T}/N \to 0$.*

The results are of first-order. To better understand the finite sample inference properties, higher-order terms in the estimation errors are of interest. To start with, the errors in the original structural parameter estimation can be expanded into three components: (I) errors pertaining to the contemporaneous coefficient matrix $\hat{S}$, (II) factor estimation errors, and (III) combinations of (I) and (II). If I take the structural IRF at horizon 0 as an example, the expansion of the original estimate is:

$$\begin{aligned}
\sqrt{T}(\hat{\varphi}_{i0} - \varphi_{i0}) \;=\; & T^{-1/2} S' H'_{NT} H_{NT} F' u_i + T^{1/2} \varepsilon' H'^{-1}_{NT} \lambda_i \\
& + \underbrace{T^{-1/2} \varepsilon' H_{NT} F' u_i}_{\text{(I): errors in } \hat{S}} \\
& + \underbrace{T^{-1/2} S' H'_{NT} \hat{F}' (F - \hat{F} H'^{-1}_{NT}) \lambda_i + T^{-1/2} S' H'_{NT} (\hat{F} - F H'_{NT})' u_i}_{\text{(II): factor estimation errors}} \\
& + \underbrace{T^{-1/2} \varepsilon' \hat{F}' (F - \hat{F} H'^{-1}_{NT}) \lambda_i + T^{-1/2} \varepsilon' (\hat{F} - F H'_{NT})' u_i}_{\text{(III): (I) and (II)}}. \qquad (8)
\end{aligned}$$

with $\varepsilon = \hat{S} - H_{NT} S$. In the original estimate, the terms in (I), (II), and (III) are of $o_p(1)$, $O_p(\sqrt{T}/\delta^2)$, and $o_p(1)$ respectively. Note $O_p(\sqrt{T}/\delta^2) = o_p(1)$ when $\sqrt{T}/N \to 0$. When I follow Procedure A, the bootstrap parameter estimates take the same form in the bootstrap space so that:

$$\begin{aligned}
\sqrt{T}(\widetilde{\varphi}^*_{i0} - \hat{\varphi}_{i0}) \;=\; & T^{-1/2} \hat{S}' H^{*\prime}_{NT} H^*_{NT} F^{*\prime} u^*_i + T^{1/2} \varepsilon^{*\prime} H^{*\prime -1}_{NT} \hat{\lambda}_i \\
& + \underbrace{T^{-1/2} \varepsilon^{*\prime} H^*_{NT} F^{*\prime} u^*_i}_{\text{(I): errors in } \hat{S}^*} \\
& + \underbrace{T^{-1/2} \hat{S}' H^{*\prime}_{NT} \hat{F}^{*\prime} (F^* - \hat{F}^* H^{*-1\prime}_{NT}) \hat{\lambda}_i + T^{-1/2} \hat{S}' H^{*\prime}_{NT} (\hat{F}^* - F^* H^{*\prime}_{NT})' u^*_i}_{\text{(II): factor estimation errors}} \\
& + \underbrace{T^{-1/2} \varepsilon^{*\prime} \hat{F}^{*\prime} (F^* - \hat{F}^* H^{*\prime -1}_{NT}) \hat{\lambda}_i + T^{-1/2} \varepsilon^{*\prime} (\hat{F}^* - F^* H^{*\prime}_{NT})' u^*_i}_{\text{(III): (I) and (II)}}. \qquad (9)
\end{aligned}$$

with

$$H^*_{NT} = \hat{V}^{-1} (\hat{F}^{*\prime} F^* / T)(\hat{\Lambda}' \hat{\Lambda} / N) \qquad (10)$$

where $\hat{V}$ is a diagonal matrix with its elements being eigenvalues of $X^* X^{*\prime}/(TN)$ in descending order. The validity is shown under the stated conditions, which guarantee that all the terms in (9) are of the same probability order under $P^*$ as those in (8) under $P$. Hence (I) and (III) disappear as $N, T \to \infty$, and so does (II) with an additional condition $\sqrt{T}/N \to 0$. I now provide the validity of Procedure B.

**Theorem 4** *(Asymptotic validity of Procedure B) Under Assumptions 1–6 and Conditions 1, 2, BT1(i), BT1(j), BT2, and BT3(b):*

$$\sup_{x \in \boldsymbol{R}} \left| P^* \left[ (\hat{\varphi}^{**}_{i,h} - \hat{\varphi}_{i,h}) \leq x \right] - P \left[ (\hat{\varphi}_{i,h} - \varphi_{i,h}) \leq x \right] \right| \xrightarrow{p} 0,$$

*for all $i$ and uniformly in $h = 0, 1, 2, \cdots$ as $N, T \to \infty$, $\sqrt{T}/N \to 0$.*

When one uses Procedure B, the bootstrap estimate of the structural IRF at time 0 is expanded in the bootstrap space as follows:

$$\begin{aligned} \sqrt{T}(\hat{\varphi}^{**}_{i0} - \hat{\varphi}_{i0}) \equiv\ & T^{-1/2} \hat{S}' F^{*\prime} u^*_i + T^{1/2} \varepsilon^{**\prime} \hat{\lambda}_i \\ & + \underbrace{T^{-1/2} \varepsilon^{**\prime} F^{*\prime} u^*_i}_{\text{(I): errors in } \hat{S}^{**}}, \end{aligned} \tag{11}$$

with $\varepsilon^{**} = \hat{S}^{**} - S$. The higher-order terms associated with factor estimation errors (II) and (III) in (9) do not appear with Procedure B in (11). Hence I expect that the intervals constructed using Procedure B are generally tighter than those obtained using Procedure A because of the factor estimation errors. More importantly, the intervals given by Procedure B may not be as accurate as those using Procedure A, especially when $N$ is not significantly larger than $\sqrt{T}$ ($\sqrt{T}/N \to 0$ is not appropriate) as the terms in (II) that are not present with Procedure B are relevant[5]. I also note that when the errors in the contemporaneous matrix estimate $\varepsilon$ are not small, the terms in (I) and (III) can also play a significant role. This leads to an error in coverage over short horizons as the effect of $\varepsilon$ will diminish over long horizons. Finally, the asymptotic normal approximation neither accounts for the factor estimation errors (II) and (III), nor is it able to capture the effect of (I) well in finite samples.

[5]Although the validity of Procedure A requires $\sqrt{T}/N \to 0$, Gonçalves and Perron (2011) shows that the finite sample coverage properties are quite well if factor estimation is involved in the bootstrap. They also propose bias correction procedure to further improve finite sample properties.

## 5 Finite sample properties

### 5.1 Monte Carlo simulations

In this section, I provide simulation results to assess the finite sample properties of the proposed inference procedures. For simplicity I consider a two-factor VAR(1) model so that the observable variables $x_{i,t}$ are generated as:

$$x_{i,t} = \lambda_i f_t + u_{i,t},$$

and the factors ($f_t : 2 \times 1$) evolve such that:

$$f_t = \Phi f_{t-1} + e_t,$$

for $i = 1, ..., N$ and $t = 1, ..., T$ with $\lambda_i = [\lambda_{i,1}, \lambda_{i,2}]$ and $\Phi$ is a $2 \times 2$ matrix. I consider three types of structural identifications, hence, $e_t = S\zeta_t$ and $S$ is:

$$S = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}.$$

for ID1 and ID2 cases and the identity matrix for ID3 case. I consider the IRF of the third observation[6] to a structural shock to the first factor. The loadings are $\lambda_{ij} \sim i.i.d.U(0,1)$ and $\lambda_{21} = 0$ for ID1 and ID2 so that I can use the triangular structure of the first two IRFs $\lambda_{ij} \sim i.i.d.N(0,1)$ for ID3 to meet Assumption 6(c)'.2. The VAR parameter also $\Phi$ respects identification restrictions so that:

$$\begin{aligned} \Phi &= \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} && \text{for ID1 and ID2,} \\ \Phi &= \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix} && \text{for ID3,} \end{aligned}$$

so that $\Sigma_F$ is diagonal in ID3 case to satisfy Assumption 6(c)'.1.

I generate quasi-random variables $\zeta_{j,t}$ and $u_{i,t}$ following i.i.d. standard normal ("Gaussian errors") or a centered chi-square distribution with one degree of freedom ("chi-squared errors") with unit variance. To eliminate the effect of the initial value, I generate a sample with a size of $2 \times T$ and discard the first $T$ sample.

[6] The first two observations are used for the structural identification in ID1 and ID2 cases. Since the loadings are homogeneous in this experiment, any observations besides the first two must give the same results.

Since the effect of the sample size on the inference results is of major interest, I compare the results of the four $(N, T)$ combinations of $N = \{50, 200\}$ and $T = \{40, 120\}$. The bootstrap inference method is conducted for $1,000$ replications and I report the results for equal-sided confidence intervals of the 95% and 85% nominal levels. By default, the bias correction in the spirit of Kilian (1998) is applied where the bias for $\hat{\Phi}$ is estimated by another $R_b = 1,000$ times bootstrap loop and evaluated by $R_b^{-1} \sum_{j=1}^{R_b} \left[ \hat{\Phi}_j^* - H^* \hat{\Phi}_j H^{*-1} \right]$ with $j = 1, ..., R_b$. The number of replications in the Monte Carlo simulations for evaluating the coverage ratio is $1,000$ and the impulse responses are considered up to five periods ahead. The coverage ratios and the median of the lengths of the confidence intervals are reported.

The coverage results of Procedures A and B and the asymptotic approximation at 95% and 85% levels when Gaussian errors are used are shown in Tables 1a and 1b. The first observation to note is that throughout the experiments Procedure A exhibits coverage probabilities close to the nominal levels, however, Procedure B undercovers in many situations. The undercoverage of Procedure B is most distinct in the case of $(N, T) = (50, 120)$, where the condition$\sqrt{T}/N \to 0$ is least relevant among four cases. The second notable result is that the asymptotic normal intervals work quite poorly in every case. The undercoverage of the asymptotic approximation is prominent in short horizons when ID1 and ID2 are used with small $N$, although it improves for longer horizons especially when $T$ is large. This reflects the fact that the finite distribution of $\hat{S}$ is contaminated with ID1 and ID2 than ID3. However, even if ID3 is used, I see lower coverage rates especially in longer horizons. Finally, these findings are robust to the chi-squared errors (Tables 2-a and 2-b).

In order to further investigate the difference between Procedures A and B, cases of smaller sample sizes are of more interest as the factors are estimated less precisely as discussed in the previous section. To this end, the sample sizes are now chosen $(N, T) = \{(10, 120), (30, 120), (50, 120)\}$ and I also consider more persistent factors and set the VAR parameters with the diagonal elements 0.7 instead of 0.4. Everything else is the same as the baseline simulation and the results for the 95% nominal level and Gaussian errors are reported in Table 3. The table shows that Procedure A is still able to provide intervals with coverage ratios very close to the nominal levels, however, the coverages using Procedure B are remarkably smaller than the nominal level. This is first because, for smaller sample sizes, the effects of neglecting factor estimation uncertainty with Procedure B becomes more distinct. Second, as frequently shown in the empirical data, when factors are more persistent and have more variability (the diagonal elements are larger) the difference in the two procedures becomes more distinct[7].

Finally, I report the coverage properties using three different confidence intervals (labeling Hall's percentile, Efron's percentile, and percentile-$t$ as PER-H, PER-E, and PER-T, respectively) in Table 4 and the results without using the Kilian (1998)'s bias correction in Table 5. In both cases, I report the results of Procedure A under the Gaussian errors and ID1 identification is used at the 95% level. Table 4 illustrates an advantage of the suggested PER-H method over PER-E and PER-T methods as I conjectured in the previous section. Table 5 confirms that the coverage deteriorates in long horizons if bias correction is not conducted. This effect becomes more distinct when the sample size is small and the factor process is more persistent.

### 5.2 Monte Carlo simulation using empirical data

Finally, I present an empirical experiment to ascertain the robustness of the proposed Procedure A to actual economic data. To this end, I employ 110 US macroeconomic series investigated by Stock and Watson (2008). The data are a mixture of quarterly and monthly frequencies, spanning the period from 1959$Q$1 to 2006$Q$4. I conducted the following treatment, as in the original paper by Stock and Watson (2008). First, monthly data are converted into quarterly data by taking a simple average over three months. Second, all series are transformed into stationary processes following Stock and Watson's (2008) guidelines. In addition, the data are demeaned and standardized to have unit standard deviations. A brief description of the data set is given in Table 8, with more details available in Stock and Watson (2008). I choose two factors, which is justified by the ICP2 criteria in Bai and Ng (2002), though moderate variations in the lag order and the number of factors do not affect the qualitative results. I also find that the first factor is closely related to medium-run real economic activity measures (e.g. production) and the second factor has a stronger correlation with price variables. This is consistent with the findings in Sargent and Sims (1977) and Stock and Watson (2005). Hence, for identification, I select the assumption that the producer price index is contemporaneously affected only by the second factor. I select the order of the vector autoregression to be four. The observation and VAR equations are then identical to those described in the previous subsection, except with a higher lag order.

The aim here is to evaluate the coverage properties for the IRFs of Procedure A. However, the coverage probabilities of the confidence intervals constructed from actual data cannot

[7]Shintani and Guo (2011) also observe that when the factors are persistent, the asymptotic approximation is more clearly dominated by the bootstrap inference.

be calculated. Hence, I use the following calibration experiment in order to replicate an approximation of the actual data-generating process.

1. Estimate the model using the PC method to obtain coefficient estimates and residuals.

2. Generate quasi-observations from the calibrated model with the error terms resampled from $\{\hat{u}_t\}$ and $\{\hat{e}_t\}$ with replacement. Note that $\{\hat{e}_t\}$ are orthonormalized by $\zeta_t = \hat{e}_t\hat{\Sigma}_e{}^{-1/2}$, where $\hat{\Sigma}_e{}^{-1/2}$ is the Cholesky decomposed covariance matrix of $\hat{e}_t$. This allows $\zeta_t$ to be interpreted as a structural innovation.

3. Using each generated data set, construct 95% confidence intervals for the IRFs using the proposed bootstrap procedure and see if the true (calibrated) IRFs are included in the estimated interval.

4. Repeat Steps 2 and 3 $1,000$ times to evaluate the coverage probabilities.

The considered IRFs are for prices (the personal consumption expenditures price index, which is composed of nondurables excluding food, clothing and oil: `GDP275_4`), long-term interest rates (the 10-year US treasury bill interest rate: `FYGM10`), a production index (the industrial production index: `IPS43`) and the unemployment rate (the unemployment rate for all workers 16 years & over: `LHUR`). Table 6-a provides the results for the impulse responses in the first 8 periods. To examine the impact of the sample size, I conduct this experiment using the full data set ($T = 190$), as shown in Table 6-a, and post-1984 data ($1984Q1 : 2006Q4$, $T = 90$), as shown in Table 6-b. The results for both cases generally yield values very close to the 95% nominal level for all four variables when using the full sample data set. This finding also holds true for the smaller sample size comprising post-1984 data. Therefore, the good finite sample properties of the bootstrap procedure are confirmed by this calibrated experiment.

Finally, Table 7 compares the results with those for Procedure B. Here, I use a smaller data set by selecting only aggregate series from Stock and Watson's (2008) data set. For example, I use the total industrial production index instead of the separate industrial production indexes for durable and nondurable goods, and so on. This procedure leaves us with 47 data series (see Table 9). However, the basic structure of the data set remains unchanged and it yields clearer results. I consider the full time length ($T = 190$). I also show that if bootstrapping is applied without considering the uncertainty associated with factor estimation (Procedure B), the resulting confidence intervals become narrower and the coverage ratios are mostly below the 95% nominal level.

## 6 Conclusions

This paper has two major contributions. First, I explicitly consider conditions and examples of structural identification in simple FAVAR models, which account for the random factor rotation induced by the popular PC estimation method. The strategy is to impose identifying restrictions on the structural parameters or IRFs, and these identifying assumptions are widely used in empirical studies. Second, and more importantly, I investigate residual-based bootstrap procedures suggested by Gonçalves and Perron (2011) and Shintani and Guo (2011) in FAVAR context. I find that factor estimation errors play an important role in inference problems in impulse response analyses. This is a formalization of the facts pertaining to the factor estimation errors discovered by Ludvigson and Ng (2009b) and Gonçalves and Perron (2011) in FAVAR setting. Although this is in close agreement with these related studies, the effects of factor estimation errors can be more prominent in this context through structural identification schemes studied in this paper and the suggested bootstrap procedure is highly recommended.

