

Enforcing International Trade Agreements under Imperfect Private Monitoring: Private Trigger Strategies and a Possible Role for the WTO

by

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Abstract

To analyze the issue of enforcing international trade agreements in the presence of potential deviations of which countries receive *imperfect* and *private* signals, this paper analyzes a repeated bilateral trade relationship where each country can secretly raise its protection level through *concealed trade barriers*. In particular, it explores the possibility that countries adopt *private trigger strategies (PTS)* under which each country triggers an *explicit* tariff war based on its privately observed imperfect signals of the potential use of concealed trade barriers. Focusing on *simple symmetric PTS* where each country imposes its one-shot Nash tariff under any punishment phase, this paper first establishes the condition under which symmetric countries may restrain the use of concealed trade barriers: the sensitivity of private signals rises in response to an increase in concealed protection. Then, it shows that the payoff of **any** equilibrium *symmetric PTS* should be identical to the one under *simple symmetric PTS* as long as the punishment is triggered by a one-shot Nash tariff. Given this generality result of *simple symmetric PTS*, the analysis reveals that it is not optimal to push down the cooperative protection level to its minimum attainable level. The paper identifies two factors that may severely limit the effectiveness of *PTS*; one is a reduction in each country's time lag in adjusting its tariff level in response to the other country's initiation of a punishment phase, and the other is asymmetry among countries. Both of these factors may limit the level of cooperation attainable under *PTS* by reducing the lengths of tariff war phases that countries can employ. In order to analyze how an impartial third-party who receives its own *imperfect* and *private* signals of potential deviations, such as the WTO, can facilitate enforcing international trade agreements, this paper constructs and analyzes *third-party trigger strategies (TTS)* under which each country triggers an explicit tariff war based on the WTO's decision on whether any deviation has occurred. A numerical analysis of *symmetric TTS* demonstrates that the optimal *symmetric TTS* may entail a punishment phase only with an initial tariff (no tariff-war period following it) as well as the one with a permanent Nash tariff war, depending on the sensitivity of the WTO's signals of potential deviations. On the one hand, the WTO may enable countries to shorten and possibly avoid a costly tariff war, which might have been necessary to curtail the incentive to wrongly punish other countries under *PTS*. On the other hand, the WTO may also enable countries to adopt a longer punishment phase than the one under *PTS*, which in turn helps countries to attain higher levels of expected payoffs.

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1. Introduction

Enforcing international trade agreements often involves disputes where countries present different opinions about potential deviations from the agreements. Differences in opinions may take various forms, such as disagreement over the existence of *concealed trade barriers* in disputes between the U.S. and Japan during 1980s or disagreement over legitimacy of antidumping duties, a frequent theme in the dispute settlement procedure of the World Trade Organization (WTO). These disagreements reflect imperfectness of information about deviations from trade agreements. In addition to being *imperfect*, each country's opinion of potential deviations can be *private* in the sense that its true opinion is not known to other countries. For example, when the U.S. Trade Representative (USTR) engages in a negotiation with China to curtail piracy and counterfeiting that impede the U.S. intellectual property rights, China and the USTR may not know each other's true belief regarding the Chinese government's effort level to curtail such practices, which in turn may contribute to a breakdown in the negotiation.¹

To analyze the issue of enforcing international trade agreements in the presence of potential deviations of which countries receive imperfect and private signals, this paper analyzes a repeated bilateral trade relationship where each country can secretly raise its protection level through concealed trade barriers. In particular, it explores the possibility that countries adopt *private trigger strategies (PTS)* under which each country triggers an *explicit* tariff war based on its privately observed imperfect signals of the potential use of concealed trade barriers. The analysis specifies the condition under which countries can restrain deviations based on *simple symmetric PTS* where each country imposes its one-shot Nash tariff under any punishment phase: the sensitivity of private signals rises in response to an increase in concealed protection. This paper then establishes that the payoff of **any** equilibrium *symmetric PTS* should be identical to the one under *simple symmetric PTS*, as long as the punishment is triggered by a one-shot Nash tariff. Given this generality result, it characterizes optimal *symmetric PTS* that maximize symmetric countries' expected payoffs under *PTS*. This paper also identifies factors that may severely limit the effectiveness of *PTS* in restraining deviations

¹ The signals that the USTR receives regarding potential deviations from trade agreements often come from the U.S. companies whose interests are affected by deviations. Such signals may involve companies' private information of which public revelation can be costly for those companies, forcing the signals to be private.

from trade agreements. In order to analyze how an impartial third-party who receives its own *imperfect* and *private* signals of potential deviations, such as the WTO, can facilitate enforcing international trade agreements, this paper constructs and analyzes *third-party trigger strategies* (*TTS*) under which each country triggers an explicit tariff war based on the WTO's decision on whether any deviation has occurred.

This paper differs from previous studies on the enforcement of trade agreements by focusing on how private nature of signals about deviations may limit the use of (private) trigger strategies.² Many studies have analyzed the enforcement issue of trade agreements using trigger strategies in a repeated game setup. With regard to observable actions of imposing tariffs, for example, Dixit (1987) established that countries can sustain the equilibrium with zero tariffs by restraining their unilateral incentive to impose tariffs based on a threat of invoking tariff wars against imposing positive tariffs.³ With regard to the issue of restraining the use of concealed trade barriers, Riezman (1991) modified “*trigger-price strategies*” of Green and Porter (1984) on collusion among firms into “*import trigger strategies*” where countries start a tariff war phase when the level of imports falls below a critical level. Because the import level is negatively correlated with countries' protection levels, countries have an incentive to hold down their concealed protection level to reduce the probability of triggering a costly tariff war. In contrast to these earlier studies focusing on *public* (observable by all countries) signals as a device that triggers tariff wars, this paper analyzes trigger strategies that rely on imperfect *private* signals of potential deviations. The private nature of signals used under *PTS* makes incentive constraints for such trigger strategies different from those under trigger strategies relying on public signals, which in turn limits the effectiveness *PTS* as discussed below.

In establishing that countries can restrict the use of concealed protection based on *PTS*, an analytical challenge emerges from the necessity to check not only one-time deviations from the specified strategy, but whole deviation paths that each country may take in order to ensure that

² Bagwell and Staiger (2005) analyzed the issue of implementing trade agreements when each government is privately informed about its own domestic political pressure for protection. Their analysis differs from this paper's because it focuses on identifying the structure of trade agreements that can induce the truthful revelation of private political pressure rather than analyzing the enforcement of trade agreements when countries privately observe imperfect signals of potential deviations.

³ Bagwell and Staiger (2002) provide a comprehensive review of studies analyzing international trade agreements as a subgame perfect equilibrium in a repeated trade relationship.

proposed *PTS* can serve as a *supergame equilibrium* of the repeated protection-setting game.⁴ To solve this issue, I provide a full characterization of an optimal (possibly deviatory) protection sequence that each country may take under *PTS* in Section 2. In the same section, I establish that there exists a stationary protection level that each country may sustain as a cooperative protection level if the sensitivity of countries' private signals rises in response to an increase in concealed protection.

In addition to establishing that symmetric countries may restrain the use of concealed trade barriers with *simple symmetric PTS*, the analysis in Section 3.1 also establishes that the payoff of any equilibrium *symmetric PTS* will be the same as the one under *simple symmetric PTS* as long as the punishment is triggered by a one-shot Nash tariff. Then, Section 3.2 characterizes *optimal symmetric PTS* under which countries maximize their expected discounted payoffs. The analysis shows that it is not optimal to push down the cooperative protection level to its minimum level attainable under *symmetric PTS* due to the cost of increasing the probability of costly tariff wars.

A potential limitation of *PTS* comes from the private nature of signals utilized under *PTS* because it limits the lengths of tariff war phases that countries can employ. Note that each country may misrepresent its private signal under *PTS* either by initiating a tariff war when its private signal does not belong to the range under which it is supposed to trigger a tariff war or by not initiating a tariff war when its private signal does belong to such a range. On the one hand, if the tariff war phase is too long so that its expected payoff is higher when it ignores its private signals to trigger a tariff war, then each country will never initiate a tariff war. On the other hand, each country will always initiate a tariff war by imposing its static optimal tariff if the following war is too short.⁵ In contrast to repeated games with public information where countries can choose any length for their tariff war phases, the private nature of signals imposes restrictions on the lengths of tariff war phases under *PTS*!

This paper identifies two factors that may severely limit the effectiveness of *PTS*; one is a reduction in each country's time lag in adjusting its tariff level in response to the other

⁴ As discussed in more details in Section 2, if private signals trigger tariff wars as under *PTS*, any deviatory action that each country might have taken in a previous period can influence its optimal deviatory action in a current period. This prohibits us from applying the logic of the well-known "one-stage-deviation principle" to *PTS*.

⁵ Each country benefits in the period that it unilaterally initiates a tariff war phase because it imposes its static optimal tariff. If there is no actual tariff war phase to be followed, then imposing the static optimal tariff becomes a dominant strategy for each country.

country's initiation of a tariff war, and the other is asymmetry among countries. Both of these factors may limit the level of cooperation attainable under *PTS* by reducing the length of a tariff war phase. One of factors that determine the length of a tariff war phase under *PTS* is the one-period gain that it can realize during the period that it initiates a tariff war phase. If there is a reduction in each country's time lag in adjusting its tariff level in response to the other country's initiation of an explicit tariff war, the tariff-war-initiating country's one-period gain from imposing its static optimal tariff decreases as the other country reacts more quickly with its own static optimal tariff. Such a reduction in the gain from initiating a tariff war decreases the length of the tariff war phase, which in turn reduces the level of cooperation sustainable under *PTS*. Asymmetry among countries reduces the length of a tariff war phase that a small country can initiate against a large country in a similar way. As asymmetry in countries' relative market size increases, the small country's ability to change the terms of trade in its favor by imposing tariffs decreases, reducing the one-period gain that it can realize by imposing its static optimal tariff. This shortens the length of a tariff war phase that the small country can initiate against the large country's potential use of concealed protection.