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## Appendix A : Proof of Theorems

In the appendix, I suppress the subscript $NT$ for the PC rotation matrix $H_{NT}$ and use $H$.

**Proof of Theorem 1:** I show the results of the individual structural parameters $\lambda_i^s$ and $\Phi^s$. Then the continuous mapping theorem immediately yields the result of the structural IRF. First, the reduced-form estimate $\hat{\lambda}_i$ is expanded into the following form (see Bai, 2003, proof of Theorem 2).

$$\hat{\lambda}_i = H'^{-1}\lambda_i + T^{-1}HF'u_i + T^{-1}\hat{F}'(F - \hat{F}H'^{-1})\lambda_i + T^{-1}(\hat{F} - FH')'u_i. \tag{A.1}$$

Under Condition 1, I let $\varepsilon = \hat{S} - HS$ with $\varepsilon \xrightarrow{p} 0$. This implies that $\xi = \hat{S}^{-1} - S^{-1}H^{-1}$ with $\xi \xrightarrow{p} 0$. Then, the estimate for the structural parameter $\lambda_i^s = S'\lambda_i$ is given by:

$$\begin{aligned}\hat{\lambda}_i^s &= \hat{S}'\hat{\lambda}_i = S'\lambda_i + \varepsilon' H'^{-1}\lambda_i + T^{-1}S'H'HF'u_i + T^{-1}\varepsilon' HF'u_i \\ &\quad + T^{-1}\hat{S}'\hat{F}'(F - \hat{F}H'^{-1})\lambda_i + T^{-1}\hat{S}'(\hat{F} - FH')'u_i. \end{aligned}\tag{A.2}$$

Rearranging the terms in (A.2) gives:

$$\begin{aligned}\hat{\lambda}_i^s - \lambda_i^s &= T^{-1}S'H'HF'u_i + \varepsilon' H'^{-1}\lambda_i + T^{-1}\varepsilon' HF'u_i \\ &\quad + T^{-1}\hat{S}'\hat{F}'(F - \hat{F}H'^{-1})\lambda_i + T^{-1}\hat{S}'(\hat{F} - FH')'u_i, \\ &= I + II + III + IV + V.\end{aligned}$$

Since $I = O_p(T^{-1/2})$, $II = o_p(1)$, $III = O_p(T^{-1/2})$ by Condition 1 and Assumption 2(i), and $IV, V = O_p(\delta^{-2})$ by Bai and Ng (2006b) Lemma A1 it is shown that $\hat{\lambda}_i^s - \lambda_i^s \xrightarrow{p} 0$.

For $\Phi^s$, the least squares estimate for $\Phi$ is given by:

$$\begin{aligned}\hat{\Phi} &= \left(\hat{Z}'\hat{Z}\right)^{-1}\left(\hat{Z}'\hat{F}\right), \\ &= \left(\hat{Z}'\hat{Z}\right)^{-1}\left(\hat{Z}'FH'\right) + \left(\hat{Z}'\hat{Z}\right)^{-1}\hat{Z}'(\hat{F} - FH'), \\ &= \left(\hat{Z}'\hat{Z}\right)^{-1}\left[\hat{Z}'Z(I_p \otimes H')(I_p \otimes H'^{-1})\Phi H' + \hat{Z}'eH'\right] + \left(\hat{Z}'\hat{Z}\right)^{-1}\hat{Z}'(\hat{F} - FH'), \\ &= \left(\hat{Z}'\hat{Z}\right)^{-1}\left[\hat{Z}'\hat{Z}(I_p \otimes H'^{-1})\Phi H'\right] + \left(\hat{Z}'\hat{Z}\right)^{-1}\left(\hat{Z}'eH'\right) \\ &\quad + \left(\hat{Z}'\hat{Z}\right)^{-1}\left[\hat{Z}'(Z(I_p \otimes H') - \hat{Z})(I_p \otimes H'^{-1})\Phi H'\right] + \left(\hat{Z}'\hat{Z}\right)^{-1}\hat{Z}'(\hat{F} - FH'), \\ &= (I_p \otimes H'^{-1})\Phi H' + \left(T^{-1}\hat{Z}'\hat{Z}\right)^{-1}\left[T^{-1}(I_p \otimes H)Z'eH'\right] \\ &\quad + \left(T^{-1}\hat{Z}'\hat{Z}\right)^{-1}\left[T^{-1}(\hat{Z} - Z(I_p \otimes H'))'eH'\right] \\ &\quad + \left(T^{-1}\hat{Z}'\hat{Z}\right)^{-1}\left[T^{-1}\hat{Z}'(Z(I_p \otimes H') - \hat{Z})(I_p \otimes H'^{-1})\Phi H'\right] \\ &\quad + \left(T^{-1}\hat{Z}'\hat{Z}\right)^{-1}\left[T^{-1}\hat{Z}'(\hat{F} - FH')\right]. \end{aligned}\tag{A.3}$$

Since $T^{-1}\hat{Z}'\hat{Z} = O_p(1)$, the last three terms in (A.3) are $O_p(\delta^{-2})$ using Lemma A1 in Bai and Ng (2006b). The estimate for the structural parameter $\Phi^s = (I_p \otimes S)\Phi S^{-1}$ is then,

$$\begin{aligned}
(I_p \otimes \hat{S}')\hat{\Phi}\hat{S}'^{-1} &= (I_p \otimes S')\Phi S'^{-1} + (I_p \otimes S'H')(T^{-1}\hat{Z}'\hat{Z})^{-1}\left[T^{-1}(I_p \otimes HZ'eS'^{-1}\right] \\
&\quad +(I_p \otimes S')\left[\Phi H' + (T^{-1}\hat{Z}'\hat{Z})^{-1}T^{-1}(I_p \otimes H)Z'eH'\right]\xi \\
&\quad +(I_p \otimes \varepsilon')[(I_p \otimes H'^{-1})\Phi + (T^{-1}\hat{Z}'\hat{Z})^{-1}T^{-1}(I_p \otimes H)Z'e]S^{-1} \\
&\quad +(I_p \otimes \varepsilon')\left[(I_p \otimes H'^{-1})\Phi + (T^{-1}\hat{Z}'\hat{Z})^{-1}T^{-1}(I_p \otimes H)Z'eH'\right]\xi \\
&\quad +(I_p \otimes S)O_p(\delta^{-2})S^{-1}, \qquad \text{(A.4)}
\end{aligned}$$

or

$$\begin{aligned}
\hat{\Phi}^s - \Phi^s &= (I_p \otimes S'H')(T^{-1}\hat{Z}'\hat{Z})^{-1}\left(T^{-1}(I_p \otimes H)\,Z'eS'^{-1}\right) \\
&\quad +(I_p \otimes S')\left[\Phi H' + (T^{-1}\hat{Z}'\hat{Z})^{-1}T^{-1}(I_p \otimes H)Z'eH'\right]\xi \\
&\quad +(I_p \otimes \varepsilon')[(I_p \otimes H'^{-1})\Phi + (T^{-1}\hat{Z}'\hat{Z})^{-1}T^{-1}(I_p \otimes H)Z'e]S^{-1} \\
&\quad +(I_p \otimes \varepsilon')\left[(I_p \otimes H'^{-1})\Phi + (T^{-1}\hat{Z}'\hat{Z})^{-1}T^{-1}(I_p \otimes H)Z'eH'\right]\xi + O_p(\delta^{-2}), \\
&= I + II + III + IV + O_p(\delta^{-2}).
\end{aligned}$$

Since $I = O_p(T^{-1/2})$ and $II$, $III, IV = o_p(1)$ by Condition 1, I obtain $\hat{\Phi}^s - \Phi^s \xrightarrow{p} 0$. These imply the result for a continuous mapping of structural parameters, i.e. the structural IRF estimate $\hat{\varphi}_{i,h} - \varphi_{i,h} \xrightarrow{p} 0$, uniformly in $h$ for all $i$.

**Proof of Theorem 2:** Theorem 1 shows that the $\varphi_{ih}$ is consistently estimated by $\hat{\varphi}_{ih}$ and I know from (5) that it is also a function of the reduced-form parameters ($\lambda_i$ and $\Phi$) and $S$. Given that the reduced-form parameter estimates $\hat{\lambda}_i$ and $\hat{\Phi}$ are asymptotically normal as $N, T \to \infty$ and $\sqrt{T}/N \to 0$ under Assumptions 1–3 (see Bai, 2003 and Bai and Ng, 2006b, who prove under weaker conditions) up to rotation and the asymptotic normality for $\hat{S}$ by Condition 2, the delta method yields the asymptotic normality for the structural IRF with the variance which is given in the theorem.

The following Conditions A1 and A2 are high-level assumptions ascertaining the fact that several key convergence results in the original space are obtained in the bootstrap space as well. The validity of Condition A1 under more primitive assumptions (Conditions BT1 and BT2 in the main text) is shown by closely following the proofs provided in Gonçalves and Perron (2011) and lengthy hence is only provided per requests. Conditions 1 and A2 are shown in Appendix B.

**Condition A1.** Under Assumptions 1-3 and Conditions BT1 and BT2, the following conditions hold for $j = 1, \cdots, p$:

a. $T^{-1}\sum_{t=1}^{T}\left\|\hat{F}_t^* - H^*F_t^*\right\|^2 = O_{p*}(\delta^{-2})$;

b. $T^{-1}\sum_{t=1}^{T}(\hat{F}_t^* - H^* F_t^*)F_t^{*\prime} = O_{p*}(\delta^{-2})$;
c. $T^{-1}\sum_{t=1}^{T}(\hat{F}_t^* - H^* F_t^*)\hat{F}_t^{*\prime} = O_{p*}(\delta^{-2})$;
d. $T^{-1}\sum_{t=j+1}^{T}(\hat{F}_{t-j}^* - H^* F_{t-j}^*)e_t^{*\prime} = O_{p*}(\frac{1}{\delta\sqrt{T}})$;
e. $T^{-1}\sum_{t=1}^{T}(\hat{F}_t^* - H^* F_t^*)u_{it}^* = O_{p*}(\delta^{-2})$;
in probability.

**Condition A2.** Under Assumptions 1-6 and Conditions 1 and BT1 the following conditions on the contemporaneous coefficient matrix estimate in Procedures A and B hold:
a. $\hat{S}^* - H^*\hat{S} \xrightarrow{p*} 0$, in probability, as $N, T \to \infty$;
b. $\hat{S}^{**} - \hat{S} \xrightarrow{p*} 0$, in probability, as $N, T \to \infty$.

**Lemma A1.** Under Assumptions 1-3 and Conditions BT1 and BT2, $H^{*\prime}H^* \xrightarrow{p*} I_r$, in probability, as $N, T \to \infty$.

**Proof of Lemma A1:** First,

$$\hat{F}^{*\prime}F^*/T = (\hat{F}^* - F^*H^{*\prime})'F^*/T + H^*F^{*\prime}F^*/T,$$

then,

$$H^{*\prime} = H^{*\prime} - H^{*\prime}F^{*\prime}F^*/T + \hat{F}^{*\prime}F^*/T - (\hat{F}^* - F^*H^{*\prime})'F^*/T. \tag{A.5}$$

Right multiplying (A.5) by $H^*$ will yield:

$$\begin{aligned} H^{*\prime}H^* &= [H^{*\prime}H^* - H^{*\prime}(F^{*\prime}F^*/T)H^*] + \hat{F}^{*\prime}\hat{F}^*/T \\ &\quad + \hat{F}^{*\prime}(F^*H^{*\prime} - \hat{F}^*)/T - (\hat{F}^* - F^*H^{*\prime})'F^*H^*/T, \\ &= I + II + III + IV. \end{aligned}$$

Since $\frac{F^{*\prime}F^*}{T} \xrightarrow{p*} \frac{\hat{F}'\hat{F}}{T} = I_r$, in probability, $I = o_{p*}(1)$, $II = I_r$ by construction, and $III, IV = O_{p*}(\delta^{-2})$ by Conditions A1(b) and A1(c).

**Proof of Theorem 3:** I equivalently show that $\sqrt{T}(\hat{\varphi}_{ih} - \varphi_{ih}) \xrightarrow{d} N(0, \Sigma_{\varphi ih})$ and $\sqrt{T}(\hat{\varphi}_{ih}^* - \hat{\varphi}_{ih}) \xrightarrow{d*} N(0, \Sigma_{\varphi ih})$, in probability, as $N, T \to \infty$ and $\sqrt{T}/N \to 0$. In the first step, I confirm that $\hat{\varphi}_{ih}^* - \hat{\varphi}_{ih} = o_{p*}(1)$, in probability, by showing it for the individual parameters $\lambda_i^s$ and $\Phi^s$. In the second step, I show that the asymptotic variance for $\sqrt{T}(\hat{\varphi}_{ih}^* - \hat{\varphi}_{ih})$ converges to that of $\sqrt{T}(\hat{\varphi}_{ih} - \varphi_{ih})$, in probability.

First step: let $\varepsilon^* = \hat{S}^* - H^*\hat{S} = o_{p*}(1)$ by Condition A2(a) and this implies $\xi^* = o_{p*}(1)$. For $\lambda_i^s$,

$$\begin{aligned} \hat{\lambda}_i^{s*} - \hat{\lambda}_i^s &= T^{-1}\hat{S}^{*\prime}H^{*\prime}H^*F^{*\prime}u_i^* + \varepsilon^{*\prime}H^{*\prime-1}\hat{\lambda}_i + T^{-1}\varepsilon^{*\prime}H^*F^{*\prime}u_i^* \\ &\quad + T^{-1}\hat{S}^{*\prime}\hat{F}^{*\prime}(F^* - \hat{F}^*H^{*\prime-1})\hat{\lambda}_i + T^{-1}\hat{S}^{*\prime}(\hat{F}^* - F^*H^{*\prime})'u_i^*, \qquad (A.6) \\ &= I + II + III + IV + V. \end{aligned}$$

Note that I have the same expression as (A.2) after replacing the original parameters, factors, rotation, and errors with their bootstrap counterparts. Hence $I = O_{p*}(T^{-1/2})$ by Condition BT1(i), $II = o_{p*}(1)$ under Condition A2(a), $III = O_{p*}(T^{-1/2})$ by Condition BT1(i) and A2(a), $IV$, $V = O_{p*}(\delta^{-2})$ by Condition A1(c) and A1(e), in probability. Hence the RHS of (A.6) is $o_{p*}(1)$, in probability, as $N, T \to \infty$. For $\Phi^s$, it follows that:

$$\begin{aligned}
\hat{\Phi}^{s*} - \hat{\Phi}^s &= (I_p \otimes \hat{S}' H^{*\prime})(T^{-1}\hat{Z}^{*\prime}\hat{Z}^*)^{-1}\left(T^{-1}(I_p \otimes H^*)\, Z^{*\prime} e^* \hat{S}'^{-1}\right) \\
&\quad + (I_p \otimes \hat{S}')\left[\hat{\Phi} H^{*\prime} + (T^{-1}\hat{Z}^{*\prime}\hat{Z}^*)^{-1}(I_p \otimes H^*) Z^{*\prime} e^* H^{*\prime}\right]\xi^* \\
&\quad + (I_p \otimes \varepsilon^{*\prime})[(I_p \otimes H^{*\prime -1})\hat{\Phi} + (T^{-1}\hat{Z}^{*\prime}\hat{Z}^*)^{-1} T^{-1}(I_p \otimes H^*) Z^{*\prime} e^*]\hat{S}'^{-1} \\
&\quad + (I_p \otimes \varepsilon^{*\prime})\left[(I_p \otimes H^{*\prime -1})\hat{\Phi} + (T^{-1}\hat{Z}^{*\prime}\hat{Z}^*)^{-1} T^{-1}(I_p \otimes H^*) Z^{*\prime} e^* H^{*\prime}\right]\xi^* \\
&\quad + O_{p*}(\delta^{-2}), \qquad\qquad (A.7) \\
&= I + II + III + IV + O_{p*}(\delta^{-2})
\end{aligned}$$

in probability, under Condition A1. Now (A.7) is analogous to (A.4). It is similarly shown that $I = o_{p*}(T^{-1/2})$ and $II$, $III$, $IV$ are all $o_{p*}(1)$, in probability, under Conditions A1 and A2. Hence $\hat{\Phi}^{s*} - \hat{\Phi}^s = o_{p*}(1)$, in probability. This two facts imply $\hat{\varphi}^*_{ih} - \hat{\varphi}_{ih} \xrightarrow{p*} 0$, in probability, as $N, T \to \infty$.

The second step involves the limit distributions. I hypothetically consider the "rotation-adjusted" version of the reduced-form parameter estimates induced by $H^*$ in the bootstrap space. Specifically, the rotation-adjusted estimates of $\hat{\lambda}^*_i$ is $H^{*\prime}\hat{\lambda}^*_i$ and that of $\hat{\Phi}^*$ is $(I_p \otimes H^{*\prime})\hat{\Phi}^* H^{*\prime -1}$ and that of $\hat{S}$ is $H^{*-1}\hat{S}$. Note that given the fact that the structural IRF is identified, all $H^*$s are eventually cancelled out in the structural IRF estimate.

For $\lambda_i$, I construct a bootstrap analogue of (A.1) left-multiplied by $H^{*\prime}$ and scaled by $\sqrt{T}$:

$$\begin{aligned}
&\sqrt{T}(H^{*\prime}\hat{\lambda}^*_i - H^{*\prime}H^{*\prime -1}\hat{\lambda}_i) \\
&= T^{-1/2} H^{*\prime} H^* F^{*\prime} u^*_i + T^{-1/2} H^{*\prime} \hat{F}^{*\prime}(F^* - \hat{F}^* H^{*\prime -1})\hat{\lambda}_i + T^{-1/2} H^{*\prime}(\hat{F}^* - F^* H^{*\prime})' u^*_i, \\
&= T^{-1/2} H^{*\prime} H^* F^{*\prime} u^*_i + O_{p*}(T^{1/2}\delta^{-2}) + O_{p*}(T^{1/2}\delta^{-2}), \qquad (A.8)
\end{aligned}$$

in probability, under Conditions A1(c) and A1(e), so that:

$$\sqrt{T}(H^{*\prime}\hat{\lambda}^*_i - \hat{\lambda}_i) \xrightarrow{d*} N(0, Q^{-1\prime}\Theta_i Q^{-1}),$$

in probability, as $N, T \to \infty$ and $\sqrt{T}/N \to 0$ under Condition BT1(i) and BT1(j) and Lemma A1. For $\Phi$, constructing a bootstrap analogue of (A.3) left-multiplied by $(I_p \otimes H^{*\prime})$, righ-multiplied by $H^{*\prime -1}$ and scaled by $\sqrt{T}$ gives

$$\begin{aligned}
& \sqrt{T}[(I_p \otimes H^{*\prime})\hat{\Phi}^* H^{*\prime -1} - (I_p \otimes H^{*\prime} H^{*\prime -1})\hat{\Phi} H^{*\prime} H^{*\prime -1}] \\
= \; & T^{-1/2}(I_p \otimes H^{*\prime}) \left(T^{-1}\hat{Z}^{*\prime}\hat{Z}^*\right)^{-1} [Z^*(I_p \otimes H^{*\prime})]' \, e^* H^{*\prime} H^{*\prime -1} \\
& + T^{-1/2}(I_p \otimes H^{*\prime}) \left(T^{-1}\hat{Z}^{*\prime}\hat{Z}^*\right)^{-1} \left[\hat{Z}^* - Z^*(I_p \otimes H^{*\prime})\right]' e^* H^{*\prime} H^{*\prime -1} \\
& + T^{-1/2}(I_p \otimes H^{*\prime}) \left(T^{-1}\hat{Z}^{*\prime}\hat{Z}^*\right)^{-1} \hat{Z}^{*\prime}(Z^*(I_p \otimes H^{*\prime}) - \hat{Z}^*)(I_p \otimes H^{*\prime -1})\hat{\Phi} H^{*\prime} H^{*\prime -1} \\
& + T^{-1/2}(I_p \otimes H^{*\prime}) \left(T^{-1}\hat{Z}^{*\prime}\hat{Z}^*\right)^{-1} \hat{Z}^{*\prime}(\hat{F}^* - F^* H^{*\prime})H^{*\prime -1}, \\
= \; & T^{-1/2}(I_p \otimes H^{*\prime}) \left(T^{-1}\hat{Z}^{*\prime}\hat{Z}^*\right)^{-1} [Z^*(I_p \otimes H^{*\prime})]' \, e^* \\
& + O_{p*}(\delta^{-1}) + O_{p*}(T^{1/2}\delta^{-2}) + O_{p*}(T^{1/2}\delta^{-2}),
\end{aligned}$$

in probability, under Conditions A1(c) and A1(d). Since

$$\begin{aligned}
T^{-1}\hat{Z}^{*\prime}\hat{Z}^* \; = \; & T^{-1}[Z^*(I_p \otimes H^{*\prime})]' Z^*(I_p \otimes H^{*\prime}) \\
& + T^{-1}\left[\hat{Z}^* - Z^*(I_p \otimes H^{*\prime})\right]' Z^*(I_p \otimes H^{*\prime}) \\
& + T^{-1}\left[Z^*(I_p \otimes H^{*\prime})\right]' \left[\hat{Z}^* - (I_p \otimes H^{*\prime})Z^*\right], \\
= \; & T^{-1}(I_p \otimes H^*)Z^{*\prime}Z^*(I_p \otimes H^{*\prime}) + O_{p*}(T^{1/2}\delta^{-2}),
\end{aligned}$$

in probability, I obtain

$$\begin{aligned}
& \sqrt{T}\left[(I_p \otimes H^{*\prime})\hat{\Phi}^* H^{*\prime -1} - \hat{\Phi}\right], \\
= \; & T^{-1/2}(I_p \otimes H^{*\prime})(I_p \otimes H^*)(Z^{*\prime}Z^*/T)(I_p \otimes H^{*\prime})(I_p \otimes H^*)Z^{*\prime}e^* + O_{p*}(T^{1/2}\delta^{-2}) + O_{p*}(\delta^{-2}), \\
= \; & \left(T^{-1}Z^{*\prime}Z^*\right)^{-1} T^{-1/2}Z^{*\prime}e^* + O_{p*}(T^{1/2}\delta^{-2}) + O_{p*}(\delta^{-2}),
\end{aligned}$$

in probability, using Lemma A1. Hence,

$$\sqrt{T} vec\left[(I_p \otimes H^{*\prime})\hat{\Phi}^* H^{*\prime -1} - \hat{\Phi}\right] \xrightarrow{d*} N(0, [I_r \otimes \Sigma_{Z*}^{-1}]'\Sigma^*[I_r \otimes \Sigma_{Z*}^{-1}]),$$

with $\Sigma_{Z*} = p\lim Z^{*\prime}Z^*/T$ as $N, T \to \infty$ and $\sqrt{T}/N \to 0$ under Condition BT2(d) and BT2(e). Since

$$\Sigma_{Z*} \xrightarrow{p} (I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1}),$$

in probability, and the probability limit of $\Sigma^*$ is given in Condition BT2(e),

$$\begin{aligned}
& \sqrt{T} vec\left[(I_p \otimes H^{*\prime})\hat{\Phi}^* H^{*\prime -1} - \hat{\Phi}\right] \\
& \xrightarrow{d*} N(0, [(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})]^{-1} \\
& \times \left[(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})\right] \otimes \left[Q^{-1\prime}\Sigma_e Q^{-1}\right] \\
& \times [(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})]^{-1\prime}),
\end{aligned}$$

in probability. Finally, for $S$, the rotation adjusted version of $\hat{S}^*$ is $H^{*-1}\hat{S}^*$ and $\sqrt{T}vec(H^{*-1}\hat{S}^* - \hat{S}) \xrightarrow{d*} N(0, \Sigma_S)$, in probability from Condition BT3(a).

For the original estimate, it is straightforward to show from (A.1), (A.3) and Condition 2 that:

$$\begin{aligned}
&\sqrt{T}(\hat{\lambda}_i - H'^{-1}\lambda_i) \xrightarrow{d} N(0, Q^{-1\prime}\Theta_i Q^{-1}),\\
&\sqrt{T}vec\left[\hat{\Phi} - (I_p \otimes H'^{-1})\hat{\Phi}H'\right] \xrightarrow{d} N(0, [(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})]^{-1}\\
&\times[(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})] \otimes [Q^{-1\prime}\Sigma_e Q^{-1}]\\
&\times[(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})]^{-1\prime}),\\
&\sqrt{T}vec(\hat{S} - HS) \xrightarrow{d} N(0, \Sigma_S).
\end{aligned}$$

**Proof of Theorem 4:** With Procedure B, $H^*$ does not show up. Hence, I do not need to introduce the rotation-adjusted parameter estimates. In addition, expansions of the bootstrap parameter estimates will have fewer terms in the absence of the factor estimation errors. For $\lambda_i^s$,

$$\hat{\lambda}_i^{s**} - \hat{\lambda}_i^s = T^{-1}\hat{S}'F^{*\prime}u_i^* + \varepsilon^{**\prime}\hat{\lambda}_i + T^{-1}\varepsilon^{**\prime}F^{*\prime}u_i^* = I + II + III. \tag{A.9}$$

$I = O_{p*}(T^{-1/2})$ under Condition BT1(i) and $II, III = o_{p*}(1)$, in probability, under Condition A2(b). For $\Phi^s$,

$$\begin{aligned}
\hat{\Phi}^{s**} - \hat{\Phi}^s &= (I_p \otimes \hat{S})(Z^{*\prime}Z^*)^{-1}(Z^{*\prime}e^*\hat{S}^{-1})\\
&\quad +(I_p \otimes \hat{S})\left[\hat{\Phi} + (Z^{*\prime}Z^*)^{-1}Z^{*\prime}e^*\right]\xi^*\\
&\quad +(I_p \otimes \varepsilon^*)[\hat{\Phi} + (Z^{*\prime}Z^*)^{-1}Z^{*\prime}e^*]\hat{S}^{-1}\\
&\quad +(I_p \otimes \varepsilon^*)\left[\hat{\Phi} + (T^{-1}Z^{*\prime}Z^*)^{-1}T^{-1}Z^{*\prime}e^*\right]\xi^*\\
&\quad \xrightarrow{p*} 0,
\end{aligned}$$

in probability, under Condition BT2(d) and Condition A2(b). Next, consider the asymptotic distributions. For $\lambda_i$,

$$\sqrt{T}(\hat{\lambda}_i^{**} - \hat{\lambda}_i) = T^{-1/2}F^*u_i^* \xrightarrow{d*} N(0, Q^{-1\prime}\Theta_i Q^{-1}),$$

in probability, as $N, T \to \infty$ by Condition BT1(i) and BT1(j). For $\Phi$, since

$$\hat{\Phi}^{**} - \hat{\Phi} = (Z^{*\prime}Z^*/T)^{-1}(Z^{*\prime}e^*/T),$$

I obtain:

$$\begin{aligned}
\sqrt{T}vec(\hat{\Phi}^{**} - \hat{\Phi}) \xrightarrow{d*} N(0, &[(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})]^{-1}\\
&\times\left[(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})\right] \otimes \left[Q^{-1\prime}\Sigma_e Q^{-1}\right]\\
&\times[(I_p \otimes Q^{-1\prime})\Sigma_Z(I_p \otimes Q^{-1})]^{-1\prime}),
\end{aligned}$$

in probability, as $N, T \to \infty$, follows the proof of Theorem 3. Finally, Condition BT3(b) guarantees $\sqrt{T} vec(\hat{S}^{**} - \hat{S}) \xrightarrow{d*} N(0, \Sigma_S)$, in probability, as $N, T \to \infty$.

## Appendix B: Proof of Conditions 1 and A2

**Lemma B1.** Under Assumptions 1–3, $\frac{\hat{e}'\hat{e}}{T} - Q^{-1\prime}\Sigma_e Q^{-1} = O_p(\delta^{-2})$.

**Proof of Lemma B1:** First I expand the residuals $\hat{e}$.

$$\begin{aligned}
\hat{e} &= \hat{F} - \hat{Z}\hat{\Phi} = \hat{F} - \hat{Z}\hat{\Phi} \\
&= FH' + (\hat{F} - FH') - \hat{Z}\hat{\Phi} \\
&= eH' + Z\Phi H' + (F - FH') - \hat{Z}\hat{\Phi} \\
&= eH' + (\hat{F} - FH') + \left[Z(I_p \otimes H') - \hat{Z}\right](I_p \otimes H'^{-1})\Phi H' \\
&\quad + \hat{Z}\left[(I_p \otimes H'^{-1})\Phi H' - \hat{\Phi}\right]. \qquad \text{(A.10)}
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{\hat{e}'\hat{e}}{T} &= H\frac{e'e}{T}H' + \frac{1}{T}(\hat{F} - FH')'(\hat{F} - FH') \\
&\quad + H\Phi'(I_p \otimes H'^{-1})'\frac{1}{T}\left[(Z(I_p \otimes H') - \hat{Z})'(Z(I_p \otimes H') - \hat{Z})\right](I_p \otimes H'^{-1})\Phi H' \\
&\quad + \left[(I_p \otimes H'^{-1})\Phi H' - \hat{\Phi}\right]'\frac{1}{T}\hat{Z}'\hat{Z}\left[(I_p \otimes H'^{-1})\Phi H' - \hat{\Phi}\right] \\
&= H\frac{e'e}{T}H' + I + II + III + \text{cross terms} \qquad \text{(A.11)}
\end{aligned}$$

Using Cauchy-Schwartz inequality, $I \leq O_p(\delta^{-2})$ and $II \leq O_p(\delta^{-2})$. Also, $III = O_p(T^{-1})$.

The cross terms are $C + C'$ where

$$\begin{aligned}
C &= H\underbrace{e'(\hat{F} - FH')/T}_{O_p\left(\frac{1}{\sqrt{T}\delta}\right)} \\
&\quad + H\underbrace{\left(e'\left[Z(I_p \otimes H') - \hat{Z}\right]/T\right)}_{O_p\left(\frac{1}{\sqrt{T}\delta}\right)}(I_p \otimes H'^{-1})\Phi H' \\
&\quad + H\underbrace{\underbrace{\left(e'\hat{Z}/T\right)}_{O_p(T^{1/2}\delta^{-2})}\underbrace{\left[(I_p \otimes H'^{-1})\Phi H' - \hat{\Phi}\right]}_{O_p(T^{-1/2})}}_{O_p(\delta^{-2})} \\
&\quad + \underbrace{\left((\hat{F} - FH')'\left[Z(I_p \otimes H') - \hat{Z}\right]/T\right)}_{O_p(\delta^{-2})}(I_p \otimes H'^{-1})\Phi H' \\
&\quad + \underbrace{\left[(\hat{F} - FH')\hat{Z}/T\right]}_{O_p(\delta^{-2})}\underbrace{\left[(I_p \otimes H'^{-1})\Phi H' - \hat{\Phi}\right]}_{O_p(T^{-1/2})} \\
&\quad + H\Phi'(I_p \otimes H'^{-1})'\underbrace{\left(\left[Z(I_p \otimes H') - \hat{Z}\right]'\hat{Z}/T\right)}_{<O_p(\delta^{-2})}\left[(I_p \otimes H'^{-1})\Phi H' - \hat{\Phi}\right]
\end{aligned}$$

hence $C = O_p(\delta^{-2})$ using Lemma A1 in Bai and Ng (2006b).