The analysis of *PTS* provides useful insights on the unilateral approach of the U.S. towards enforcing international trade agreements. According to the analysis, the fact that the U.S. is the largest trading economy in the world makes her a country that is most capable of using *PTS* to deter its trading partners' use of concealed trade barriers. Indeed, the U.S. is the only country in the world that has the legislation, Section 301, authorizing its government to invoke a tariff war based on its own unilateral judgment of potential deviations from trade agreements. As emphasized in the analysis, the credibility of invoking a tariff war is crucial for the success of such a unilateral approach. To raise the credibility of a punitive tariff war, the U.S. Congress modified Section 301 in 1988 by shifting the retaliation authority from the President (who is likely to care about diplomatic relationships) to the USTR and by mandating retaliation against unjustifiable practices. Note also that the USTR has often taken its Section 301 investigation cases to the WTO panel investigation.⁶ Such an action seemingly respecting the multilateral enforcement mechanism of the WTO, can be an attempt to make the threat of invoking a tariff war more credible by delegating the judgment of deviations to a third party panel who would

⁶ See Bayard and Elliott (1992) for a detailed discussion of Section 301 cases.

presumably care less about the cost or the benefit from triggering a tariff war that may result from the judgment.

In order to analyze how an impartial third-party who receive its own *imperfect* and *private* signals of potential deviations, such as the WTO, can facilitate enforcing international trade agreements, this paper constructs and analyzes *third-party trigger strategies (TTS)* under which each country triggers an explicit tariff war based on the WTO's decision on whether any deviation has occurred. A numerical analysis of *symmetric TTS* demonstrates that the optimal *TTS* may entail a punishment phase only with an initial tariff (no tariff-war period following it) as well as the one with a permanent Nash tariff war, depending on the sensitivity of the WTO's signals of potential deviations. On the one hand, the WTO may enable countries to shorten and possibly avoid a costly tariff war, which might have been necessary to curtail each country's incentive to wrongly punish the other country with its static optimal tariff. On the other hand, the WTO may also enable countries to adopt a longer punishment phase than the one under *PTS*, which in turn helps countries to attain higher levels of expected payoffs

While this paper focuses on the issue of enforcing international agreements, the analysis of *PTS* is applicable to a broader range of problems. The protection-setting game analyzed in this paper belongs to repeated games under *imperfect private monitoring*. It is well known in the game theory literature that analyzing repeated games with imperfect private monitoring is difficult because utilization of privately observed signals in determining continuation plays can destroy the recursive structure of repeated games.⁷ As a way to overcome this problem in repeated games with imperfect private monitoring, Kandori and Matsushima (1998) and Compte (1998) allow players to communicate their privately observed information regarding potential deviations. Such communication serves as imperfect *public* information (in the sense that every player can observe the communication), thus, restoring a recursive structure to the repeated game.⁸ *PTS* considered in this paper show an alternative way to restore a recursive structure to a repeated game with imperfect private monitoring. Under *PTS*, each country initiates a tariff war phase by imposing *explicit* tariffs if it receives private signals that are highly correlated with other countries' defective behaviors. Because all trading partners can

⁷ Kandori (2002) discusses this point and recent developments in repeated game with private monitoring in detail.

⁸ In these studies, the communication among players entails no cost (so that it is "cheap talk") and each country's revealed private information does not affect its own continuation payoff in order to ensure truthful revelation of private information.

perfectly observe explicit tariffs, countries can avoid potential confusion between punishment phases and non-punishment phases, which ensures a “recursive” structure of the repeated game along the equilibrium path. In the context of collusion among firms that can engage in secret-price cuttings, firms can employ *advertised* (thus *public*) sales to initiate a punishment phase against potential defections from collusive pricing as a way to coordinate their punishments. According to this paper’s analysis of *PTS*, the expansion of internet commerce can negatively affect collusive behaviors based on such *PTS* because it reduces each firm’s time lag in effectively advertising its sales over the internet in response to the other firm’s initiation of advertised sales.

The paper is organized as follows. Section 2.1 develops a bilateral trade model where each country receives imperfect private signals of each other’s use of concealed trade barriers and specifies *simple PTS* that countries employ to deter such protection. Section 2.2 describes incentive constraints under *simple PTS*, providing conditions under which those incentive constraints are satisfied. Section 3.1 proves the existence of *simple symmetric PTS* as a supergame equilibrium of the repeated protection-setting game. Section 3.2 proves that the payoff of any equilibrium symmetric PTS will be the same as the one under *simple symmetric PTS* as long as the punishment is triggered by a one-shot Nash tariff. Section 3.2 then characterizes optimal *symmetric PTS* under which countries maximize their joint expected discounted payoffs. Section 4.1 analyzes how a reduction in the time lag in adjusting tariff levels in response to an initiation of an explicit tariff war affects effectiveness of *PTS*, and Section 4.2 provides an analysis of the effect of introducing asymmetry among countries on *PTS*. To analyze the role of the WTO in enforcing international trade agreements in the presence of imperfect private monitoring, Section 5 specifies *symmetric TTS*, providing a comparison between *optimal symmetric PTS* and *optimal symmetric TTS* through a numerical analysis. Section 6 concludes by providing a summary of the results together with a discussion about a possible extension of this paper’s analysis towards understanding the workings of dispute settlement procedure of the WTO.

2. Private Trigger Strategies

2.1. A Trade Model with Concealed Trade Barriers and Private Trigger Strategies

The basic bilateral trade model comes from Dixit (1987) with concealed trade barriers being introduced in a similar way as in Riezman (1991). There exist two countries, home (H) and foreign (F), producing and trading two products, good 1 and good 2, under perfect competition. H imports good 2 and F imports good 1. In each period, each country simultaneously chooses its explicit tariff level, $e^i \geq 0$ and its (total) import protection level, $\tau^i \geq 0$, where $i = *$ or none, having variables with and without superscripts $*$ respectively denote foreign and home variables henceforth. Then, $\tau - e \geq 0$ and $\tau^* - e^* \geq 0$ represent the concealed protection levels of H and F, respectively, and the local prices, p_1 , p_2 , p_1^* , and p_2^* are related as follows: $p_2 = p_2^*(1 + \tau)$ and $p_1^* = p_1(1 + \tau^*)$.⁹ Given the assumption of perfect competition, I can define each country's one-period payoff function as a function of the terms of trade, denoted by $\pi (\equiv p_1 / p_2^*)$, and its own protection level. Such a payoff function, represented by $w^i(\pi, \tau^i)$ with $i = *$ or none, induces a corresponding import demand function, denoted by $m^i(\pi, \tau^i)$. If there exists no uncertainty (random elements) in this world, each country's amount of imports is a deterministic function of each country's protection level and the term of trade. This implies that each country may figure out the exact level of the other country's protection based on the information about the terms of trade and the amount of imports, even in the presence of concealed trade barriers.

However, when I introduce uncertainty into the model as a way of representing shocks to technology or preferences, the exact derivation of other countries' protection levels based on the amount of imports and the terms of trade may become impossible. Uncertainty caused by random shocks can be modeled into random components in countries' import demand functions as follows:

$$(1) \quad m_t^i = m^i(\pi_t, \tau_t^i, \theta_t^i) \text{ with } i = * \text{ or none,}$$

where $\theta_t^i \in \Theta^i$ denotes each country's random components affecting its import demand at period t (subscript t denotes the variables determined in period t), which follow a joint density function, $f(\theta_t, \theta_t^*)$ that is iid across periods. In equilibrium, the following balance of payment condition should be satisfied:

$$(2) \quad \pi_t \cdot m(\pi_t, \tau_t, \theta_t) = m^*(\pi_t, \tau_t^*, \theta_t^*),$$

⁹ Thus, this paper does not consider the possibility of using negative or prohibitive protection.

which determines the equilibrium values for π_t , m_t , and m_t^* as functions of τ_t , τ_t^* , θ_t , and θ_t^* .