- **Proof of Condition 1:**

  - **ID1**: Using the notation $\Lambda_{1:r} = \psi_{1:r,0}$, the part inside the Cholesky factorization in the step 1 becomes

$$\begin{aligned}
\hat{\Lambda}_{1:r}(\hat{e}'\hat{e}/T)\hat{\Lambda}'_{1:r} &= \hat{\Lambda}_{1:r}HSS'H'\hat{\Lambda}'_{1:r} + \hat{\Lambda}_{1:r}\left(\hat{e}'\hat{e}/T - HSS'H'\right)\hat{\Lambda}'_{1:r}, \\
&= \Lambda_{1:r}H^{-1}HSS'H'H'^{-1}\Lambda'_{1:r} - \left[\hat{\Lambda}_{1:r} - \Lambda_{1:r}H^{-1}\right]HSS'H'\left[\hat{\Lambda}_{1:r} - \Lambda_{1:r}H^{-1}\right]' \\
&\quad - \hat{\Lambda}_{1:r}HSS'H'\left[\hat{\Lambda}_{1:r} - \Lambda_{1:r}H^{-1}\right]' - \left[\hat{\Lambda}_{1:r} - \Lambda_{1:r}H^{-1}\right]HSS'H'\hat{\Lambda}'_{1:r} \\
&\quad + \hat{\Lambda}_{1:r}\left(\hat{e}'\hat{e}/T - HSS'H'\right)\hat{\Lambda}'_{1:r}, \\
&= \Lambda_{1:r}SS'\Lambda'_{1:r} + I + II + III + IV. \qquad \text{(A.12)}
\end{aligned}$$

    Since $\hat{\Lambda}_{1:r} - \Lambda_{1:r}H^{-1} = O_p(T^{-1/2})$, $I = O_p(T^{-1})$ and $II, III = O_p(T^{-1/2})$. For $IV$,

$$\hat{e}'\hat{e}/T - HSS'H' = \underbrace{\hat{e}'\hat{e}/T - H(e'e/T)H'}_{\to O_p(\delta^{-2}) \text{ by LemmaB1}} + \underbrace{H(e'e/T)H' - HSS'H'}_{\to 0 \text{ by Assumption 5}},$$

hence I obtain $\hat{\Lambda}_{1:r}(\hat{e}'\hat{e}/T)\hat{\Lambda}'_{1:r} \xrightarrow{p} \Lambda^s_{1:r}\Lambda^{s\prime}_{1:r}$. Since $\Lambda^s_{1:r}$ is a triangular matrix with positive diagonal elements by Assumption 6(a)

$$\hat{S} - HS = \underbrace{\underbrace{\hat{\Lambda}^{-1}_{1:r}}_{\to Q^{-1\prime}\Lambda^{-1}_{1:r}} \; Chol\left[\hat{\Lambda}_{1:r}(\hat{e}'\hat{e}/T)\hat{\Lambda}'_{1:r}\right]}_{\to \Lambda_{1:r}S} - \underbrace{HS}_{\to Q^{-1\prime}S} \xrightarrow{p} 0.$$

- **ID2:** I exactly follow ID1 by replacing the short-run IRF with the long-run IRF. As $\hat{\varphi}_{1:r,\infty} - \varphi_{1:r,\infty} = O_p(T^{-1/2})$ is straightforward from Theorem 1, the entire discussion for ID1 goes through.

- **ID3:** I first show that Assumption 6(c)' implies Assumption 6(c). To this end, I show that under Assumptions 6(c)'.1 and 6(c)'.2, $Q^{-1}$ becomes a diagonal matrix. Combining this fact and $S$ being triangular by Assumption 6(c)'.3 will yield the result. The proof is similar to Bai and Ng (2010)'s PC1 condition. I start with:

$$\hat{F}'F/T = (\hat{F} - FH')'F + HF'F/T = HF'F/T + O_p(\delta^{-2}).$$

Define $H_F = (F'F/T)H$. From the definition of $H$ in (7),

$$H = V^{-1}_{NT}(\hat{F}'F/T)(\Lambda'\Lambda/N) = V^{-1}_{NT}H'_F(\Lambda'\Lambda/N) + O_p(\delta^{-2}),$$

so that multiplying $F'F/T$ on both side will give:

$$H_F = V^{-1}_{NT}H'_F(\Lambda'\Lambda/N)(F'F/T) + O_p(\delta^{-2}).$$

Multiplying $V_{NT}$ on each side and taking the transpose:

$$(F'F/T)(\Lambda'\Lambda/N)H_F = H_F V_{NT} + O_p(\delta^{-2}).$$

Denote $Q^{-1}_F = \Sigma_F Q^{-1}$. In the limit,

$$\Sigma_F \Sigma_\Lambda Q^{-1}_F = Q^{-1}_F V,$$

with $V \equiv \underset{N,T\to\infty}{p\lim}\, V_{NT}$. This equation suggests that $Q^{-1}_F$ is a matrix consisting of eigenvectors of $\Sigma_F\Sigma_\Lambda$. Since $\Sigma_F\Sigma_\Lambda$ is diagonal by Assumptions 6(c)'.1 and 6(c)'.2 and it has distinct eigenvalues by Assumption 1(c), each eigenvalue is associated with a unique eigenvector and each eigenvector has a single nonzero element. This implies that $Q^{-1}_F$ is diagonal. Now since $\Sigma_F$ is diagonal and so is $Q^{-1}_F$ as shown above, $Q^{-1} = \Sigma_F Q^{-1}_F$ is a diagonal matrix. Finally, it is straightforward to obtain:

$$\hat{S} - HS = \underbrace{Chol\ \underbrace{(\hat{e}'\hat{e}/T)}_{\to Q^{-1\prime}SS'Q^{-1}}}_{\to Q^{-1\prime}S} - \underbrace{HS}_{\to Q^{-1\prime}S} \xrightarrow{p} 0,$$

by triangularity of $Q^{-1\prime}S$ and the sign restriction.

- **Proof of Conditions A2:**

  The proof requires two steps. Step 1 shows that the identifying restrictions hold in the bootstrap space, i.e. the same restrictions are relevant in the limits of the original estimates. Step 2 confirms all the convergence results which are used to prove Condition 1 hold in the bootstrap space.

  - **ID1 and ID2:**

    * **Step 1** is trivial since all the identifying restrictions are on structural IRFs which are consistently estimated by the original estimates. Hence the restrictions on $\varphi_{0,1:r}$ carry over to the limit of $\hat{\varphi}_{0,1:r}$, in probability.
    * **Step 2 (Procedure A):** The goal is to show:

    $$\hat{\Lambda}^*_{1:r}(\hat{e}^{*\prime}\hat{e}^*/T)\hat{\Lambda}^{*\prime}_{1:r} \xrightarrow{p*} \hat{\Lambda}^s_{1:r}\hat{\Lambda}^{s\prime}_{1:r},$$

    in probability. Given the fact that I have the same expansion as (A.12) with the bootstrap counterpart, $I = O_{p*}(T^{-1})$, $II$, $III = O_{p*}(T^{-1/2})$ since $\hat{\Lambda}^*_{1:r} - \hat{\Lambda}_{1:r}H^{*-1} = O_{p*}(T^{-1/2})$. I also have

    $$\hat{e}^{*\prime}\hat{e}^*/T - H^*(e^{*\prime}e^*/T)H^{*\prime} = O_{p*}(T^{-1/2}),$$

    by the same argument of Lemma B1 using Condition A1. Finally,

    $$H^*(e^{*\prime}e^*/T)H^{*\prime} - H^*\hat{S}\hat{S}'H^{*\prime} \xrightarrow{p*} 0,$$

    in probability, is shown by using $\frac{e^{*\prime}e^*}{T} \xrightarrow{p*} \frac{\hat{e}'\hat{e}}{T}$ and the fact both $\hat{e}'\hat{e}/T - Q^{-1\prime}SS'Q^{-1} = O_p(\delta^{-2})$ by Lemma B1 and $\hat{S}\hat{S}' - Q^{-1\prime}SS'Q^{-1} \xrightarrow{p} 0$ by Condition 1 yield

    $$\hat{e}'\hat{e}/T - \hat{S}\hat{S}' \xrightarrow{p} 0. \tag{A.13}$$

    Hence,

    $$\hat{S}^* - H^{*\prime}\hat{S} = \underbrace{\hat{\Lambda}^{*-1}_{1:r}}_{\to Q^{*-1\prime}\hat{\Lambda}^{-1}_{1:r}} Chol\underbrace{\left[\hat{\Lambda}^*_{1:r}(\hat{e}^{*\prime}\hat{e}^*/T)\hat{\Lambda}^{*\prime}_{1:r}\right]}_{\to\hat{\Lambda}_{1:r}\hat{S}} - \underbrace{H^{*\prime}\hat{S}}_{\to Q^{*-1\prime}\hat{S}} \xrightarrow{p*} 0,$$

    in probability.
    * **Step 2 (Procedure B):** I need to show $\hat{\Lambda}^{**}_{1:r}(\hat{e}^{**\prime}\hat{e}^{**}/T)\hat{\Lambda}^{**\prime}_{1:r} \xrightarrow{p*} \hat{\Lambda}^s_{1:r}\hat{\Lambda}^{s\prime}_{1:r}$. Since $\hat{\Lambda}^{**}_{1:r}$ and $\hat{e}^{**}$ are simple least square estimates of $\hat{\Lambda}_{1:r}$ and $e^*$, $\hat{\Lambda}^{**}_{1:r} - \hat{\Lambda}_{1:r} = O_{p*}(T^{-1/2})$. Also,

    $$\hat{e}^{**\prime}\hat{e}^{**}/T - e^{*\prime}e^*/T \xrightarrow{p*} 0,$$

    in probability, or

    $$\hat{e}^{**\prime}\hat{e}^{**}/T - \hat{e}'\hat{e}/T \xrightarrow{p*} 0, \tag{A.14}$$

in probability. It is suggested by (A.13) and (A.14) that:

$$\hat{S}^{**} - \hat{S} = \underbrace{\hat{\Lambda}^{**-1}_{1:r}}_{\to\hat{\Lambda}^{-1}_{1:r}} Chol\underbrace{\left[\hat{\Lambda}^{**}_{1:r}(\hat{e}^{**\prime}\hat{e}^{**}/T)\hat{\Lambda}^{**\prime}_{1:r}\right]}_{\to\hat{\Lambda}_{1:r}\hat{S}} - \hat{S} \xrightarrow{p*} 0,$$

in probability. Note that the same discussion goes through with ID2 case with replacing the short-run IRFs with long-run IRFs.

- **ID3:**
  - **Step 1** is not as trivial as the previous cases since the restrictions of Assumption 6(c) are on reduced-form parameters. However, I show that the same restrictions on the original estimates hold in the bootstrap world incidentally. That is, I show that the following conditions analogous to Assumption 6(c)' hold:
    1. $F^{*\prime}F^*/T \xrightarrow{p*} I_r$, in probability;
    2. $\hat{\Lambda}'\hat{\Lambda}/N$ is a diagonal matrix;
    3. $\hat{S}$ is an (upper or) lower triangular matrix and the signs of diagonal elements of $Q^{*-1\prime}\hat{S}$ are known.

    1. is trivial since:

    $$F^{*\prime}F^*/T \xrightarrow{p*} \hat{F}'\hat{F}/T = I_r.$$

    2. is also trivial since $\hat{\Lambda}'\hat{\Lambda}/N = V_{NT}$ by construction of the principal components and $V_{NT}$ is a diagonal matrix. Triangularity of $\hat{S}$ in 3. is because $\hat{S} = Chol\left(\frac{\hat{e}'\hat{e}}{T}\right)$. The signs are deduced in bootstrap replications through the signs of structural IRF estimates as in the original estimates. Hence Assumption 6(c) holds in the bootstrap space. This will give the fact that $Q^{*-1\prime}\hat{S}$ is a triangular matrix and the signs of its diagonal matrix are known.
  - **Step 2 (Procedure A):** I have:

    $$\hat{S}^* - H^{*\prime}\hat{S} = \underbrace{Chol\ \underbrace{(\hat{e}^{*\prime}\hat{e}^*/T)}_{\to Q^{*-1\prime}\hat{S}\hat{S}'Q^{*-1}}}_{Q^{*-1\prime}\hat{S}} - \underbrace{H^{*\prime}\hat{S}}_{\to Q^{*-1\prime}\hat{S}} \xrightarrow{p*} 0,$$

    in probability.
  - **Step 2 (Procedure B):** I use (A.13) and (A.14) to get:

    $$\hat{S}^{**} - \hat{S} = \underbrace{Chol\underbrace{(\hat{e}^{**\prime}\hat{e}^{**}/T)}_{\to\hat{S}\hat{S}'}}_{\hat{S}} - \hat{S} \xrightarrow{p*} 0,$$

    in probability.

## Appendix C: A Discussion on Conditions 2 and BT3

This appendix provides a discussion about the relevance of Conditions 2 and BT3. I also consider the effects of the identification schemes on the distribution of $\hat{S}$. First, I provide a more primitive condition of Condition 2:

**Condition C1.** Let $\hat{\sigma} = vec\left(\frac{\hat{e}'\hat{e}}{T}\right)$ and $\sigma = vec\left(Q^{-1\prime}SS'Q^{-1}\right)$. Then,

$$\sqrt{T}\left(\hat{\sigma} - \sigma\right) \rightarrow N(0, \Sigma_\sigma),$$

as $N, T \rightarrow \infty$ and $\sqrt{T}/N \rightarrow 0$.

This condition in conventional structural VARs is proven in Lütkepohl (2005) based on the assumption of Gaussian VAR errors with the Cholesky identification. If I simply apply this results in our model, the explicit form of $\Sigma_\sigma$ is given by:

$$\Sigma_\sigma = 2\left[(Q^{-1\prime}SS'Q^{-1})^{-1} \otimes (Q^{-1\prime}SS'Q^{-1})^{-1}\right]^{-1}.$$

The difference in our context is that our $\hat{e}$ involves factor estimation errors, however, as in Lemma B1, it is shown that the terms associated with factor estimation errors are at most $O_p(\sqrt{T}\delta^{-2})$ and negligible under $\sqrt{T}/N \rightarrow 0$. I next need a bootstrap counterpart of Condition C1.

**Condition C1-BT.**

a. Let $\hat{\sigma}^* = vec\left(H^{*-1}\frac{\hat{e}^{*\prime}\hat{e}^*}{T}H^{*\prime-1}\right)$ and $\widetilde{\sigma} = vec\left(\hat{S}\hat{S}'\right)$. Then,

$$\sqrt{T}\Sigma_\sigma^{*-1/2}\left(\hat{\sigma}^* - \widetilde{\sigma}\right) \rightarrow N(0, I_{r^2}),$$

in probability, as $N, T \rightarrow \infty$ and $\sqrt{T}/N \rightarrow 0$. Moreover, $p\lim(\Sigma_\sigma^*) = \Sigma_\sigma$.

b. Let $\hat{\sigma}^{**} = vec\left(\frac{\hat{e}^{**\prime}\hat{e}^{**}}{T}\right)$. Then,

$$\sqrt{T}\Sigma_\sigma^{*-1/2}\left(\hat{\sigma}^* - \widetilde{\sigma}\right) \rightarrow N(0, I_{r^2}),$$

in probability, as $N, T \rightarrow \infty$. Moreover, $p\lim(\Sigma_\sigma^{**}) = \Sigma_\sigma$.

The probability limits of $\Sigma_\sigma^*$ and $\Sigma_\sigma^{**}$ are similarly derived as:

$$p\lim 2\left[(\hat{S}\hat{S}')^{-1} \otimes (\hat{S}\hat{S}')^{-1}\right]^{-1} = 2\left[(Q^{-1\prime}SS'Q^{-1})^{-1} \otimes (Q^{-1\prime}SS'Q^{-1})^{-1}\right]^{-1},$$

so that the stated conditions are valid. Conditions C1-BT(a) and C1-BT(b) correspond to Conditions BT3(a) and BT3(b) respectively. Hence given that Conditions C1 and C1-BT hold, Conditions 2 and BT3 are straightforward if a simple Choleskey identification (ID3) is used (See Lütkepohl, 2005, Proposition C15(3)). In this case, a particular form of $\Sigma_S$ is given by:

$$\Sigma_S = G\Sigma_\sigma G',$$

with

$$G = [(I_{r^2} + K_{r^2 r^2})(S \otimes I_r)]^{-1},$$

where $K_{mn}$ is a commutation matrix such that $vec(A') = K_{mn} vec(A)$ for an $m \times n$ matrix $A$ in general. When ID1 or ID2 schemes are used, $\hat{S}$ involves the short-run or long-run reduced-form IRF estimates. However, if the following conditions hold, the asymptotic distributions of $\hat{S}$ estimated based on ID1 and ID2 are identical to that of ID3.

**Condition C2.** Let $\psi$ and $S$ be $r \times r$ fixed matrices respectively and the multiplication $\psi S$ is a lower triangular matrix with positive diagonal elements. Then $Chol(\hat{\psi}\hat{S}\hat{S}'\hat{\psi}')$ and $\hat{\psi}\hat{S}$ scaled by $\sqrt{T}$ have the same limit distribution as $N, T \to \infty$ and $\sqrt{T}/N \to 0$.

Note that $\psi$ is interpreted as the short-run or long-run reduced-form IRFs. The bootstrap counterparts of Condition C2 are given the following Condition C2-BT.

**Condition C2-BT.**

a. The bootstrap estimates $Chol(\hat{\psi}^* \hat{S}^* \hat{S}^{*\prime} \hat{\psi}^{*\prime})$ and $\hat{\psi}^* \hat{S}^*$ have the same limit distribution in probability as $N, T \to \infty$ and $\sqrt{T}/N \to 0$.

b. The bootstrap estimates $Chol(\hat{\psi}^{**} \hat{S}^{**} \hat{S}^{**\prime} \hat{\psi}^{**\prime})$ and $\hat{\psi}^{**} \hat{S}^{**}$ have the same limit distribution in probability as $N, T \to \infty$.

In the bootstrap space, these are straightforward conditions. For part a, $\hat{\psi}^* \hat{S}^*$ is a consistent estimate for $\hat{\psi}\hat{S}$ since the object is structural (without rotations). For part b, construction of $\hat{\psi}^{**} \hat{S}^{**}$ does not involve any factor estimations thus rotations. There are a few remarks regarding this condition. First $\psi$ is used as a catalyzer only to make the argument in the Choleskey operator a lower triangular matrix, which is guaranteed by Assumptions 6(a) or 6(b). The common idea of ID1 and ID2 is to obtain $\hat{S}$ such that $\hat{S} = \hat{\psi}^{-1}\hat{\psi}\hat{S}$ given $\hat{\psi}$ is invertible. Second, since $\hat{\psi}\hat{S}$ is not exactly triangular in its estimate, the finite sample distributions can be contaminated. This effects on the finite sample coverage ratios are investigated in Monte Carlo simulations in Section 5.

## Appendix D: Bias-correction in the bootstrap procedures

For the simulation studies presented in this paper, I applied the following bias-correction procedure in the spirit of Kilian (1998), for the VAR parameter $\Phi$. The important difference for our setup from Kilian (1998) is to estimate the bias by using $\hat{\Phi}_j^* - H^* \hat{\Phi}_j H^{*-1}$ instead of $\hat{\Phi}_j^* - \hat{\Phi}_j$ $(j = 1, ..., p)$ in Procedure A. For Procedure B, the bias is simply $\hat{\Phi}_j^{**} - \hat{\Phi}_j$. This estimation should be straightforward to implement given the asymptotic results for the two-step PC estimates described in the main text.

1. In Procedure A, estimate the model and generate $R_b$ bootstrap replications $\hat{\Phi}^{*,k}$, $k = 1, 2, \cdots, R_b$. Then approximate the bias $B_j = E(\hat{\Phi}_j^* - H^*\hat{\Phi}_j H^{*-1})$ by $B_j^b = \frac{1}{R_b}\sum_{k=1}^{R_b}(\hat{\Phi}_j^{*,ib} - H^*\hat{\Phi}_j H^{*-1})$ where $H^*$ is estimated by regressing $\hat{F}_t^*$ on $F_t^*$. In Procedure B, the bias is $B_j^b = \frac{1}{R_b}\sum_{k=1}^{R_b}(\hat{\Phi}_j^{**,ib} - \hat{\Phi}_j)$.

2. Calculate the modulus of the largest eigenvalues of the companion matrix:

$$\begin{bmatrix} \hat{\Phi}_1 - B_1^b & \cdots & \hat{\Phi}_{p-1} - B_{p-1}^b & \hat{\Phi}_p - B_p^b \\ I_k & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & I_k & 0 \end{bmatrix},$$

   and if it is less than 1, construct the bias-corrected coefficient estimate $\widetilde{\Phi} = \hat{\Phi} - B^b$. If not, let $\widetilde{\Phi} = \hat{\Phi}$. This will preserve the stationarity of the generated process.

3. Generate the bias-corrected bootstrap replications for the IRFs by using $\hat{\Lambda}$, $\widetilde{\Phi}$, $\hat{e}_t$, and $\hat{u}_t$.

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## Table 1-a. Coverage properties of impulse response functions (Gaussian errors, 95% level)

ID1: short-run restriction

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 40 | 50 | 94.3 | 95.2 | 95.6 | 95.8 | 95.7 | 94.5 | 9.06 | 5.87 | 3.60 | 2.29 | 1.51 | 1.04 |
| | 40 | 200 | 94.6 | 95.8 | 95.7 | 95.8 | 95.3 | 95.1 | 8.98 | 5.87 | 3.69 | 2.41 | 1.62 | 1.15 |
| | 120 | 50 | 94.4 | 95.3 | 95.7 | 95.4 | 94.5 | 93.4 | 5.08 | 2.63 | 1.64 | 1.02 | 0.65 | 0.42 |
| | 120 | 200 | 92.9 | 95.9 | 97.2 | 97.1 | 96.6 | 95.6 | 3.41 | 2.09 | 1.43 | 0.93 | 0.61 | 0.41 |
| Proc. B | 40 | 50 | 92.4 | 95.2 | 95.8 | 94.4 | 92.7 | 91.2 | 4.36 | 3.03 | 1.89 | 1.31 | 0.91 | 0.69 |
| | 40 | 200 | 92.6 | 94.2 | 94.6 | 94.9 | 93.8 | 93.3 | 5.00 | 3.15 | 2.00 | 1.38 | 0.97 | 0.72 |
| | 120 | 50 | 82.3 | 91.9 | 95.5 | 96.1 | 95.5 | 93.2 | 1.78 | 1.09 | 0.77 | 0.55 | 0.38 | 0.28 |
| | 120 | 200 | 88.4 | 92.8 | 95.7 | 96.3 | 95.6 | 94.4 | 2.08 | 1.37 | 0.97 | 0.68 | 0.48 | 0.33 |
| Normal | 40 | 50 | 45.8 | 66.0 | 75.6 | 75.8 | 75.2 | 72.6 | 0.81 | 0.65 | 0.49 | 0.34 | 0.23 | 0.15 |
| | 40 | 200 | 52.9 | 66.0 | 75.9 | 77.9 | 77.3 | 74.9 | 0.84 | 0.67 | 0.50 | 0.36 | 0.25 | 0.16 |
| | 120 | 50 | 30.0 | 44.6 | 60.6 | 71.1 | 75.5 | 79.4 | 0.46 | 0.37 | 0.30 | 0.22 | 0.16 | 0.11 |
| | 120 | 200 | 34.3 | 44.3 | 57.4 | 66.9 | 73.7 | 77.9 | 0.49 | 0.39 | 0.32 | 0.24 | 0.17 | 0.12 |

ID2: long-run restriction

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 40 | 50 | 95.7 | 95.8 | 95.7 | 95.2 | 94.4 | 93.5 | 19.13 | 8.74 | 5.06 | 3.53 | 2.37 | 1.73 |
| | 40 | 200 | 94.8 | 95.0 | 95.1 | 95.5 | 94.9 | 94.1 | 16.42 | 7.72 | 4.69 | 3.18 | 2.17 | 1.58 |
| | 120 | 50 | 94.9 | 95.8 | 96.2 | 95.9 | 95.1 | 94.5 | 8.52 | 3.33 | 2.02 | 1.22 | 0.80 | 0.53 |
| | 120 | 200 | 94.8 | 96.3 | 96.9 | 96.7 | 96.3 | 95.0 | 6.06 | 2.59 | 1.80 | 1.17 | 0.80 | 0.54 |
| Proc. B | 40 | 50 | 97.2 | 96.4 | 95.2 | 94.1 | 92.4 | 91.6 | 10.48 | 3.52 | 1.80 | 1.19 | 0.86 | 0.64 |
| | 40 | 200 | 96.9 | 96.5 | 94.9 | 95.1 | 93.0 | 92.5 | 10.32 | 3.36 | 1.88 | 1.29 | 0.95 | 0.70 |
| | 120 | 50 | 95.4 | 91.6 | 93.5 | 94.6 | 94.2 | 92.7 | 3.76 | 1.14 | 0.78 | 0.55 | 0.38 | 0.28 |
| | 120 | 200 | 97.0 | 93.6 | 95.2 | 95.3 | 94.8 | 93.8 | 4.26 | 1.34 | 0.91 | 0.65 | 0.45 | 0.32 |
| Normal | 40 | 50 | 63.4 | 90.3 | 89.9 | 87.1 | 83.9 | 80.6 | 1.98 | 1.29 | 0.87 | 0.58 | 0.38 | 0.24 |
| | 40 | 200 | 65.3 | 92.2 | 92.8 | 90.0 | 86.3 | 83.0 | 1.88 | 1.23 | 0.85 | 0.58 | 0.37 | 0.24 |
| | 120 | 50 | 48.7 | 72.5 | 84.1 | 86.9 | 88.8 | 88.7 | 1.08 | 0.74 | 0.52 | 0.37 | 0.25 | 0.17 |
| | 120 | 200 | 53.7 | 75.6 | 84.5 | 87.7 | 89.0 | 88.7 | 1.14 | 0.76 | 0.54 | 0.38 | 0.26 | 0.18 |

ID3: contemporaneous restriction

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 40 | 50 | 89.2 | 95.6 | 92.4 | 93.8 | 94.2 | 94.5 | 1.08 | 1.19 | 0.89 | 0.64 | 0.45 | 0.32 |
| | 40 | 200 | 89.7 | 95.5 | 92.6 | 94.9 | 95.3 | 96.4 | 1.08 | 1.23 | 0.90 | 0.66 | 0.47 | 0.34 |
| | 120 | 50 | 90.3 | 96.9 | 95.1 | 92.9 | 92.6 | 93.1 | 0.68 | 0.73 | 0.54 | 0.34 | 0.20 | 0.12 |
| | 120 | 200 | 91.8 | 97.3 | 94.1 | 93.0 | 92.5 | 93.0 | 0.67 | 0.71 | 0.52 | 0.33 | 0.20 | 0.12 |
| Proc. B | 40 | 50 | 82.7 | 89.4 | 88.9 | 94.2 | 95.4 | 96.0 | 0.74 | 1.01 | 0.77 | 0.55 | 0.39 | 0.28 |
| | 40 | 200 | 82.9 | 89.3 | 87.5 | 93.1 | 95.2 | 96.2 | 0.75 | 1.04 | 0.75 | 0.55 | 0.41 | 0.29 |
| | 120 | 50 | 83.7 | 90.4 | 90.2 | 90.7 | 91.3 | 93.2 | 0.46 | 0.62 | 0.46 | 0.29 | 0.18 | 0.10 |
| | 120 | 200 | 85.6 | 91.9 | 90.5 | 91.3 | 92.8 | 93.4 | 0.45 | 0.59 | 0.44 | 0.28 | 0.17 | 0.10 |
| Normal | 40 | 50 | 94.9 | 93.3 | 89.7 | 88.8 | 87.9 | 87.0 | 1.07 | 1.09 | 0.69 | 0.38 | 0.21 | 0.11 |
| | 40 | 200 | 94.9 | 92.8 | 88.9 | 87.6 | 86.9 | 86.1 | 1.09 | 1.11 | 0.70 | 0.39 | 0.21 | 0.11 |
| | 120 | 50 | 95.0 | 92.0 | 91.8 | 91.4 | 89.3 | 89.2 | 0.63 | 0.65 | 0.44 | 0.25 | 0.13 | 0.07 |
| | 120 | 200 | 94.8 | 94.0 | 91.7 | 91.4 | 90.9 | 90.1 | 0.60 | 0.61 | 0.43 | 0.24 | 0.13 | 0.07 |

## Table 1-b. Coverage properties of impulse response functions (Gaussian errors, 85% level)

ID1: short-run restriction

| | | | Coverage Ratio (%) | | | | | | Length of C.I. (Median) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *T* | *N* | *h* =0 | 1 | 2 | 3 | 4 | 5 | *h* =0 | 1 | 2 | 3 | 4 | 5 |
| Proc. A | 40 | 50 | 79.2 | 84.6 | 86.4 | 85.1 | 83.1 | 81.6 | 3.01 | 2.01 | 1.24 | 0.84 | 0.58 | 0.42 |
| | 40 | 200 | 81.5 | 85.5 | 85.8 | 87.2 | 86.2 | 83.4 | 2.88 | 1.99 | 1.27 | 0.88 | 0.63 | 0.45 |
| | 120 | 50 | 76.3 | 82.9 | 87.0 | 88.0 | 87.8 | 86.2 | 1.73 | 1.00 | 0.67 | 0.44 | 0.30 | 0.20 |
| | 120 | 200 | 76.1 | 83.7 | 88.3 | 90.0 | 90.0 | 88.4 | 1.33 | 0.93 | 0.66 | 0.46 | 0.31 | 0.21 |
| Proc. B | 40 | 50 | 78.0 | 83.6 | 85.5 | 85.4 | 84.6 | 82.0 | 1.68 | 1.22 | 0.87 | 0.64 | 0.47 | 0.36 |
| | 40 | 200 | 77.1 | 84.6 | 85.7 | 85.7 | 84.8 | 83.3 | 1.77 | 1.26 | 0.88 | 0.65 | 0.49 | 0.36 |
| | 120 | 50 | 64.0 | 73.3 | 82.6 | 87.2 | 88.1 | 87.1 | 0.91 | 0.61 | 0.47 | 0.33 | 0.24 | 0.17 |
| | 120 | 200 | 66.4 | 74.3 | 82.0 | 86.3 | 88.4 | 88.4 | 1.02 | 0.71 | 0.52 | 0.38 | 0.27 | 0.19 |
| Normal | 40 | 50 | 45.8 | 66.0 | 75.6 | 75.8 | 75.2 | 72.6 | 0.81 | 0.65 | 0.49 | 0.34 | 0.23 | 0.15 |
| | 40 | 200 | 52.9 | 66.0 | 75.9 | 77.9 | 77.3 | 74.9 | 0.84 | 0.67 | 0.50 | 0.36 | 0.25 | 0.16 |
| | 120 | 50 | 30.0 | 44.6 | 60.6 | 71.1 | 75.5 | 79.4 | 0.46 | 0.37 | 0.30 | 0.22 | 0.16 | 0.11 |
| | 120 | 200 | 34.3 | 44.3 | 57.4 | 66.9 | 73.7 | 77.9 | 0.49 | 0.39 | 0.32 | 0.24 | 0.17 | 0.12 |

ID2: long-run restriction

| | | | Coverage Ratio (%) | | | | | | Length of C.I. (Median) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *T* | *N* | *h* =0 | 1 | 2 | 3 | 4 | 5 | *h* =0 | 1 | 2 | 3 | 4 | 5 |
| Proc. A | 40 | 50 | 82.5 | 84.6 | 84.4 | 85.5 | 82.8 | 81.4 | 6.07 | 2.82 | 1.71 | 1.21 | 0.83 | 0.61 |
| | 40 | 200 | 80.1 | 84.5 | 85.1 | 85.8 | 83.6 | 82.8 | 5.43 | 2.61 | 1.62 | 1.11 | 0.77 | 0.56 |
| | 120 | 50 | 80.2 | 81.9 | 85.0 | 87.6 | 87.1 | 85.4 | 3.10 | 1.26 | 0.80 | 0.53 | 0.34 | 0.23 |
| | 120 | 200 | 80.4 | 80.2 | 87.1 | 89.2 | 89.2 | 88.3 | 2.51 | 1.17 | 0.81 | 0.54 | 0.37 | 0.26 |
| Proc. B | 40 | 50 | 88.7 | 85.4 | 83.8 | 84.8 | 81.7 | 79.9 | 3.96 | 1.41 | 0.79 | 0.59 | 0.44 | 0.32 |
| | 40 | 200 | 88.0 | 85.6 | 83.7 | 83.5 | 82.0 | 80.5 | 3.71 | 1.30 | 0.82 | 0.61 | 0.45 | 0.34 |
| | 120 | 50 | 81.4 | 73.2 | 78.7 | 84.3 | 85.3 | 85.4 | 2.00 | 0.63 | 0.45 | 0.33 | 0.24 | 0.17 |
| | 120 | 200 | 85.8 | 75.9 | 83.1 | 86.5 | 88.3 | 87.5 | 2.11 | 0.67 | 0.50 | 0.36 | 0.26 | 0.18 |
| Normal | 40 | 50 | 48.9 | 78.1 | 80.8 | 79.8 | 77.4 | 75.4 | 1.46 | 0.95 | 0.64 | 0.42 | 0.28 | 0.17 |
| | 40 | 200 | 53.7 | 83.0 | 85.6 | 83.8 | 80.9 | 77.6 | 1.38 | 0.91 | 0.63 | 0.42 | 0.27 | 0.17 |
| | 120 | 50 | 37.8 | 60.8 | 69.9 | 76.2 | 79.1 | 79.0 | 0.79 | 0.54 | 0.38 | 0.27 | 0.18 | 0.12 |
| | 120 | 200 | 40.2 | 61.3 | 70.7 | 76.6 | 79.4 | 80.3 | 0.83 | 0.56 | 0.40 | 0.28 | 0.19 | 0.13 |