With random shocks realizing after τ and τ^* being determined, each country's one-period expected payoff, denoted by u^i , is a function of both countries' protection levels:

$$(3) u^i(\tau_t^i, \tau_t^j) = \iint_{(\theta_t, \theta_t^*) \in (\Theta, \Theta^*)} w^i(\pi_t(\tau_t, \tau_t^*, \theta_t, \theta_t^*), \tau_t^i; \theta_t^i) f(\theta_t, \theta_t^*) d\theta_t d\theta_t^* \text{ with } i, j = * \text{ or none, and } i \neq j,$$

where $w^i(\pi, \tau^i; \theta^i)$ represents each country's one-period payoff function that is affected by random shocks, θ^i . Regarding derivatives of $u^i(\tau^i, \tau^j)$ with respect to τ^i and τ^j , I assume the following standard trade-theoretic results continue to hold in the presence of these random variables: $\partial u^i / \partial \tau^i > 0$ at $\tau^i = 0$ (each country has an incentive to raise its protection level above zero); $\partial u^j / \partial \tau^i > 0$ (such protection hurts the other country); $\partial u^i / \partial \tau^i + \partial u^j / \partial \tau^i < 0$ (such protection reduces the total payoff of H and F as it creates distortional loss) with $i, j = *$ or none, and $i \neq j$. For analytical simplicity, I introduce the following additional assumptions: $\partial^2 u^i / \partial \tau^{i^2} < 0$ (the marginal gain from protection decreases as the protection level increases); $\partial^2 u^i / \partial \tau^i \partial \tau^j = 0$ (the marginal gain from protection is not affected by the other country's protection level) with $i, j = *$ or none, and $i \neq j$.¹⁰ These additional assumptions guarantee the existence of a unique static optimal protection level for each country which I denote by $h^i (> 0)$ with $i = *$ or none. The one-shot protection setting game between H and F then generates a Nash equilibrium where $(\tau, \tau^*) = (h, h^*)$.

At the end of period t , each country privately observes realized values of its payoff and its own random variable, (u_t, θ_t) by H and (u_t^*, θ_t^*) by F, and both countries observe a pair of explicit tariffs, (e_t, e_t^*) . Each country cannot infer the exact level of the other country's concealed protection because it does not know the realized value of the other's random variable. However, note that the privately observed information, denoted by $\omega_t^i = (u_t^i, \theta_t^i, \tau_t^i) \in \Omega^i$ with $i = *$ or none, can serve as a measure for detecting the other country's potential use of

¹⁰ These properties of a social utility function can be derived from a two good, partial equilibrium model of trade with linear demand and supply curves. See Bond and Park (2002) for derivation of such properties.

concealed protection.¹¹ More specifically, each country can choose a subset of its possible private signals, denoted by Ω^{D^i} , such that $\partial Pr(\omega_t^i \in \Omega^{D^i}) / \partial \tau_t^j > 0$ where $Pr(B)$ denotes the probability of event B occurring with $i, j = *$ or none, and $i \neq j$. For example, H can assign values of u_t that are less than a certain critical value as the payoff part of Ω^D . This can induce $\partial Pr(\omega_t \in \Omega^D) / \partial \tau_t^* > 0$ because $\partial u_t / \partial \tau_t^* < 0$, and the sensitivity of u_t against τ_t^* can improve once it is properly controlled for θ_t and τ_t .

Given the stage game depicted above, I can describe an infinitely repeated protection-setting game between H and F with their privately observed signals serving as measures to detect the potential use of concealed protection as follows. A strategy for each country is defined by $s^i = (s^i(t))_{t=1}^\infty$ with

$$(4) \quad s(t) : A^{t-1} \times \Omega^{t-1} \times (E^*)^{t-1} \rightarrow A \text{ and } s^*(t) : (A^*)^{t-1} \times (\Omega^*)^{t-1} \times E^{t-1} \rightarrow A^*$$

where A^i denotes the set of possible actions that each country can take in a period with $a^i \equiv (\tau^i, e^i) \in A^i$, and E^i denotes the set of possible explicit tariffs that each country can impose in a period with $e^i \in E^i$, having $i = *$ or none. $s(t)$, the strategy of H in period t , assigns its action (τ_t, e_t) based on the history of its own previous actions, $(a_1, \dots, a_{t-1}) \in A^{t-1}$, the history of its own private information, $(\omega_1, \omega_2, \dots, \omega_{t-1}) \in \Omega^{t-1}$, and the history of the other country's explicit tariffs, $(e_1^*, e_2^*, \dots, e_{t-1}^*) \in (E^*)^{t-1}$, while $s^*(t)$ assigns the action for F in a similar manner. If each country conforms to its strategy defined in (4), then the expected discounted payoff is given by:

$$(5) \quad V^i(s^i, s^j) = E \left[\sum_{t=1}^{\infty} u^i(\tau_t^i, \tau_t^j) (\delta^C)^{t-1} \middle| (s, s^*) \right],$$

where $E[\cdot | (s, s^*)]$ is the expectation with respect to the probability measure on histories induced by strategy profile (s, s^*) , and $\delta^C \in [0, 1)$ denotes the common discount factor with $i, j = *$ or none, and $i \neq j$. Now, I define a *supergame equilibrium* in this infinitely repeated protection setting game as follows:

¹¹ Once H observes u_t , θ_t , and τ_t , for example, H can calculate the probability of $\tau_t^* \leq l$ (a certain protection level) by $Pr(\tau_t^* \leq l | u_t, \theta_t, \tau_t) = \int_0^l \left(\int_{\theta^* \in \Theta^*(u_t, \theta_t, \tau_t, \tau^*)} f(\theta_t, \theta^*) d\theta^* \right) d\tau^*$ where $\Theta^*(u_t, \theta_t, \tau_t, \tau^*) = \{\theta^* \in \Theta^* | u(\tau_t, \tau^*, \theta_t, \theta^*) = u_t\}$.

Definition 1. A strategy profile (s, s^*) is a *supergame equilibrium* in the repeated game between H and F, if $V(s, s^*) \geq V(s', s^*)$ and $V^*(s, s^*) \geq V^*(s, s^{*/})$ for all $s' \neq s$ and $s^{*/} \neq s^*$.¹²

To explore the possibility of supporting cooperative protection levels, denoted by l and l^* , that are lower than the one-shot Nash protection levels (h and h^*) as a supergame equilibrium of the repeated game described above, I consider following strategies under which each country uses its private signal, ω and ω^* , as a device to trigger an explicit tariff war against the other country's potential use of concealed protections.¹³

- (i) Given that period $t - 1$ was a “*cooperative*” period with $(e_{t-1}, e_{t-1}^*) = (0, 0)$, each country keeps cooperating by setting $(\tau_t^i, e_t^i) = (l^i, 0)$ if $\omega_{t-1}^i \notin \Omega^{D^i}$, and it initiates a tariff war by setting $(\tau_t^i, e_t^i) = (h^i, h^i)$ if $\omega_{t-1}^i \in \Omega^{D^i}$ with $i = *$ or none.
- (ii) Given that a “*tariff war phase*” was initiated in period $t - 1$ with $(e_{t-1}, e_{t-1}^*) \neq (0, 0)$, each country sets $(\tau^i, e^i) = (h^i, h^i)$ for the following $(T - 2)$ periods and it continues to do so one more period with probability λ if $e_{t-1}^* = 0$; each country sets $(\tau^i, e^i) = (h^i, h^i)$ for the following $(T^* - 2)$ periods and it continues to do so one more period with probability λ^* if $e_{t-1} = 0$; each country sets $(\tau^i, e^i) = (h^i, h^i)$ for the following $(T^S - 2)$ periods and it continues to do so one more period with probability λ^S if $e_{t-1} > 0$ and $e_{t-1}^* > 0$, with $T, T^*,$ and T^S being integer numbers that are greater than or equal to 2, $\lambda, \lambda^*,$ and λ^S belonging to $[0, 1]$, and $i = *$ or none.
- (iii) In period 1 and other “*initial*” periods right after the end of any tariff war phase, each country sets $(\tau^i, e^i) = (l^i, 0)$ with probability $(1 - Pr^i)$ but initiates a tariff war by setting $(\tau^i, e^i) = (h^i, h^i)$ with probability Pr^i , where $Pr^i = Pr(\omega_t^i \in \Omega^{D^i})$ with $(\tau_t, e_t) = (l, 0), (\tau_t^*, e_t^*) = (l^*, 0)$, and $i = *$ or none.

¹² This definition of a *supergame equilibrium* of a repeated game with privately observed signals of other players' actions follows Matsushima (1991).

¹³ One trivial supergame equilibrium strategy profile is to assign a one-shot Nash protection level for all periods because that would assign the static optimal behavior for each country.

Note that the absence or presence of explicit tariffs classifies any period into either a “*cooperative*” period (with no explicit tariffs) or a “*tariff war*” period (with some positive tariffs). While H and F cannot observe each other’s concealed protection levels, they use their explicit tariffs as public signals to coordinate tariff war phases as described in (i) and (ii). Extending a tariff war phase one more period with a certain probability as specified in (ii) is an instrument to make the length of a tariff war phase to behave as if it were a continuous variable.¹⁴ Also note that the actions for period 1 and other “initial periods” described in (iii) are designed to make them mimic those in a period that immediately follows a “cooperative” one, which in turn simplifies the analysis of the trigger strategies defined above.¹⁵ Finally, note that the sets of private signals that trigger tariff wars (Ω^D, Ω^{D^*}) and the lengths of different tariff war phases ($T - 1$ if H triggers, $T^* - 1$ if F triggers, and $T^S - 1$ if H and F trigger simultaneously) with corresponding probabilities to extend the tariff war phases one more period ($\lambda, \lambda^*, \lambda^S$) characterize the strategy profile defined by (i), (ii) and (iii), together with the cooperative protection levels (l, l^*). Thus, I define *simple private trigger strategies (PTS)* as follows:

Definition 2. If (i), (ii), and (iii) describe a strategy profile $(\underline{s}, \underline{s}^*)$, then, $(\underline{s}, \underline{s}^*)$ are *simple private trigger strategies (PTS)* with $(l, l^*, \Omega^D, \Omega^{D^*}, T, T^*, T^S, \lambda, \lambda^*, \lambda^S)$ as characterizing parameters.