ID3: contemporaneous restriction

| | | | Coverage Ratio (%) | | | | | | Length of C.I. (Median) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *T* | *N* | *h* =0 | 1 | 2 | 3 | 4 | 5 | *h* =0 | 1 | 2 | 3 | 4 | 5 |
| Proc. A | 40 | 50 | 80.7 | 86.5 | 84.2 | 86.5 | 87.7 | 87.4 | 0.80 | 0.88 | 0.64 | 0.44 | 0.28 | 0.19 |
| | 40 | 200 | 80.2 | 87.2 | 86.1 | 88.6 | 89.3 | 90.0 | 0.80 | 0.91 | 0.66 | 0.45 | 0.30 | 0.19 |
| | 120 | 50 | 81.5 | 91.3 | 89.9 | 88.5 | 87.6 | 87.8 | 0.51 | 0.55 | 0.40 | 0.24 | 0.14 | 0.08 |
| | 120 | 200 | 82.2 | 91.4 | 89.9 | 88.5 | 87.9 | 88.1 | 0.49 | 0.52 | 0.39 | 0.23 | 0.14 | 0.08 |
| Proc. B | 40 | 50 | 72.8 | 76.7 | 77.6 | 83.1 | 86.5 | 89.2 | 0.55 | 0.75 | 0.57 | 0.38 | 0.25 | 0.17 |
| | 40 | 200 | 71.1 | 78.4 | 78.5 | 83.5 | 88.3 | 91.0 | 0.56 | 0.76 | 0.55 | 0.39 | 0.27 | 0.18 |
| | 120 | 50 | 73.4 | 78.8 | 80.3 | 82.1 | 83.5 | 84.7 | 0.34 | 0.46 | 0.34 | 0.21 | 0.12 | 0.07 |
| | 120 | 200 | 75.0 | 79.9 | 81.4 | 82.3 | 85.0 | 86.3 | 0.33 | 0.43 | 0.32 | 0.20 | 0.12 | 0.07 |
| Normal | 40 | 50 | 86.2 | 83.1 | 80.7 | 79.6 | 80.6 | 81.1 | 0.78 | 0.80 | 0.51 | 0.28 | 0.15 | 0.08 |
| | 40 | 200 | 86.7 | 83.6 | 80.7 | 81.5 | 81.1 | 81.1 | 0.80 | 0.82 | 0.51 | 0.29 | 0.15 | 0.08 |
| | 120 | 50 | 83.2 | 81.9 | 81.8 | 82.4 | 82.0 | 81.9 | 0.46 | 0.48 | 0.33 | 0.19 | 0.10 | 0.05 |
| | 120 | 200 | 86.5 | 83.0 | 83.8 | 83.0 | 83.4 | 83.4 | 0.44 | 0.45 | 0.31 | 0.18 | 0.09 | 0.05 |

## Table 2-a. Coverage properties of impulse response functions (chi-squared errors, 95% level)

ID1: short-run restriction

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 40 | 50 | 93.0 | 94.2 | 94.7 | 94.7 | 93.7 | 93.6 | 12.08 | 7.57 | 4.27 | 2.66 | 1.72 | 1.20 |
| | 40 | 200 | 95.4 | 95.0 | 95.1 | 95.1 | 94.7 | 94.5 | 12.17 | 8.87 | 5.26 | 3.49 | 2.38 | 1.66 |
| | 120 | 50 | 95.2 | 95.2 | 95.4 | 95.8 | 95.7 | 95.8 | 11.48 | 6.82 | 3.87 | 2.33 | 1.42 | 0.88 |
| | 120 | 200 | 94.5 | 94.7 | 94.5 | 94.8 | 94.9 | 95.2 | 9.04 | 5.48 | 3.36 | 2.01 | 1.26 | 0.80 |
| Proc. B | 40 | 50 | 93.5 | 94.6 | 94.4 | 94.0 | 92.7 | 91.4 | 7.95 | 4.39 | 2.58 | 1.66 | 1.07 | 0.79 |
| | 40 | 200 | 95.2 | 96.3 | 96.4 | 96.2 | 95.5 | 94.6 | 7.82 | 4.33 | 2.67 | 1.75 | 1.21 | 0.86 |
| | 120 | 50 | 90.4 | 95.1 | 96.6 | 96.8 | 95.6 | 94.1 | 4.17 | 2.22 | 1.31 | 0.84 | 0.54 | 0.37 |
| | 120 | 200 | 92.8 | 95.4 | 96.5 | 96.4 | 95.6 | 94.9 | 3.81 | 2.12 | 1.35 | 0.85 | 0.56 | 0.38 |
| Normal | 40 | 50 | 62.9 | 82.7 | 86.9 | 85.1 | 82.6 | 80.8 | 1.66 | 1.15 | 0.78 | 0.53 | 0.36 | 0.22 |
| | 40 | 200 | 70.0 | 83.0 | 87.8 | 87.4 | 84.6 | 82.0 | 1.55 | 1.11 | 0.79 | 0.54 | 0.34 | 0.22 |
| | 120 | 50 | 44.6 | 65.4 | 76.5 | 85.1 | 88.3 | 88.2 | 0.90 | 0.64 | 0.48 | 0.35 | 0.24 | 0.16 |
| | 120 | 200 | 50.3 | 67.0 | 80.6 | 87.1 | 91.2 | 92.5 | 0.85 | 0.62 | 0.47 | 0.34 | 0.24 | 0.17 |

ID2: long-run restriction

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 40 | 50 | 94.7 | 94.4 | 94.6 | 95.2 | 95.1 | 93.9 | 22.59 | 9.94 | 5.48 | 3.54 | 2.30 | 1.58 |
| | 40 | 200 | 93.1 | 92.7 | 92.8 | 93.3 | 93.6 | 93.8 | 24.50 | 13.41 | 7.62 | 5.11 | 3.52 | 2.53 |
| | 120 | 50 | 95.5 | 94.5 | 94.9 | 94.9 | 95.1 | 95.0 | 20.48 | 8.48 | 4.78 | 2.90 | 1.70 | 1.04 |
| | 120 | 200 | 96.0 | 96.5 | 96.2 | 96.3 | 96.4 | 95.8 | 16.89 | 6.66 | 3.90 | 2.52 | 1.55 | 1.01 |
| Proc. B | 40 | 50 | 96.2 | 96.1 | 95.4 | 94.6 | 93.7 | 92.6 | 14.88 | 4.79 | 2.36 | 1.53 | 1.01 | 0.69 |
| | 40 | 200 | 94.6 | 94.2 | 94.2 | 93.8 | 92.8 | 92.2 | 15.34 | 4.83 | 2.54 | 1.66 | 1.13 | 0.78 |
| | 120 | 50 | 95.5 | 94.1 | 95.7 | 95.2 | 93.7 | 91.1 | 6.42 | 2.03 | 1.08 | 0.68 | 0.43 | 0.29 |
| | 120 | 200 | 97.6 | 96.7 | 97.2 | 96.8 | 96.0 | 94.9 | 6.37 | 1.95 | 1.11 | 0.74 | 0.50 | 0.33 |
| Normal | 40 | 50 | 71.6 | 92.3 | 92.9 | 88.9 | 85.0 | 82.0 | 2.73 | 1.58 | 1.00 | 0.66 | 0.41 | 0.24 |
| | 40 | 200 | 71.5 | 92.9 | 93.7 | 91.9 | 89.0 | 85.1 | 2.69 | 1.65 | 1.11 | 0.71 | 0.47 | 0.29 |
| | 120 | 50 | 55.9 | 81.3 | 88.9 | 92.5 | 92.2 | 90.8 | 1.53 | 0.93 | 0.62 | 0.41 | 0.27 | 0.17 |
| | 120 | 200 | 58.0 | 83.0 | 88.1 | 90.3 | 91.1 | 90.8 | 1.44 | 0.90 | 0.60 | 0.42 | 0.28 | 0.18 |

ID3: contemporaneous restriction

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 40 | 50 | 86.9 | 94.1 | 92.3 | 94.2 | 94.9 | 94.8 | 1.55 | 1.31 | 0.95 | 0.66 | 0.45 | 0.32 |
| | 40 | 200 | 86.6 | 94.4 | 92.8 | 94.9 | 95.8 | 96.6 | 1.49 | 1.23 | 0.88 | 0.61 | 0.44 | 0.31 |
| | 120 | 50 | 91.0 | 96.3 | 94.0 | 93.1 | 93.7 | 93.3 | 1.03 | 0.78 | 0.53 | 0.33 | 0.20 | 0.12 |
| | 120 | 200 | 92.3 | 96.5 | 95.7 | 95.0 | 94.3 | 94.5 | 1.05 | 0.81 | 0.57 | 0.35 | 0.21 | 0.13 |
| Proc. B | 40 | 50 | 82.7 | 89.1 | 89.4 | 93.9 | 95.0 | 95.9 | 1.17 | 1.11 | 0.80 | 0.57 | 0.40 | 0.29 |
| | 40 | 200 | 81.7 | 89.5 | 89.8 | 94.1 | 96.3 | 97.5 | 1.15 | 1.06 | 0.76 | 0.55 | 0.40 | 0.28 |
| | 120 | 50 | 86.7 | 89.7 | 88.1 | 89.2 | 90.7 | 92.7 | 0.81 | 0.67 | 0.47 | 0.30 | 0.18 | 0.10 |
| | 120 | 200 | 88.4 | 92.1 | 91.0 | 91.8 | 92.8 | 94.2 | 0.78 | 0.67 | 0.49 | 0.31 | 0.19 | 0.11 |
| Normal | 40 | 50 | 87.8 | 91.4 | 89.9 | 88.1 | 87.6 | 87.2 | 1.25 | 1.15 | 0.72 | 0.40 | 0.21 | 0.11 |
| | 40 | 200 | 87.1 | 91.4 | 89.5 | 89.0 | 87.8 | 86.5 | 1.19 | 1.09 | 0.68 | 0.38 | 0.20 | 0.11 |
| | 120 | 50 | 87.1 | 89.9 | 89.6 | 89.0 | 89.0 | 88.6 | 0.70 | 0.65 | 0.44 | 0.25 | 0.13 | 0.06 |
| | 120 | 200 | 85.6 | 92.7 | 93.0 | 91.7 | 91.4 | 91.8 | 0.70 | 0.65 | 0.47 | 0.27 | 0.14 | 0.07 |

## Table 2-b. Coverage properties of impulse response functions (chi-squared errors, 85% level)

ID1: short-run restriction

| | | | Coverage Ratio (%) | | | | | | Length of C.I. (Median) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *T* | *N* | *h* =0 | 1 | 2 | 3 | 4 | 5 | *h* =0 | 1 | 2 | 3 | 4 | 5 |
| Proc. A | 40 | 50 | 76.5 | 82.0 | 81.9 | 82.4 | 80.5 | 79.1 | 3.97 | 2.45 | 1.41 | 0.94 | 0.61 | 0.42 |
| | 40 | 200 | 80.0 | 83.9 | 83.6 | 82.4 | 81.2 | 80.5 | 3.97 | 2.92 | 1.76 | 1.20 | 0.82 | 0.58 |
| | 120 | 50 | 79.0 | 82.2 | 83.8 | 85.0 | 85.0 | 83.3 | 3.78 | 2.24 | 1.30 | 0.81 | 0.51 | 0.33 |
| | 120 | 200 | 80.8 | 83.9 | 84.2 | 85.1 | 85.1 | 84.9 | 2.97 | 1.84 | 1.13 | 0.70 | 0.44 | 0.30 |
| Proc. B | 40 | 50 | 76.7 | 82.2 | 84.0 | 83.0 | 80.9 | 78.8 | 2.71 | 1.59 | 0.97 | 0.68 | 0.47 | 0.35 |
| | 40 | 200 | 80.7 | 84.9 | 84.3 | 84.7 | 81.6 | 80.7 | 2.60 | 1.57 | 1.02 | 0.73 | 0.51 | 0.36 |
| | 120 | 50 | 70.8 | 77.7 | 83.9 | 87.2 | 86.8 | 84.8 | 1.62 | 0.91 | 0.60 | 0.42 | 0.29 | 0.19 |
| | 120 | 200 | 75.9 | 81.7 | 86.2 | 88.0 | 88.3 | 87.6 | 1.48 | 0.89 | 0.60 | 0.41 | 0.28 | 0.20 |
| Normal | 40 | 50 | 48.8 | 67.6 | 75.9 | 75.4 | 75.5 | 74.1 | 1.22 | 0.84 | 0.58 | 0.39 | 0.26 | 0.16 |
| | 40 | 200 | 56.8 | 70.5 | 74.9 | 76.7 | 76.8 | 75.2 | 1.14 | 0.81 | 0.58 | 0.40 | 0.25 | 0.16 |
| | 120 | 50 | 35.0 | 51.9 | 63.4 | 72.1 | 76.6 | 78.3 | 0.66 | 0.47 | 0.36 | 0.25 | 0.18 | 0.12 |
| | 120 | 200 | 38.9 | 53.8 | 66.8 | 75.6 | 80.3 | 83.2 | 0.63 | 0.46 | 0.35 | 0.25 | 0.18 | 0.12 |

ID2: long-run restriction

| | | | Coverage Ratio (%) | | | | | | Length of C.I. (Median) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *T* | *N* | *h* =0 | 1 | 2 | 3 | 4 | 5 | *h* =0 | 1 | 2 | 3 | 4 | 5 |
| Proc. A | 40 | 50 | 81.7 | 83.1 | 82.1 | 82.0 | 80.6 | 80.4 | 7.24 | 3.23 | 1.76 | 1.17 | 0.77 | 0.54 |
| | 40 | 200 | 81.7 | 80.3 | 81.0 | 80.8 | 80.5 | 79.0 | 7.60 | 4.28 | 2.51 | 1.71 | 1.16 | 0.84 |
| | 120 | 50 | 83.5 | 82.1 | 83.3 | 83.6 | 82.9 | 81.9 | 6.60 | 2.72 | 1.58 | 0.93 | 0.57 | 0.35 |
| | 120 | 200 | 85.8 | 86.1 | 86.9 | 87.5 | 87.2 | 86.2 | 5.35 | 2.22 | 1.33 | 0.83 | 0.53 | 0.35 |
| Proc. B | 40 | 50 | 87.4 | 84.6 | 82.0 | 81.5 | 77.9 | 77.8 | 5.07 | 1.71 | 0.92 | 0.63 | 0.42 | 0.29 |
| | 40 | 200 | 85.2 | 82.6 | 79.2 | 80.9 | 79.8 | 78.0 | 5.10 | 1.70 | 0.97 | 0.67 | 0.46 | 0.34 |
| | 120 | 50 | 81.8 | 81.0 | 83.6 | 84.8 | 84.3 | 81.9 | 2.61 | 0.86 | 0.50 | 0.34 | 0.23 | 0.16 |
| | 120 | 200 | 87.3 | 82.2 | 85.5 | 86.9 | 86.9 | 85.0 | 2.68 | 0.82 | 0.53 | 0.36 | 0.26 | 0.18 |
| Normal | 40 | 50 | 58.0 | 82.9 | 85.8 | 82.6 | 79.7 | 77.0 | 2.01 | 1.16 | 0.73 | 0.49 | 0.30 | 0.18 |
| | 40 | 200 | 58.5 | 84.5 | 87.1 | 86.6 | 83.8 | 79.1 | 1.97 | 1.21 | 0.81 | 0.52 | 0.34 | 0.21 |
| | 120 | 50 | 42.6 | 69.7 | 78.2 | 82.7 | 85.5 | 84.2 | 1.13 | 0.69 | 0.45 | 0.30 | 0.20 | 0.13 |
| | 120 | 200 | 44.6 | 69.3 | 77.0 | 80.3 | 81.8 | 82.5 | 1.06 | 0.66 | 0.44 | 0.31 | 0.21 | 0.13 |

ID3: contemporaneous restriction

| | | | Coverage Ratio (%) | | | | | | Length of C.I. (Median) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *T* | *N* | *h* =0 | 1 | 2 | 3 | 4 | 5 | *h* =0 | 1 | 2 | 3 | 4 | 5 |
| Proc. A | 40 | 50 | 77.0 | 86.3 | 85.7 | 86.2 | 86.5 | 86.7 | 1.14 | 0.97 | 0.66 | 0.43 | 0.28 | 0.18 |
| | 40 | 200 | 77.9 | 85.0 | 84.0 | 85.6 | 86.6 | 88.0 | 1.10 | 0.90 | 0.61 | 0.41 | 0.27 | 0.17 |
| | 120 | 50 | 82.3 | 89.2 | 87.5 | 86.9 | 87.4 | 86.9 | 0.75 | 0.58 | 0.38 | 0.23 | 0.13 | 0.07 |
| | 120 | 200 | 83.9 | 91.3 | 90.0 | 89.9 | 90.1 | 89.9 | 0.76 | 0.59 | 0.41 | 0.25 | 0.14 | 0.08 |
| Proc. B | 40 | 50 | 70.2 | 76.3 | 75.8 | 82.1 | 87.3 | 89.8 | 0.87 | 0.82 | 0.57 | 0.37 | 0.25 | 0.17 |
| | 40 | 200 | 72.3 | 76.1 | 76.9 | 83.8 | 87.5 | 89.7 | 0.85 | 0.77 | 0.55 | 0.37 | 0.25 | 0.16 |
| | 120 | 50 | 77.5 | 79.2 | 79.3 | 80.2 | 82.0 | 84.6 | 0.59 | 0.49 | 0.34 | 0.21 | 0.12 | 0.06 |
| | 120 | 200 | 80.2 | 83.3 | 82.0 | 84.3 | 86.2 | 86.9 | 0.58 | 0.49 | 0.36 | 0.22 | 0.13 | 0.07 |
| Normal | 40 | 50 | 76.5 | 79.2 | 79.2 | 79.7 | 79.8 | 81.1 | 0.92 | 0.85 | 0.53 | 0.29 | 0.16 | 0.08 |
| | 40 | 200 | 76.7 | 79.9 | 79.8 | 80.7 | 81.1 | 81.8 | 0.88 | 0.80 | 0.50 | 0.28 | 0.15 | 0.08 |
| | 120 | 50 | 73.6 | 79.6 | 80.8 | 80.2 | 80.9 | 81.0 | 0.52 | 0.48 | 0.33 | 0.19 | 0.09 | 0.05 |
| | 120 | 200 | 71.9 | 82.8 | 83.4 | 83.5 | 84.9 | 84.7 | 0.52 | 0.48 | 0.34 | 0.20 | 0.10 | 0.05 |

**Table 3. Coverage properties of Procedure A with small $N$ and persistent factors (Gaussian errors, 95% level)**

ID1

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 120 | 10 | 91.9 | 90.6 | 91.9 | 93.2 | 93.2 | 93.9 | 6.42 | 3.86 | 3.23 | 2.85 | 2.54 | 2.31 |
| | 120 | 30 | 92.3 | 92.1 | 93.8 | 94.4 | 95.1 | 95.2 | 3.88 | 2.70 | 2.50 | 2.31 | 2.11 | 1.95 |
| | 120 | 50 | 92.8 | 92.6 | 94.8 | 96.4 | 97.1 | 97.2 | 3.30 | 2.46 | 2.34 | 2.23 | 2.07 | 1.93 |
| Proc. B | 120 | 10 | 59.9 | 65.2 | 72.9 | 79.2 | 84.4 | 88.9 | 1.01 | 0.85 | 0.85 | 0.86 | 0.87 | 0.87 |
| | 120 | 30 | 73.7 | 76.3 | 82.7 | 87.0 | 90.0 | 91.7 | 1.09 | 0.94 | 1.00 | 0.99 | 0.99 | 0.98 |
| | 120 | 50 | 78.5 | 81.2 | 87.6 | 90.8 | 92.1 | 94.2 | 1.32 | 1.08 | 1.13 | 1.12 | 1.12 | 1.09 |
| Normal | 120 | 10 | 26.3 | 34.4 | 44.0 | 54.4 | 62.2 | 69.9 | 0.55 | 0.53 | 0.56 | 0.57 | 0.57 | 0.57 |
| | 120 | 30 | 35.0 | 42.5 | 51.9 | 59.6 | 67.7 | 73.9 | 0.58 | 0.53 | 0.53 | 0.54 | 0.55 | 0.54 |
| | 120 | 50 | 39.1 | 44.2 | 52.0 | 58.6 | 65.4 | 71.8 | 0.61 | 0.55 | 0.56 | 0.57 | 0.57 | 0.57 |

ID2

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 120 | 10 | 95.2 | 95.0 | 94.5 | 95.1 | 95.1 | 94.6 | 28.43 | 8.39 | 6.53 | 5.98 | 5.23 | 4.64 |
| | 120 | 30 | 96.9 | 96.6 | 97.1 | 97.4 | 97.1 | 96.9 | 21.96 | 9.27 | 7.75 | 7.28 | 6.90 | 6.47 |
| | 120 | 50 | 95.7 | 96.8 | 97.2 | 96.9 | 97.0 | 97.1 | 17.88 | 8.77 | 7.83 | 7.52 | 7.19 | 6.75 |
| Proc. B | 120 | 10 | 76.3 | 75.3 | 73.4 | 76.9 | 80.5 | 84.0 | 7.21 | 1.54 | 0.78 | 0.70 | 0.71 | 0.71 |
| | 120 | 30 | 78.5 | 87.3 | 90.3 | 89.2 | 91.1 | 92.5 | 9.51 | 2.91 | 1.37 | 1.05 | 0.97 | 0.93 |
| | 120 | 50 | 88.3 | 97.3 | 92.1 | 90.1 | 90.4 | 91.7 | 10.37 | 3.57 | 1.73 | 1.22 | 1.10 | 1.06 |
| Normal | 120 | 10 | 35.3 | 76.2 | 85.5 | 90.1 | 92.6 | 92.2 | 2.33 | 1.73 | 1.52 | 1.33 | 1.19 | 1.04 |
| | 120 | 30 | 45.5 | 80.2 | 91.5 | 94.3 | 95.4 | 95.7 | 2.64 | 2.11 | 1.86 | 1.65 | 1.49 | 1.35 |
| | 120 | 50 | 47.2 | 77.8 | 90.3 | 93.2 | 95.2 | 95.7 | 2.50 | 2.07 | 1.85 | 1.64 | 1.48 | 1.34 |

ID3

| | $T$ | $N$ | Coverage Ratio (%) $h$=0 | 1 | 2 | 3 | 4 | 5 | Length of C.I. (Median) $h$=0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Proc. A | 120 | 10 | 83.4 | 91.5 | 96.0 | 95.9 | 95.4 | 94.3 | 0.80 | 0.87 | 0.93 | 0.89 | 0.82 | 0.72 |
| | 120 | 30 | 84.3 | 92.3 | 95.3 | 95.5 | 95.1 | 94.4 | 0.78 | 0.83 | 0.89 | 0.87 | 0.80 | 0.71 |
| | 120 | 50 | 84.3 | 92.5 | 96.1 | 96.4 | 95.6 | 95.8 | 0.77 | 0.82 | 0.88 | 0.86 | 0.79 | 0.71 |
| Proc. B | 120 | 10 | 74.1 | 82.1 | 85.7 | 88.2 | 90.2 | 91.2 | 0.43 | 0.61 | 0.73 | 0.74 | 0.68 | 0.60 |
| | 120 | 30 | 75.3 | 83.2 | 87.5 | 88.6 | 88.2 | 89.4 | 0.42 | 0.58 | 0.70 | 0.71 | 0.65 | 0.59 |
| | 120 | 50 | 76.0 | 84.6 | 89.0 | 90.6 | 91.8 | 92.3 | 0.42 | 0.58 | 0.70 | 0.70 | 0.66 | 0.58 |
| Normal | 120 | 10 | 87.2 | 85.5 | 87.4 | 87.7 | 87.7 | 87.2 | 0.62 | 0.66 | 0.72 | 0.68 | 0.59 | 0.49 |
| | 120 | 30 | 87.6 | 87.0 | 88.0 | 87.6 | 87.0 | 86.1 | 0.61 | 0.64 | 0.69 | 0.65 | 0.58 | 0.48 |
| | 120 | 50 | 86.7 | 87.9 | 90.2 | 89.9 | 89.4 | 88.7 | 0.58 | 0.62 | 0.69 | 0.65 | 0.57 | 0.47 |

## Table 4: Coverage properties of Procedure A under various bootstrap confidence intervals (Gaussian errors, ID1, 95% level)

| | | | Coverage Ratio (%) | | | | | | Length of C.I. (Median) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *T* | *N* | *h* =0 | 1 | 2 | 3 | 4 | 5 | h=0 | 1 | 2 | 3 | 4 | 5 |
| PER-H | 40 | 50 | 92.3 | 94.4 | 94.5 | 94.8 | 94.4 | 93.2 | 9.52 | 6.05 | 3.74 | 2.50 | 1.66 | 1.12 |
| | 40 | 200 | 95.0 | 95.3 | 95.4 | 95.5 | 94.7 | 94.7 | 8.77 | 5.73 | 3.66 | 2.38 | 1.62 | 1.12 |
| | 120 | 50 | 94.6 | 95.5 | 95.5 | 96.1 | 95.9 | 94.9 | 9.12 | 5.95 | 3.79 | 2.53 | 1.69 | 1.20 |
| | 120 | 200 | 92.5 | 96.3 | 97.3 | 97.1 | 96.2 | 95.3 | 3.24 | 2.11 | 1.50 | 0.99 | 0.65 | 0.44 |
| PER-E | 40 | 50 | 99.7 | 100.0 | 100.0 | 100.0 | 99.7 | 99.4 | 9.52 | 6.05 | 3.74 | 2.50 | 1.66 | 1.12 |
| | 40 | 200 | 99.8 | 100.0 | 100.0 | 99.7 | 99.3 | 99.0 | 8.77 | 5.73 | 3.66 | 2.38 | 1.62 | 1.12 |
| | 120 | 50 | 99.8 | 100.0 | 100.0 | 100.0 | 99.9 | 99.6 | 9.12 | 5.95 | 3.79 | 2.53 | 1.69 | 1.20 |
| | 120 | 200 | 89.9 | 96.6 | 98.4 | 98.8 | 99.1 | 98.9 | 3.24 | 2.11 | 1.50 | 0.99 | 0.65 | 0.44 |
| PER-T | 40 | 50 | 86.7 | 90.0 | 89.6 | 90.7 | 87.5 | 86.9 | 9.88 | 6.46 | 3.76 | 2.85 | 1.97 | 1.61 |
| | 40 | 200 | 90.2 | 90.9 | 91.8 | 91.5 | 90.7 | 89.3 | 8.22 | 5.79 | 3.76 | 2.71 | 1.82 | 1.51 |
| | 120 | 50 | 90.3 | 91.8 | 91.5 | 91.2 | 89.9 | 88.1 | 8.44 | 6.06 | 4.22 | 2.61 | 1.98 | 1.57 |
| | 120 | 200 | 79.7 | 86.5 | 90.7 | 93.4 | 94.3 | 95.1 | 2.75 | 2.12 | 1.57 | 1.11 | 0.73 | 0.51 |

## Table 5: Coverage properties of Procedure A without bias corrections (Gaussian errors, ID1, 95% level)

| | | | Coverage Ratio (%) | | | | | | Length of C.I. (Median) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *T* | *N* | *h* =0 | 1 | 2 | 3 | 4 | 5 | *h* =0 | 1 | 2 | 3 | 4 | 5 |
| diag 0.4 | 40 | 50 | 94.0 | 93.7 | 93.0 | 90.8 | 87.2 | 84.3 | 9.48 | 4.30 | 2.00 | 1.09 | 0.60 | 0.37 |
| | 40 | 200 | 95.6 | 95.4 | 94.4 | 93.3 | 89.4 | 86.4 | 8.59 | 4.28 | 2.11 | 1.17 | 0.66 | 0.39 |
| | 120 | 50 | 93.9 | 94.2 | 94.5 | 94.9 | 92.9 | 91.1 | 4.88 | 2.15 | 1.22 | 0.73 | 0.46 | 0.28 |
| | 120 | 200 | 93.7 | 95.2 | 95.9 | 95.5 | 94.4 | 92.5 | 3.72 | 1.83 | 1.17 | 0.68 | 0.41 | 0.26 |
| diag 0.7 | 40 | 50 | 93.8 | 91.6 | 91.1 | 89.5 | 86.5 | 83.1 | 9.02 | 5.19 | 3.75 | 2.84 | 2.12 | 1.62 |
| | 40 | 200 | 95.7 | 95.0 | 94.2 | 92.9 | 90.2 | 86.6 | 7.74 | 4.76 | 3.54 | 2.65 | 2.07 | 1.63 |
| | 120 | 50 | 91.3 | 91.3 | 91.9 | 91.8 | 91.6 | 91.3 | 3.11 | 2.16 | 1.88 | 1.61 | 1.42 | 1.23 |
| | 120 | 200 | 91.4 | 91.4 | 92.1 | 93.3 | 93.4 | 93.1 | 2.61 | 1.99 | 1.85 | 1.65 | 1.42 | 1.27 |

**Table 6-a. Coverage properties for calibrated model by Procedure A (full sample, 95% level)**

| | | $h$=0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| Price | coverage (%) | 94.6 | 94.7 | 96.3 | 95.8 | 95.2 | 94.5 | 94.3 | 95.4 |
| | length | 4.11 | 1.55 | 0.77 | 0.80 | 0.97 | 1.16 | 1.15 | 1.09 |
| 10-yr Tbill rate | coverage (%) | 94.6 | 94.8 | 95.0 | 94.6 | 94.5 | 94.8 | 94.3 | 94.3 |
| | length | 6.79 | 1.25 | 2.01 | 3.41 | 3.34 | 3.10 | 2.64 | 2.16 |
| Production | coverage (%) | 94.5 | 94.3 | 94.7 | 94.3 | 94.0 | 94.4 | 94.4 | 95.0 |
| | length | 2.22 | 8.38 | 10.04 | 9.63 | 6.50 | 4.24 | 2.72 | 1.59 |
| Unemployment rate | coverage (%) | 93.5 | 94.4 | 94.6 | 94.6 | 94.5 | 94.3 | 94.5 | 94.6 |
| | length | 0.27 | 7.32 | 9.64 | 9.72 | 7.02 | 4.79 | 3.28 | 1.82 |