Given this definition, I can derive the expected discounted payoff under $(\underline{s}, \underline{s}^*)$ with $(l, l^*, \Omega^D, \Omega^{D^*}, T, T^*, T^S, \lambda, \lambda^*, \lambda^S)$, denoted by $V^i(\underline{s}, \underline{s}^*)$, as follows:

¹⁴ For example, $\lambda = 0$ implies that H and F play the one-shot Nash tariff war for $(T - 2)$ periods, and $\lambda = 1$ implies that they play the one-shot Nash tariff war for $(T - 1)$ periods, with each $\lambda \in (0, 1)$ being equivalent to a case where they play the one-shot Nash tariff war for some intermediate length of periods between $(T - 2)$ and $(T - 1)$. This allows the expected discounted payoff from invoking a tariff war phase to vary smoothly (by varying the length of a tariff war phase “*smoothly*”) so that it can be set to equal the expected discounted payoff from not invoking a tariff war phase, an important requirement for incentive constraints considered in Section 2.2.1.

¹⁵ If, for example, $Pr = 0 \neq Pr(\omega_t \in \Omega^D)$ with $(\tau_t, e_t) = (l, 0)$ and $(\tau_t^*, e_t^*) = (l, 0)$, then the expected one-period payoffs for period 1 and other initial periods will be different from those for any period immediately following a cooperative one, making the expected discounted payoffs along the equilibrium path more complicated than those in (6). Furthermore, having actions in period 1 and in other initial periods different from those in periods immediately following a cooperative period will make deviation incentives different across these periods, which in turn complicates characterization of the optimal protection sequence in Section 2.2.2.

$$(6) \quad V^i(\underline{s}, \underline{s}^*) = \frac{(1 - Pr^i Pr^j)[u^i(l^i, l^j) - u^i(h^i, h^j)]}{1 - \delta^C + Pr(1 - Pr^*)(\delta^C - \delta) + Pr^*(1 - Pr)(\delta^C - \delta^*) + PrPr^*(\delta^C - \delta^S)} \\ + \frac{Pr^j(1 - Pr^i)[u^i(l^i, h^j) - u^i(l^i, l^j)] + Pr^i(1 - Pr^j)[u^i(h^i, l^j) - u^i(l^i, l^j)]}{1 - \delta^C + Pr(1 - Pr^*)(\delta^C - \delta) + Pr^*(1 - Pr)(\delta^C - \delta^*) + PrPr^*(\delta^C - \delta^S)} + \frac{u^i(h^i, h^j)}{1 - \delta^C}$$

where $\delta^k = \lambda^k (\delta^C)^{T^k} + (1 - \lambda^k)(\delta^C)^{T^k - 1}$ with $k = *, s$, or none, $i = *$ or none, and $i \neq j$, having $(\delta^C - \delta)$, $(\delta^C - \delta^*)$, and $(\delta^C - \delta^S)$ respectively represent the relative length of the tariff war phase initiated by H alone, by F alone, and by H and F simultaneously. Because $(T, T^*, T^S, \lambda, \lambda^*, \lambda^S)$ uniquely defines $(\delta, \delta^*, \delta^S)$ as shown above, I will describe *PTS* using $(l, l^*, \mathcal{D}, \mathcal{D}^*, \delta, \delta^*, \delta^S)$ instead of using $(l, l^*, \mathcal{D}, \mathcal{D}^*, T, T^*, T^S, \lambda, \lambda^*, \lambda^S)$ from now on.

Note that *simple PTS* defined above is *simple* in the sense that each country imposes its one-shot Nash tariff under any punishment phase. *PTS* may involve more complex punishment phases such as imposing lower tariffs if the signal indicates weaker violations or/and employing a stronger punishment, such as autarky, against presumably more severe violations. As shown later, the payoff of any equilibrium *symmetric PTS* will be identical to the one under *simple symmetric PTS* defined above with $l = l^*, \mathcal{D} = \mathcal{D}^*$, and $\delta = \delta^*$, as long as the punishment is triggered by a static Nash tariff. Henceforth, I denote *simple PTS* by *PTS* unless it is necessary to distinguish them.

2.2. Incentive Constraints under Private Trigger Strategies

In this section, I analyze incentive constraints for *PTS* to be a supergame equilibrium in the repeated game defined in Section 2.1. The private nature of signals that trigger tariff wars under *PTS* makes such incentive constraints different from the incentive constraints for trigger strategies under which public signals trigger punishment phases in two distinctive ways. First, the private nature of signals imposes restrictions on the lengths of tariff war phases under *PTS*, which contrasts with the repeated game with public information where countries can choose any length for their tariff war phases. Section 2.2.1 analyzes such limits on the lengths of tariff war phases under *PTS*. Second, to check the absence of $s' (\neq \underline{s})$ or $s^{*'} (\neq \underline{s}^*)$ such that $V(s', \underline{s}^*) > V(\underline{s}, \underline{s}^*)$ and $V^*(\underline{s}, s^{*'}) > V^*(\underline{s}, \underline{s}^*)$, one needs to check not only one-time

deviations from the specified strategy, but whole deviation paths that each country may take.¹⁶ If private signals trigger tariff wars as under *PTS*, any deviatory action that each country might have taken in a previous period can influence its optimal deviatory action in a current period: the previous period defection affects the probability of a tariff war being initiated in the current period, which in turn influences the current-period optimal action. This necessitates characterization of an optimal (potentially deviatory) protection sequence that each country may take against $(\underline{s}, \underline{s}^*)$ in analyzing the incentive constraints for *PTS*. Section 2.2.2 characterizes such a sequence for H under *PTS*, and shows that H's optimal protection sequence can be a stationary one of setting τ at l (the cooperative protection level) in all periods until a tariff war starts, a prerequisite for *PTS* to be a supergame equilibrium.

2.2.1. Constraints on Lengths of Tariff War Phases

In any period that immediately follows a cooperative period with $(e, e^*) = (0, 0)$ and in any initial periods (period 1 and a period right after the end of any tariff war phase), each country faces the choice of whether or not to initiate a tariff war phase by imposing its static optimal tariff. To eliminate the incentive to misrepresent private signals in such periods, the expected payoff from initiating a tariff war phase should be identical to the expected payoff from not initiating it for each country. Denote such conditions that equate those expected payoffs by *ICP* for H and *ICP** for F. Then,

ICPⁱ:

$$(7) \quad (1 - Pr^j)[u^i(l^i, l^j) + \delta^C V_C^i] + Pr^j[u^i(l^i, h^j) + (\delta^C - \delta^j)V_N^i + \delta^j V_C^i] = \\ (1 - Pr^j)[u^i(h^i, l^j) + (\delta^C - \delta^i)V_N^i + \delta^i V_C^i] + Pr^j[u^i(h^i, h^j) + (\delta^C - \delta^S)V_N^i + \delta^S V_C^i],$$

where $V_C^i \equiv V^i(\underline{s}, \underline{s}^*)$ and $V_N^i \equiv u(h^i, h^j)/(1 - \delta^C)$ with $i = *$ or none, and $i \neq j$. For each country in period t that follows a cooperative period or in an initial period, the left side of the equality in (7) represents the expected discounted payoff from not initiating a tariff war phase, setting $(\tau_t^i, e_t^i) = (l^i, 0)$. The right side of the equality represents the expected discounted payoff from initiating a tariff war phase, setting $(\tau_t^i, e_t^i) = (h^i, h^j)$. In calculating these expected

¹⁶ When a public signal triggers tariff wars, any deviatory actions that each country might have taken in any previous periods will not affect its optimal deviatory action in the current period for a given history of public signals up to the current period. This is because one country's defections in the previous periods affect the other country's current and future actions only through affecting the history of public signals. Therefore, we can apply

discounted payoffs in (7), it is assumed that the other country initiates a tariff war phase with a certain probability that conforms PTS , Pr^* by F and Pr by H.

Using $u^i(l^i, l^j) - u^i(l^i, h^j) = u^i(h^i, l^j) - u^i(h^i, h^j)$ implied by $\partial^2 u^i / \partial \tau^i \partial \tau^j = 0$, I simplify (7) into

$$(ICP^i) \quad u^i(l^i, l^j) - u^i(h^i, l^j) + (\delta^C - \delta^i)(V_C^i - V_N^i) = Pr^j [(\delta^C - \delta^j) - (\delta^i - \delta^S)](V_C^i - V_N^i),$$

where $i = *$ or none, and $i \neq j$. For any given cooperative protection levels (l, l^*) and any given ranges of private signals that trigger tariff war phases (Ω^D, Ω^{D*}) , I have three variables $(\delta, \delta^*, \delta^S)$ to be determined with two equations (ICP and ICP^*), potentially having infinite combinations of $(\delta, \delta^*, \delta^S)$ that satisfies ICP and ICP^* . To have ICP and ICP^* satisfied regardless of values that Pr^* and Pr may take, however, $\delta^C + \delta^S = (\delta^* + \delta)$ needs to hold.¹⁷ Then, $\delta^C - \delta = [u(h, l^*) - u(l, l^*)]/(V_C - V_N)$ from ICP and $\delta^C - \delta^* = [u^*(h^*, l) - u^*(l^*, l)]/(V_C^* - V_N^*)$ from ICP^* . The following lemma provides a sufficient condition for H and F not having any incentive to misrepresent its private signals along the equilibrium path under PTS :

Lemma 1.

- (a) If $\delta^C - \delta = [u(h, l^*) - u(l, l^*)]/(V_C - V_N)$, $\delta^C - \delta^* = [u^*(h^*, l) - u^*(l^*, l)]/(V_C^* - V_N^*)$, and $\delta^C + \delta^S = (\delta^* + \delta)$, then, ICP and ICP^* are satisfied for any values of Pr^* and Pr .
- (b) If H and F value their future payoffs high enough (δ^C is close enough to 1) and the probability of a tariff war being triggered along the equilibrium path is low enough (Pr and Pr^* are close enough to 0), then, for any given combination of $(l, l^*, \Omega^D, \Omega^{D*})$ with $(l, l^*) < (h, h^*)$ there exists a unique combination of δ , δ^* , and δ^S that satisfies the sufficient condition for ICP and ICP^* defined in Lemma 1 (a).

Proof (a) is obvious from simplified ICP and ICP^* . See Appendix A for the proof of (b).

the logic of one-stage-deviation principle for the subgame perfect equilibrium with observable actions (Theorem 4.1. and Theorem 4.2 in Fudenberg and Tirole, 1991) to the perfect public equilibrium (with unobservable actions).¹⁷ Generally speaking, ICP and ICP^* need not be satisfied for any values of Pr^* and Pr because $Pr = Pr(\omega_i \in \Omega^D)$ and $Pr^* = Pr(\omega_i^* \in \Omega^{D*})$ with $(\tau_i, e_i) = (l, 0)$ and $(\tau_i^*, e_i^*) = (l, 0)$ along the equilibrium path under PTS . As discussed in Section 2.2.2, however, countries may misrepresent private signals along their deviation path under which Pr^* and Pr may not take the same values as those along the equilibrium path. Lemma 4 (b) in Section 2.2.2 establishes that the sufficient condition in Lemma 1 (a) indeed guarantees countries have no strict incentives to misrepresent their private signals even along their deviation paths.

For a given combination of $(l, l^*, \Omega^D, \Omega^{D*})$, ICP and ICP^* specify the lengths of tariff war phases that countries can employ under PTS . Since $\delta = \lambda(\delta^C)^T + (1 - \lambda)(\delta^C)^{T-1}$, $\delta^* = \lambda^*(\delta^C)^{T^*} + (1 - \lambda^*)(\delta^C)^{T^*-1}$ and $\delta^S = \lambda^S(\delta^C)^{T^S} + (1 - \lambda^S)(\delta^C)^{T^S-1}$, ICP limits the length of a tariff war phase that H can initiate (T, λ) ; ICP^* limits the length of a tariff war phase that F can initiate (T^*, λ^*) ; and $\delta^C + \delta^S = (\delta^* + \delta)$ determines the length of a tariff war phase that H and F initiate simultaneously (T^S, λ^S) . Note that the length of a tariff war phase that H can initiate $(\delta^C - \delta)$ increases in its expected gain in the initial period of the tariff war phase from imposing its static optimal tariff $(u(h, l^*) - u(l, l^*))$ and decreases in its expected loss in the tariff war periods that will follow $(V_C - V_N)$. The expected gain in the initial period provides H with the incentive to start a tariff war phase despite the expected loss from engaging in a tariff war. Thus, the larger the expected gain in the initial period, the longer a tariff war that H can tolerate (without violating ICP) and the larger the expected loss from a tariff war, the shorter a tariff war that H can tolerate (without violating ICP). The same logic applies in determining the length of a tariff war phase that F can initiate through ICP^* .

2.2.2. Optimal Protection Sequence and Existence of a Stationary Protection Level

To characterize the optimal protection sequence, I analyze the dynamic optimization problem for H to maximize its expected discounted payoff by choosing a protection sequence $\{\tau_{d+1}\}_{d=0}^{\infty}$, given that F follows its specified strategy under PTS . The dynamic optimization problem for H is

$$(8) \quad \text{Sup}_{\{\tau_{d+1}\}_{d=0}^{\infty}} \sum_{d=0}^{\infty} \left\{ (\delta^C)^d \cdot \left[\prod_{t=0}^{d-1} [1 - Pr^*(\tau_t)] \right] \cdot F(\tau_d, \tau_{d+1}) \right\}$$

where $\prod_{t=0}^{d-1} [1 - Pr^*(\tau_t)] = [1 - Pr^*(\tau_0)] \times [1 - Pr^*(\tau_1)] \times \dots \times [1 - Pr^*(\tau_{d-1})]$ with $\prod_{t=0}^{-1} [1 - Pr^*(\tau_t)] = 1$, $Pr^*(\tau_t) = Pr(\omega_t^* \in \Omega^{D*})$ given $(\tau_t, e_t) = (\tau_t, 0)$ and $(\tau_t^*, e_t^*) = (l^*, 0)$, and $\tau_0 = l$; and $F(\tau_d, \tau_{d+1}) = Pr^*(\tau_d)[u(\tau_{d+1}, h^*) + (\delta^C - \delta^*)V_N + \delta^*V_{CO}] + [1 - Pr^*(\tau_d)]u(\tau_{d+1}, l^*)$ with $V_{CO} = V_C$. Note that the protection sequence $\{\tau_{d+1}\}_{d=0}^{\infty}$ in (8) specifies the protection levels only until F triggers an initial tariff war phase and the optimization in (8) assumes that H will follow its specified strategy under PTS once F triggers an initial tariff war phase with $V_{CO} = V_C \equiv V(\underline{s}, \underline{s}^*)$.

The *full* optimization problem should characterize the optimal protection sequence after the end of each tariff war phase that may occur in the future periods. Characteristics of the optimal protection sequence derived from (8), however, will be qualitatively identical to those of the full optimization problem. This is because the optimal sequence resulting from (8) will be identical to the one from the full optimization problem if V_{CO} in (8) is set to be equal to the maximized expected discounted payoff of the full problem, having H face an identical optimization problem in determining the protection sequence after the end of each tariff war phase in the future.¹⁸ Also note that the optimal protection sequence considered in (8) excludes the possibility of using explicit tariffs as a part of its path. As shown in *Lemma 4 (b)* of this section, however, once the lengths of tariff war phases satisfy the sufficient condition for *ICP* and *ICP** in *Lemma 1 (a)*, then, H cannot increase its payoff by using explicit tariffs along its deviation path. Hence, there is no loss of generality in analyzing the optimal protection sequence for H through the optimization problem defined in (8).¹⁹

Even though the optimization problem in (8) does not take a standard form for which a dynamic programming method is typically applied, *Lemma 2 (a)* below establishes equivalency between (8) and the following (non-standard) dynamic programming problem.²⁰

$$(9) \quad V(\tau_{-1}) = \underset{\tau \in [0, h]}{\text{Sup}} \left\{ F(\tau_{-1}, \tau) + \delta^C [1 - Pr^*(\tau_{-1})] V(\tau) \right\} \text{ for all } \tau_{-1} \in [0, h],$$

¹⁸ The discounted payoff of the full optimization problem can be obtained by applying the following iterative process to the optimization problem in (8). Initially set V_{CO} in (8) to be V_C defined in (6) and solve the optimization in (8), obtaining a discounted payoff as an outcome of this initial optimization problem. Then, set the value of V_{CO} in (8) to have the value of this initially generated discounted payoff, supposedly higher than (or equal to) the initial V_{CO} ($= V_C$), which redefines the optimization problem in (8). This redefined optimization problem will generate another discounted payoff as an outcome of this second optimization problem. Then, set V_{CO} in (8) to have the value of this newly generated discounted payoff and continue this iterative process until the discounted payoff generated through this process reaches its limit. As the sequence of the discounted payoffs generated through this process is monotonically increasing and bounded, there exists such a limit. This limit will be equal to the discounted payoff of the full optimization problem.

¹⁹ While I focus on characterizing the optimal protection sequence for H under *PTS* in this section, the same characterization can be applied to the optimal protection sequence for F.

²⁰ (8) is not a standard problem in the sense that the component that corresponds to the return function of a standard problem, $\left[\prod_{t=0}^{d-1} [1 - Pr^*(\tau_t)] \right] F(\tau_d, \tau_{d+1})$, depends not only on the current choice variable and the choice made in the immediate prior period (as in the case of a usual return function of a typical dynamic programming problem) but also on all the choices made since the initial period. The dynamic programming problem in (9) is not a standard form because the current state variable, τ_{-1} , affects not only the current return function part, $F(\tau_{-1}, \tau)$, but also the future discounted payoff part through $[1 - Pr^*(\tau_{-1})]$.

where τ_{-1} and τ , respectively denote a previous-period and a current-period protection level of H.²¹ Given a solution $V(\cdot)$ to (9), the optimal policy correspondence $G: [0, h] \rightarrow [0, h]$ is defined by:

$$(10) \quad G(\tau_{-1}) = \{\tau \in [0, h]: V(\tau_{-1}) = F(\tau_{-1}, \tau) + \delta^C [1 - Pr^*(\tau_{-1})] \cdot V(\tau)\},$$

which contains values of τ that maximizes $V(\tau_{-1})$ for each $\tau_{-1} \in [0, h]$. Despite the fact that the dynamic optimization problem in (8) and the corresponding dynamic programming problem in (9) take non-standard forms, *Lemma 2* establishes the following standard results on V and G :

Lemma 2.