**Table 6-b. Coverage properties for calibrated model by Procedure A (post 1984, 95% level)**

| | | $h$=0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| Price | coverage (%) | 93.7 | 93.8 | 92.5 | 97.0 | 95.7 | 96.4 | 96.9 | 96.6 |
| | length | 2.04 | 0.42 | 0.37 | 0.35 | 0.44 | 0.26 | 0.21 | 0.21 |
| 10-yr Tbill rate | coverage (%) | 94.7 | 93.7 | 94.7 | 96.1 | 94.0 | 95.4 | 97.3 | 96.2 |
| | length | 2.13 | 1.25 | 1.06 | 1.01 | 0.97 | 0.73 | 0.58 | 0.53 |
| Production | coverage (%) | 90.8 | 92.7 | 94.8 | 95.0 | 92.9 | 93.3 | 94.6 | 93.4 |
| | length | 1.00 | 1.15 | 0.94 | 0.89 | 0.84 | 0.67 | 0.52 | 0.48 |
| Unemployment rate | coverage (%) | 91.8 | 93.5 | 93.0 | 93.5 | 94.5 | 94.0 | 93.5 | 92.9 |
| | length | 0.16 | 1.25 | 1.03 | 0.97 | 0.91 | 0.74 | 0.58 | 0.55 |

## Table 7. Comparison between Procedures A and B (95% level)

### Coverage ratios

| | *Proc* | *h* =0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| Price | A | 94.9 | 94.7 | 94.8 | 94.8 | 95.5 | 95.8 | 95.7 | 95.9 |
| | B | 81.7 | 77.5 | 80.6 | 90.4 | 89.1 | 87.0 | 89.0 | 88.9 |
| 10-yr Tbill rate | A | 92.6 | 94.7 | 96.1 | 96.5 | 98.3 | 96.0 | 94.8 | 96.5 |
| | B | 63.5 | 72.1 | 77.1 | 66.5 | 92.7 | 88.4 | 85.9 | 88.0 |
| Production | A | 93.2 | 94.2 | 96.9 | 96.7 | 98.0 | 96.1 | 96.4 | 95.1 |
| | B | 64.9 | 70.5 | 74.8 | 67.0 | 88.9 | 88.0 | 86.5 | 88.0 |
| Unemployment rate | A | 94.5 | 94.1 | 96.2 | 97.0 | 98.0 | 96.8 | 96.9 | 96.6 |
| | B | 67.9 | 69.9 | 72.2 | 69.0 | 82.8 | 87.2 | 87.1 | 88.2 |

### Length of C.I. (median)

| | *Proc* | *h* =0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| Price | A | 2.37 | 1.66 | 1.21 | 1.12 | 1.38 | 1.06 | 0.83 | 0.85 |
| | B | 0.83 | 0.63 | 0.46 | 0.41 | 0.44 | 0.31 | 0.22 | 0.21 |
| 10-yr Tbill rate | A | 8.61 | 6.05 | 3.66 | 4.01 | 3.13 | 2.79 | 2.69 | 2.53 |
| | B | 3.25 | 2.58 | 1.50 | 1.72 | 1.04 | 0.81 | 0.75 | 0.62 |
| Production | A | 7.41 | 5.11 | 3.01 | 3.51 | 2.57 | 2.27 | 2.32 | 2.12 |
| | B | 2.93 | 2.27 | 1.33 | 1.50 | 0.92 | 0.70 | 0.66 | 0.60 |
| Unemployment rate | A | 7.59 | 5.18 | 3.09 | 3.48 | 2.68 | 2.31 | 2.32 | 2.15 |
| | B | 2.97 | 2.30 | 1.40 | 1.56 | 1.08 | 0.77 | 0.70 | 0.62 |

## Table 8. Data description (disaggregate data)

1) Monthly data

| | mnemonic | transform | data description |
|---|---|---|---|
| 1 | IPS13 | 5 | INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS |
| 2 | IPS18 | 5 | INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS |
| 3 | IPS25 | 5 | INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT |
| 4 | IPS34 | 5 | INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS |
| 5 | IPS38 | 5 | INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS |
| 6 | IPS43 | 5 | INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC) |
| 7 | IPS306 | 5 | INDUSTRIAL PRODUCTION INDEX - FUELS |
| 8 | PMP | 1 | NAPM PRODUCTION INDEX (PERCENT) |
| 9 | UTL11 | 1 | CAPACITY UTILIZATION - MANUFACTURING (SIC) |
| 10 | CES277R | 5 | REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - CONSTRUCTION (CES277/PI071) |
| 11 | CES278 R | 5 | REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - MFG (CES278/PI071) |
| 12 | CES006 | 5 | EMPLOYEES, NONFARM - MINING |
| 13 | CES011 | 5 | EMPLOYEES, NONFARM - CONSTRUCTION |
| 14 | CES017 | 5 | EMPLOYEES, NONFARM - DURABLE GOODS |
| 15 | CES033 | 5 | EMPLOYEES, NONFARM - NONDURABLE GOODS |
| 16 | CES046 | 5 | EMPLOYEES, NONFARM - SERVICE-PROVIDING |
| 17 | CES048 | 5 | EMPLOYEES, NONFARM - TRADE, TRANSPORT, UTILITIES |
| 18 | CES049 | 5 | EMPLOYEES, NONFARM - WHOLESALE TRADE |
| 19 | CES053 | 5 | EMPLOYEES, NONFARM - RETAIL TRADE |
| 20 | CES088 | 5 | EMPLOYEES, NONFARM - FINANCIAL ACTIVITIES |
| 21 | CES140 | 5 | EMPLOYEES, NONFARM - GOVERNMENT |
| 22 | LHEL | 2 | INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA) |
| 23 | LHELX | 2 | EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF |
| 24 | LHNAG | 5 | CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA) |
| 25 | LHUR | 2 | UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA) |
| 26 | LHU680 | 2 | UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA) |
| 27 | LHU5 | 5 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA) |
| 28 | LHU14 | 5 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA) |
| 29 | LHU15 | 5 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA) |
| 30 | LHU26 | 5 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA) |
| 31 | LHU27 | 5 | UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA) |
| 32 | CES151 | 1 | AVG WKLY HOURS, PROD WRKRS, NONFARM - GOODS-PRODUCING |
| 33 | CES155 | 2 | AVG WKLY OVERTIME HOURS, PROD WRKRS, NONFARM - MFG |
| 34 | HSNE | 4 | HOUSING STARTS:NORTHEAST (THOUS.U.)S.A. |
| 35 | HSMW | 4 | HOUSING STARTS:MIDWEST(THOUS.U.)S.A. |
| 36 | HSSOU | 4 | HOUSING STARTS:SOUTH (THOUS.U.)S.A. |
| 37 | HSWST | 4 | HOUSING STARTS:WEST (THOUS.U.)S.A. |
| 38 | FYFF | 2 | INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA) |
| 39 | FYGM3 | 2 | INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA) |
| 40 | FYGT1 | 2 | INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA) |
| 41 | FYGT10 | 2 | INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA) |
| 42 | FYBAAC | 2 | BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM) |
| 43 | Sfygm6 | 1 | fygm6-fygm3 |
| 44 | Sfygt1 | 1 | fygt1-fygm3 |
| 45 | Sfygt10 | 1 | fygt10-fygm3 |
| 46 | sFYAAAC | 1 | FYAAAC-Fygt10 |
| 47 | sFYBAAC | 1 | FYBAAC-Fygt10 |
| 48 | FM1 | 6 | MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA) |
| 49 | MZMSL | 6 | MZM (SA) FRB St. Louis |
| 50 | FM2 | 6 | MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$, |
| 51 | FMFBA | 6 | MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA) |
| 52 | FMRRA | 6 | DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA) |
| 53 | FMRNBA | 6 | DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA) |
| 54 | BUSLOAN | 6 | Commercial and Industrial Loans at All Commercial Banks (FRED) Billions $ (SA) |
| 55 | CCINRV | 6 | CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19) |
| 56 | PSCCOMF | 5 | Real SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100) (PSCCOM/PCEPILFE) |
| 57 | PW561R | 5 | PPI Crude (Relative to Core PCE) (pw561/PCEPiLFE) |
| 58 | PMCP | 1 | NAPM COMMODITY PRICES INDEX (PERCENT) |
| 59 | EXRUS | 5 | UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.) |
| 60 | EXRSW | 5 | FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$) |

| | | | |
|---|---|---|---|
| 61 | EXRJAN | 5 | FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$) |
| 62 | EXRUK | 5 | FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND) |
| 63 | EXRCAN | 5 | FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$) |
| 64 | FSPCOM | 5 | S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10) |
| 65 | FSPIN | 5 | S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10) |
| 66 | FSDXP | 2 | S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM) |
| 67 | FSPXE | 2 | S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA) |
| 68 | FSDJ | 5 | COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE |
| 69 | HHSNTN | 2 | U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83) |
| 70 | PMI | 1 | PURCHASING MANAGERS' INDEX (SA) |
| 71 | PMNO | 1 | NAPM NEW ORDERS INDEX (PERCENT) |
| 72 | PMDEL | 1 | NAPM VENDOR DELIVERIES INDEX (PERCENT) |
| 73 | PMNV | 1 | NAPM INVENTORIES INDEX (PERCENT) |
| 74 | MOCMQ | 5 | NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1996 DOLLARS (BCI) |
| 75 | MSONDQ | 5 | NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI) |

2) Quarterly data

| | | | |
|---|---|---|---|
| 76 | GDP253 | 5 | Real Personal Consumption Expenditures - Durable Goods , Quantity Index (2000= |
| 77 | GDP254 | 5 | Real Personal Consumption Expenditures - Nondurable Goods, Quantity Index (200 |
| 78 | GDP255 | 5 | Real Personal Consumption Expenditures - Services, Quantity Index (2000=100) , |
| 79 | GDP259 | 5 | Real Gross Private Domestic Investment - Nonresidential - Structures, Quantity |
| 80 | GDP260 | 5 | Real Gross Private Domestic Investment - Nonresidential - Equipment & Software |
| 81 | GDP261 | 5 | Real Gross Private Domestic Investment - Residential, Quantity Index (2000=100 |
| 82 | GDP263 | 5 | Real Exports, Quantity Index (2000=100) , SAAR |
| 83 | GDP264 | 5 | Real Imports, Quantity Index (2000=100) , SAAR |
| 84 | GDP266 | 5 | Real Government Consumption Expenditures & Gross Investment - Federal, Quantit |
| 85 | GDP267 | 5 | Real Government Consumption Expenditures & Gross Investment - State & Local, Q |
| 86 | LBOUT | 5 | OUTPUT PER HOUR ALL PERSONS: BUSINESS SEC(1982=100,SA) |
| 87 | LBPUR7 | 5 | REAL COMPENSATION PER HOUR,EMPLOYEES:NONFARM BUSINESS(82=100,SA) |
| 88 | LBMNU | 5 | HOURS OF ALL PERSONS: NONFARM BUSINESS SEC (1982=100,SA) |
| 89 | LBLCPU | 5 | UNIT LABOR COST: NONFARM BUSINESS SEC (1982=100,SA) |
| 90 | GDP274_1 | 6 | Motor vehicles and parts Price Index |
| 91 | GDP274_2 | 6 | Furniture and household equipment Price Index |
| 92 | GDP274_3 | 6 | Other Price Index |
| 93 | GDP275_1 | 6 | Food Price Index |
| 94 | GDP275_2 | 6 | Clothing and shoes Price Index |
| 95 | GDP275_3 | 6 | Gasoline, fuel oil, and other energy goods Price Index |
| 96 | GDP275_4 | 6 | Other Price Index |
| 97 | GDP276_1 | 6 | Housing Price Index |
| 98 | GDP276_3 | 6 | Electricity and gas Price Index |
| 99 | GDP276_4 | 6 | Other household operation Price Index |
| 100 | GDP276_5 | 6 | Transportation Price Index |
| 101 | GDP276_6 | 6 | Medical care Price Index |
| 102 | GDP276_7 | 6 | Recreation Price Index |
| 103 | GDP276_8 | 6 | Other Price Index |
| 104 | GDP280A | 6 | Structures |
| 105 | GDP281A | 6 | Equipment and software Price Index |
| 106 | GDP282A | 6 | Residential Price Index |
| 107 | GDP284A | 6 | Exports Price Index |
| 108 | GDP285A | 6 | Imports Price Index |
| 109 | GDP287A | 6 | Federal Price Index |
| 110 | GDP288A | 6 | State and local Price Index |

Note : Transformation code indicates followings: 1-no transformation, 2-first difference, 3-second difference, 4-logarithms, 5-first difference after taking logarithms, 6-second difference after taking logarithms.

## Table 9. Data description (aggregate data)

1) Monthly data

| | mnemonic | transform | data description |
|---|---|---|---|
| 1 | IPS10 | 5 | INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX |
| 2 | IPS43 | 5 | INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC) |
| 3 | UTL11 | 1 | CAPACITY UTILIZATION - MANUFACTURING (SIC) |
| 4 | CES278 | 6 | AVG HRLY EARNINGS, PROD WRKRS, NONFARM - MFG |
| 5 | CES002 | 5 | EMPLOYEES, NONFARM - TOTAL PRIVATE |
| 6 | LHEL | 2 | INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA) |
| 7 | LHUR | 2 | UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA) |
| 8 | LHU680 | 2 | UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA) |
| 9 | CES151 | 1 | AVG WKLY HOURS, PROD WRKRS, NONFARM - GOODS-PRODUCING |
| 10 | CES155 | 2 | AVG WKLY OVERTIME HOURS, PROD WRKRS, NONFARM - MFG |
| 11 | HSBR | 4 | HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR) |
| 12 | HSFR | 4 | HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA |
| 13 | FYFF | 2 | INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA) |
| 14 | FYGM3 | 2 | INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA) |
| 15 | FYGT10 | 2 | INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA) |
| 16 | FYAAAC | 2 | BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM) |
| 17 | Sfygt10 | 1 | fygt10-fygm3 |
| 18 | FMFBA | 6 | MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA) |
| 19 | FMRRA | 6 | DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA) |
| 20 | BUSLOANS | 6 | Commercial and Industrial Loans at All Commercial Banks (FRED) Billions $ (SA) |
| 21 | CCINRV | 6 | CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19) |
| 22 | PWFSA | 6 | PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA) |
| 23 | PSCCOM | 6 | SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100) |
| 24 | PW561 | 6 | PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA) |
| 25 | EXRUS | 5 | UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.) |
| 26 | FSPCOM | 5 | S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10) |
| 27 | HHSNTN | 2 | U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83) |
| 28 | PMI | 1 | PURCHASING MANAGERS' INDEX (SA) |
| 29 | PMNO | 1 | NAPM NEW ORDERS INDEX (PERCENT) |
| 30 | PMDEL | 1 | NAPM VENDOR DELIVERIES INDEX (PERCENT) |
| 31 | PMNV | 1 | NAPM INVENTORIES INDEX (PERCENT) |

2) Quarterly data

| | | | |
|---|---|---|---|
| 32 | GDP251 | 5 | Real Gross Domestic Product, Quantity Index (2000=100) , SAAR |
| 33 | GDP252 | 5 | Real Personal Consumption Expenditures, Quantity Index (2000=100) , SAAR |
| 34 | GDP256 | 5 | Real Gross Private Domestic Investment, Quantity Index (2000=100) , SAAR |
| 35 | GDP263 | 5 | Real Exports, Quantity Index (2000=100) , SAAR |
| 36 | GDP264 | 5 | Real Imports, Quantity Index (2000=100) , SAAR |
| 37 | GDP265 | 5 | Real Government Consumption Expenditures & Gross Investment, Quantity Index (2 |
| 38 | GDP272 | 6 | Gross Domestic Product, Price Index (2000=100) , SAAR |
| 39 | GDP273 | 6 | Personal Consumption Expenditures, Price Index (2000=100) , SAAR |
| 40 | GDP275_4 | 6 | Other Price Index |
| 41 | GDP277 | 6 | Gross Private Domestic Investment, Price Index (2000=100) , SAAR |
| 42 | GDP284 | 6 | Exports, Price Index (2000=100) , SAAR |
| 43 | GDP285 | 6 | Imports, Price Index (2000=100) , SAAR |
| 44 | GDP286 | 6 | Government Consumption Expenditures & Gross Investment, Price Index (2000=100) |
| 45 | LBOUT | 5 | OUTPUT PER HOUR ALL PERSONS: BUSINESS SEC(1982=100,SA) |
| 46 | LBMNU | 5 | HOURS OF ALL PERSONS: NONFARM BUSINESS SEC (1982=100,SA) |
| 47 | LBLCPU | 5 | UNIT LABOR COST: NONFARM BUSINESS SEC (1982=100,SA) |

Note : Same as Table 8.