- (a) Define $V_S(\tau_0)$ be the supremum function that is generated by (8). Then, (i) V_S satisfies (9); (ii) the solution to (9) $V(\tau_{-1}) = V_S(\tau_{-1})$; (iii) every optimal protection sequence solving (8) is generated from G in (10); (iv) any protection sequence generated by G in (10) is an optimal protection sequence that solves (8).
- (b) There exists a unique continuous function V that satisfies (9).
- (c) The optimal policy correspondence G defined by (10) is compact-valued and upper hemicontinuous. (See Appendix A for Proof)

Given *Lemma 2*, I can characterize the optimal protection sequence of H by characterizing $G(\cdot)$ in (10) because any protection sequence generated by G with the initial τ_{-1} being set at l is an optimal protection sequence that solves (8). Utilizing one of generalized envelope theorems of Milgrom and Segal (2002) and a general result on the differentiability of the value function of Cotter and Park (2006), I can characterize V and G as follows:²²

Lemma 3.

²¹ Note that limiting H's protection choice to be equal or less than h as in (9) does not affect the generality of the optimization problem because H has no incentive to raise its protection level above its static optimal protection level, h .

²² In characterizing V and G , I cannot use the well-known result of Benveniste and Scheinkman (1979) on the differentiability of the value function. While Benveniste and Scheinkman established that concavity of the return function on the state and choice variables is sufficient to guarantee the differentiability of the resulting value function of a typical dynamic programming problem, the dynamic problem of choosing the optimal protection sequence analyzed in this paper does not belong to the typical dynamic programming problem, as explained earlier. Recently, Milgrom and Segal (2002) developed generalized envelope theorems for arbitrary choice sets, and Cotter and Park (2006) established differentiability of the value function on the range of the optimal policy correspondence, regardless of the curvature of the return function. I apply these results in characterizing V and G , as shown in the proof of *Lemma 3*.

Assume that the lengths of punishment phases satisfy the conditions in *Lemma 1 (a)*.

(a) $V(\tau_{-1})$ is strictly decreasing in $\tau_{-1} \in [0, h]$.

(b) $G(\tau_{-1})$ is strictly increasing in τ_{-1} in the sense that $g(\tau''_{-1}) > g(\tau'_{-1})$ for all $\tau''_{-1} > \tau'_{-1} \in [0, h]$ with $g(\tau''_{-1}) \in G(\tau''_{-1})$ and $g(\tau'_{-1}) \in G(\tau'_{-1})$. (See Appendix A for Proof)

Because a higher τ_{-1} (a higher protection level in the cooperative previous period) implies a higher probability that F triggers a tariff war phase in the current period, a higher τ_{-1} also implies a more hostile environment for H to maximize its discounted payoff. Therefore, the outcome of the maximization problem, $V(\tau_{-1})$, will get smaller as τ_{-1} increases (*Lemma 3 (a)*).

To understand *Lemma 3 (b)*, first note that choosing τ (a current-period protection level) is an act to balance the current period's loss from setting the protection level below h (the static optimal one) against the future periods' gain from reducing the probability of a tariff war. Figure 1 demonstrates this. Given the previous-period protection level τ_{-1} is equal to τ'_{-1} , setting $\tau = h$ maximizes $F(\tau'_{-1}, \tau) = Pr^*(\tau'_{-1})u(\tau, h^*) + [1 - Pr^*(\tau'_{-1})]u(\tau, l^*) + Pr^*(\tau'_{-1})[\delta^*V_C + (\delta^C - \delta^*)V_N]$, because it maximizes the expected current period payoff, $Pr^*(\tau'_{-1})u(\tau, h^*) + [1 - Pr^*(\tau'_{-1})]u(\tau, l^*)$ and τ does not affect the future expected discounted payoff contingent upon a tariff war being initiated in the current period, $(\delta^C - \delta^*)V_N + \delta^*V_C$. By reducing τ below h , however, H can increase its expected discounted payoff, $F(\tau'_{-1}, \tau) + \delta^C[1 - Pr^*(\tau'_{-1})]V(\tau)$ because $V(\tau)$ strictly decreases in τ by *Lemma 3 (a)*. As shown in Figure 1, if H lowers τ from h , $\delta^C[1 - Pr^*(\tau'_{-1})]V(\tau)$ strictly increases. Therefore, $g(\tau'_{-1})$, the optimal current-period protection with τ'_{-1} being the previous-period protection level, is lower than h .

Given this understanding of the optimal choice over τ as a balancing act between the static incentive to raise τ closer to h and the dynamic incentive to avoid a tariff war by reducing τ , I can explain why $G(\tau_{-1})$ strictly increases in τ_{-1} using Figure 1. When τ_{-1} increases from τ'_{-1} to τ''_{-1} , it may shift $F(\tau_{-1}, \tau)$ upwards as shown in Figure 1 but it will not affect $\partial F(\tau_{-1}, \tau) / \partial \tau = \partial u(\tau, l^*) / \partial \tau$, implying that the static incentive to raise τ closer to h stays the same; for example, $F(\tau''_{-1}, h) - F(\tau''_{-1}, g(\tau'_{-1})) = F(\tau'_{-1}, h) - F(\tau'_{-1}, g(\tau'_{-1}))$ in Figure 1. An increase in τ_{-1} , however, weakens the dynamic incentive for lowering τ to avoid a tariff war in a future period

because it increases the likelihood of a tariff war phase starting in the current period. Figure 1, illustrates this by $\delta^C [1 - Pr^*(\tau''_{-1})][V(g(\tau'_{-1})) - V(h)] < \delta^C [1 - Pr^*(\tau'_{-1})][V(g(\tau'_{-1})) - V(h)]$ with $Pr^*(\tau''_{-1}) > Pr^*(\tau'_{-1})$; the dynamic gains from reducing τ from h to $g(\tau'_{-1})$ decreases as τ_{-1} increases from τ'_{-1} to τ''_{-1} . As a result, a higher τ_{-1} moves the balance for choosing an optimal τ towards a higher current period protection level so that $g(\tau''_{-1}) > g(\tau'_{-1})$ as shown in Figure 1.

The fact that $G(\tau_{-1})$ is strictly increasing in τ_{-1} entails both an increasing protection sequence and a decreasing one as shown in Figure 2; if $\tau_0 = l = \tau'_0$, then the optimal protection sequence will be increasing with $\tau'_0 < \tau'_1 < \tau'_2 < \dots$; and if $\tau_0 = l = \tau''_0$, then the optimal protection sequence will be decreasing with $\tau''_0 > \tau''_1 > \tau''_2 > \dots$.²³ If $\tau_0 = l = \tau_S$, however, the resulting optimal protection sequence will be stationary with $\tau_0 = \tau_1 = \tau_2 = \dots$. If there exists such a stationary protection level, $\tau_S \in [0, h)$ under *PTS* with $G(\tau_S) = \tau_S$ and $l = \tau_S$, then H would continue to set its protection level at l until a tariff war phase begins. Therefore, the existence of such a stationary protection level, τ_S , is a prerequisite for *PTS* to be a supergame equilibrium of the repeated game. An increasing optimal policy correspondence (*Lemma 3 (b)*) itself, however, does not rule out the possibility that the only stationary protection level of the dynamic problem in (9) is h , as demonstrated by $G'(\tau_{-1})$ in Figure 2.

To address the existence issue of a stationary protection level $\tau_S \in [0, h)$ with $G(\tau_S) = \tau_S$, I analyze a necessary condition for such τ_S . If $V(\tau)$ is differentiable with respect to τ , then τ_S should satisfy the following first order condition for a stationary equilibrium, denoted by *IC*:

$$(11) \quad \mathbf{IC:} \quad \partial F(\tau_S, \tau_S)/\partial \tau + \delta^C [1 - Pr^*(\tau_S)] \cdot [\partial V(\tau_S)/\partial \tau] = 0,$$

where $\partial F(\tau_S, \tau_S)/\partial \tau = \partial u(\tau_S, l^*)/\partial \tau$ and $\partial V(\tau_S)/\partial \tau = -[\partial Pr^*(\tau_S)/\partial \tau] \{u(\tau_S, l^*) + \delta^C V(\tau_S) - [u(\tau_S, h^*) + (\delta^C - \delta^*)V_N + \delta^*V_C]\}$. While I cannot assume differentiability of $V(\tau)$ on $\tau \in [0, h]$ as explained earlier, $V(\tau)$ is differentiable on any $\tau \in G(\tau_{-1})$ and $\tau \in (0, h)$ for each $\tau_{-1} \in [0, h]$, according to a generalized differentiability result of Cotter and Park (2006). Therefore, (11) is

²³ If the cooperative protection level is set too low under *PTS* with $l = \tau'_0$, then H would keep raising the protection level above the cooperative one until it reaches a stationary level, τ_S , and the opposite is true if the cooperative protection level is too high with $l = \tau''_0$. Blonigan and Park (2004) identify that a similar dynamic behavior does emerge in the context of an exporting firm's dynamic pricing problem in the presence of antidumping policy; once an exporting firm becomes subject to antidumping duty, it would either continue to decrease its export price (thus, having the duty increase over time) or continue to increase its export price (thus, having the duty lowered over time) depending on whether the initial export pricing is higher or lower than a stationary pricing.

indeed a necessary condition for any stationary protection level that belongs to $(0, h)$, thus it serves as an incentive constraint (*IC*) for H to sustain the cooperative protection level, l , under *PTS* with $l = \tau_S$.

For τ_S to be a stationary protection level for H, its static incentive to raise its protection level, $\partial F(\tau_S, \tau_S)/\partial \tau > 0$ in (11), needs be balanced by its dynamic incentive to avoid a costly tariff war in the future, $\delta^C[1 - Pr^*(\tau_S)] \cdot [\partial V(\tau_S)/\partial \tau] < 0$ in (11). *Lemma 4 (a)* below provides a sufficient condition for the existence of such $\tau_S \in (0, h)$ with $G(\tau_S) = \tau_S$ and *Lemma 4 (b)* shows that H does not have any incentive to utilize tariffs as part of its deviation path if $l = \tau_S$.

Lemma 4.

Assume that the lengths of tariff war phases satisfy the conditions in *Lemma 1 (a)*.

(a) If $\partial^2 Pr^*(\tau)/(\partial \tau)^2 > 0$ with $[\partial^2 Pr^*(\tau)/(\partial \tau)^2][1 - Pr^*(\tau)] - \{1 + \delta^C[1 - Pr^*(\tau)]\}[\partial Pr^*(\tau)/\partial \tau]^2 > 0$ for all $\tau \in [0, h]$ and $\partial Pr^*(\tau)/\partial \tau \approx 0$ at $\tau = 0$, then there exists a unique stationary equilibrium protection level $\tau_S \in (0, h)$ with $G(\tau_S) = \tau_S$. τ_S is also a globally stable equilibrium with $G(\tau) > \tau$ for $\tau \in [0, \tau_S)$ and $G(\tau) < \tau$ for $\tau \in (\tau_S, h)$.²⁴

(b) If $l = \tau_S$, then H cannot increase its discounted payoff above $V(\underline{s}, \underline{s}^*)$ by taking any (deviatory) protection sequence that involves initiating tariff wars by imposing tariffs.

(See Appendix A for Proof)

According to *Lemma 4 (a)*, it is possible to have *IC* in (11) satisfy for some $\tau_S < h$ if the sensitivity of F's private information in detecting a rise in H's concealed protection, $\partial Pr^*(\tau_S)/\partial \tau$, increases as H's concealed protection level rises with $\partial^2 Pr^*(\tau)/(\partial \tau)^2 > 0$. On the one hand, H's static incentive to raise its protection level, $\partial F(\tau_S, \tau_S)/\partial \tau = \partial u(\tau_S, l^*)/\partial \tau$ in (11), diminishes as τ_S increases with $\partial^2 u(\tau_S, l^*)/\partial \tau^2 < 0$, reaching zero at $\tau_S = h$. On the other hand, H's dynamic incentive to avoid a future tariff war, $\delta^C[1 - Pr^*(\tau_S)] \cdot [\partial V(\tau_S)/\partial \tau]$ in (11), may diminish or intensify in response to an increase in τ_S , depending on the value that $\partial^2 Pr^*(\tau_S)/\partial \tau^2$ takes. A higher τ_S reduces H's weight on its dynamic incentive to avoid a tariff war, $1 - Pr^*(\tau_S)$, by increasing the probability of a tariff war in the current period. If $\partial^2 Pr^*(\tau_S)/\partial \tau^2 > 0$, an enhanced

²⁴ τ_S being a globally stable protection level is a contributing factor to the stability of *PTS* as an equilibrium of the repeated game. This is because H will eventually return to its globally stable behavior of setting $\tau = \tau_S (= l)$ after any arbitrary perturbations (possibly caused by errors) in its protection level choices.

sensitivity of F's private information in detecting a rise in H's protection can offset such a reduction in H's incentive to avoid a tariff war; the absolute value of $\delta^C[1 - Pr^*(\tau_S)] \cdot [\partial V(\tau_S)/\partial \tau]$ rises in response to a rise in τ_S if $[\partial^2 Pr^*(\tau)/(\partial \tau)^2][1 - Pr^*(\tau)] - \{1 + \delta^C[1 - Pr^*(\tau)]\}[\partial Pr^*(\tau)/\partial \tau]^2 > 0$ for all $\tau \in [0, h]$, as assumed in *Lemma 4 (a)*. This in turn guarantees the existence of a unique $\tau_S \in (0, h)$ that satisfies *IC* in (11) if $\partial Pr^*(\tau)/\partial \tau \approx 0$ at $\tau = 0$.

Having the sensitivity of private information rise against increasing concealed protection can be crucial in discouraging the use of concealed protection under *PTS*. If $\partial^2 Pr^*(\tau)/(\partial \tau)^2 = 0$, for example, the dynamic incentive for lowering τ below h to avoid a tariff war in a future period, $\delta^C[1 - Pr^*(\tau_S)] \cdot [\partial V(\tau_S)/\partial \tau]$ in (11), decreases as τ_S gets higher, entailing the possibility of *IC* in (11) not being satisfied for any $\tau_S < h$.

While *Lemma 4* specifies the condition under which H (and F under an analogous condition) would follow *PTS* by keeping its protection at a cooperative level until a tariff war is triggered, note that *Lemma 4* “assumes” the lengths of tariff war phases to satisfy the conditions in *Lemma 1 (a)*. Because such lengths of tariff phases “vary” with the cooperative protection levels (l, l^*) to sustain under *PTS*, it still remains to be shown whether there exist *PTS* that satisfy *ICP*, *ICP**, *IC*, and *IC** simultaneously. One obvious candidate is symmetric *PTS* for symmetric countries with $l = l^*$, $\Omega^D = \Omega^{D*}$, and $\delta = \delta^*$, which is the focus of Section 3.

3. Optimal Symmetric Private Trigger Strategies for Symmetric Countries

This section establishes that symmetric countries can sustain a symmetric cooperative protection level under symmetric *PTS* if the sensitivity of their private information satisfies certain conditions. In addition, this section proves that any equilibrium symmetric trigger strategies that initiate a punishment period by imposition of a static Nash tariff based on each country's imperfect private information should generate the same payoff as the one that the specific symmetric *PTS* defined in *Definition 2* would yield. After proving the existence and the uniqueness (in terms of payoffs, at least among a certain class of trigger strategies) of symmetric *PTS* as a supergame equilibrium in Section 3.1, I characterize optimal symmetric *PTS* under which H and F maximize their joint expected discounted payoffs in Section 3.2.

3.1. Symmetric Private Trigger Strategies and Uniqueness Results

To analyze the case where H and F are symmetric, this section assumes that $u(\tau^1, \tau^2) = u^*(\tau^1, \tau^2)$ for all τ^1 and $\tau^2 \in [0, h]$ and that $Pr(\omega_t \in \mathcal{D}) = Pr(\omega_t^* \in \mathcal{D}^*)$ for all $(\tau_t, e_t) = (\tau_t^*, e_t^*)$ and $\mathcal{D} = \mathcal{D}^*$. This section focuses on symmetric *PTS* with $l = l^*$, $\mathcal{D} = \mathcal{D}^*$, and $\delta = \delta^*$. If symmetric *PTS* satisfy *ICP* and *IC* for H, such *PTS* will also satisfy *ICP*^{*} and *IC*^{*} for F by symmetry. Without loss of generality, the following analysis focuses on proving the existence of symmetric *PTS* that satisfy *ICP* and *IC*.

Assume that there exists τ_S that satisfies *IC* in (11) with $\tau_S = l$. This implies that $V(\tau_S) = V_C$, and I can rewrite *IC* in (11) as follows:

$$(12) \quad \partial u(\tau_S, l^*) / \partial \tau = \delta^C [\partial Pr^*(\tau_S) / \partial \tau] [1 - Pr^*(\tau_S)] [u(\tau_S, l^*) - u(\tau_S, h^*) + (\delta^C - \delta^*)(V_C - V_N)].$$

As discussed in the previous section, (12) is a necessary condition for H to have no incentive to change its protection level away from the cooperative one until a tariff war phase starts.

To guarantee that *PTS* satisfy *ICP*, I assume that the lengths of tariff phases are determined by the sufficient condition for *ICP* in Lemma 1 (a); $\delta^C - \delta = [u(h, l^*) - u(l, l^*)] / (V_C - V_N)$ ($= \delta^C - \delta^* = [u^*(h^*, l) - u^*(l^*, l)] / (V_C^* - V_N^*)$ by symmetry) and $\delta^C + \delta^S = (\delta^* + \delta)$. Denoting the symmetric cooperative protection level under symmetric *PTS* by l^c ($= l = l^*$), *IC* in (12) can then be rewritten into the following implicit function, $I(l^c)$:

$$(13) \quad I(l^c) \equiv \partial u(l^c, l^c) / \partial \tau - \delta^C [\partial Pr^*(l^c) / \partial \tau] [1 - Pr^*(l^c)] [u(h, l^c) - u(l^c, h^*)] = 0$$

by substituting $\delta^C - \delta^*$ with $[u(h, l^*) - u(l, l^*)] / (V_C - V_N)$. Using $I(l^c)$, Proposition 1 provides a sufficient condition for the existence of symmetric *PTS* that countries can sustain as a supergame equilibrium of their repeated protection setting game:

Proposition 1.

If $\partial^2 Pr^*(l^c) / (\partial l^c)^2 > 0$ with $[\partial^2 Pr^*(l^c) / (\partial l^c)^2] [1 - Pr^*(l^c)] - \{1 + \delta^C [1 - Pr^*(l^c)]\} [\partial Pr^*(l^c) / \partial l^c]^2 > 0$ for all $l^c \in [0, h]$, $\partial Pr^*(l^c) / \partial l^c \approx 0$ at $l^c = 0$, and there exists at least one protection level, $l_S^c < h$ such that $I(l_S^c) = 0$, then, symmetric H and F can employ symmetric *PTS* with $l^c = l_S^c$, $\delta = \delta^* = \delta^C - [u(h, l_S^c) - u(l_S^c, l_S^c)] / (V_C - V_N)$, and $\delta^S = \delta + \delta^* - \delta^C$ as a supergame equilibrium of the repeated protection setting game. (See Appendix A for Proof)

Proposition 1 assumes the same condition regarding the sensitivity of private information as the one in Lemma 4 (a), ensuring that there exists a unique stationary equilibrium protection

level $\tau_S \in (0, h)$ with $G(\tau_S) = \tau_S$. In addition, it requires $I(l^c) = 0$ for at least one value of $l^c < h$, denoting it by l_S^c . With $l^c = l_S^c$ and the lengths of punishment phases ($T = T^*$ and T^S) defined by $\delta = \delta^* = \delta^C - [u(h, l_S^c) - u(l_S^c, l_S^c)]/(V_C - V_N)$, and $\delta^S = \delta + \delta^* - \delta^C$, $I(l_S^c) = 0$ guarantees *IC* and *ICP* to be simultaneously satisfied under such *PTS*. According to *Lemma 4*, l_S^c is the unique stationary protection level with $G(l_S^c) = l_S^c$ and countries have no incentive to deviate from such *PTS*.

The sufficient condition in *Lemma 4 (a)* does not necessarily imply that the second term of $I(l^c)$ in (13), $\delta^C [\partial Pr^*(l^c)/\partial \tau] [1 - Pr^*(l^c)] [u(h, l^c) - u(l^c, h^*)]$, representing H's dynamic incentive to avoid a tariff war, increases in response to a rise in l^c .²⁵ Thus, one may consider the case where multiple values of l^c satisfy $I(l^c) = 0$ as illustrated in Figure 3; $l^c = l_{\max}$ as well as $l^c = l_{\min}$ satisfy $I(l^c) = 0$. Denoting the minimum of such l^c by l_{\min} , then symmetric *PTS* with $l^c = l_{\min}$ will Pareto-dominate the others when $Pr^*(l^c)$ is small enough.²⁶

While the above result establishes that symmetric countries may employ *simple symmetric PTS* characterized by *Definition 2* and *Proposition 1* in restraining the use of concealed trade barriers, one may wonder whether there exist other (symmetric) private trigger strategies that may outperform this simple one. Surprisingly, the following result shows that there is no loss of generality in focusing on this *simple PTS* to characterize the optimal *PTS* as long as the initial explicit tariff that starts a punishment phase is the static optimal tariff of each country. The first part of the following result applies to a larger class of trigger strategies: any symmetric *PTS* where each country can start the initial punishment phase with an explicit tariff of any level.

Denote the level of an explicit tariff that initiates the first punishment phase by d_0 and the level of a cooperative protection level for the initial cooperative periods (prior to any punishment being triggered) by l_0 . Then, I can represent the expected discounted payoff of H of employing such a symmetric *PTS* as follows, denoting it by $V^P(l_0; d_0)$:

²⁵ For the proof of this claim, see the proof for *Proposition 1* in Appendix A.

²⁶ Note that $u(l_{\min}, l_{\min}) > u(l_{\max}, l_{\max})$ and $Pr^*(l_{\min}) < Pr^*(l_{\max})$ imply a higher cooperative-period payoff and a lower probability of tariff wars with $l^c = l_{\min}$ than with $l^c = l_{\max}$. While the lengths of tariff war phases may be longer with $l^c = l_{\min}$ than with $l^c = l_{\max}$, an increase in l^c will lower the expected discounted payoff under symmetric *PTS* if $Pr^*(l^c)$ is close enough to 0, as shown in (15) of the following section.

$$(15) \quad V^P(l_0; d_0) = (1 - Pr)(1 - Pr^*)[u(l_0, l_0) + \delta^C V_C^P] + (1 - Pr)Pr^*[u(l_0, d_0) + \delta^C V_{D^*}^P] \\ + Pr(1 - Pr^*)[u(d_0, l_0) + \delta^C V_D^P] + PrPr^*[u(d_0, d_0) + \delta^C V_{D_S}^P],$$

where $Pr^* \equiv Pr^*(l_0) = Pr(\omega_t^* \in \mathcal{D}^*)$ given $(\tau_t, e_t) = (l_0, 0)$ and $(\tau_t^*, e_t^*) = (l_0, 0)$, $V_C^P \equiv V^P(l_0; d_0)$, and V_D^P , $V_{D^*}^P$, and $V_{D_S}^P$ denote the expected discounted payoff of H after an initial punishment phase had been by triggered, respectively by H alone, by F alone, and by H and F simultaneously in the previous period, with $Pr = Pr^*$ and $V_D^P = V_{D^*}^P$ by the symmetry assumption. The payoff function in (15) implicitly assumes that each country sets its initial cooperative explicit tariff to be zero and each country starts the initial punishment phase by setting $(\tau, e) = (d_0, d_0)$. Relaxing the latter assumption by allowing $e \neq d_0$ (and $\tau = d_0$) would not affect the following result because the equilibrium value of $V^P(l_0; d_0)$ does not depend on the values of V_D^P , $V_{D^*}^P$, and $V_{D_S}^P$, as shown later. Relaxing the former assumption by allowing $e > 0$ in the initial cooperative periods would not raise the payoff in (15) as long as the sensitivity of private information of concealed trade barriers improves with a higher level of concealed trade barriers.²⁷ For the proposed *PTS* to be incentive-compatible (so that they can be supported as equilibrium behaviors), the payoff in (15) needs to be equal to the following expression:

$$(15') \quad V^P(l_0; d_0) = \frac{u(l_0, l_0)}{1 - \delta^C} - Pr \frac{u(d_0, l_0) - u(l_0, d_0)}{1 - \delta^C}.$$

Note that the payoff in (15') depends only on the values of l_0 and d_0 . Once established, the above result will remarkably simplify the job of characterizing the payoff frontier attainable under any symmetric *PTS* that rely on triggering a punishment phase with some explicit tariffs: one only need to find l_0 and d_0 that are incentive-compatible, which in turn maximize the payoff in (15').

To prove (15'), I use three incentive-compatibility conditions: (i) a generalized version of *ICP*, (ii) *IC* for setting the initial protection level to be l_0 with the action of setting $\tau = l_0$ being followed upon the contingency of no punishment phase being initiated and (iii) *IC* for setting the initial protection level to be l_0 with the action of setting $\tau = d_0$ being followed upon the contingency of no punishment phase being initiated. First, note that any symmetric *PTS*

²⁷ $\tau = \tau^* = 0$ forces each country to raise its protection level all through concealed trade barriers. If the sensitivity of private information improves with a higher level of such barriers, the effectiveness of private trigger strategies against the incentive to raise protection levels should improve with such constraints.

triggering the initial punishment phase with an explicit tariff of d_0 should satisfy the following ICP^P :

$$(ICP^P) \quad \begin{aligned} & (1 - Pr^*)[u(l_0, l_0) + \delta^C V_C^P] + Pr^*[u(l_0, d_0) + \delta^C V_{D^*}^P] \\ & = (1 - Pr^*)[u(d_0, l_0) + \delta^C V_D^P] + Pr^*[u(d_0, d_0) + \delta^C V_{D_s}^P] \end{aligned}$$

which equalizes the payoff of initiating the (initial) punishment phase with the payoff of not initiating it, similarly to ICP in the previous section. Using $u(l_0, l_0) - u(l_0, d_0) = u(d_0, l_0) - u(d_0, d_0)$ implied by $\partial^2 u / \partial \tau \partial \tau^* = 0$, once again I can simplify (ICP^P) as follows:

$$(ICP^P) \quad u(l_0, l_0) - u(d_0, l_0) + \delta^C (V_C^P - V_{D^*}^P) = \delta^C Pr^* [(V_C^P - V_D^P) + (V_C^P - V_{D^*}^P) - (V_C^P - V_{D_s}^P)].$$

To identify ICs for setting the initial protection level to be l_0 , I can write the expected discounted payoff of H of choosing its initial protection level to be τ_0 as

$$\begin{aligned} & Pr^*[u(\tau_0, d_0) + \delta^C V_D^P] \\ & + (1 - Pr^*)\{u(\tau_0, l_0) + \delta^C Pr^*(\tau_0)[u(l_0, d_0) + \delta^C V_{D^*}^P] + \delta^C [1 - Pr^*(\tau_0)][u(l_0, l_0) + \delta^C V_C^P]\}, \end{aligned}$$

or

$$\begin{aligned} & Pr^*[u(\tau_0, d_0) + \delta^C V_D^P] \\ & + (1 - Pr^*)\{u(\tau_0, l_0) + \delta^C Pr^*(\tau_0)[u(d_0, d_0) + \delta^C V_{D_s}^P] + \delta^C [1 - Pr^*(\tau_0)][u(d_0, l_0) + \delta^C V_D^P]\} \end{aligned}$$

depending on whether H sets $\tau = l_0$ or $\tau = d_0$, respectively, upon the contingency of no punishment phase being initiated after setting $\tau = \tau_0$ as the initial protection level. To be able to support $\tau_0 = l_0$ as an equilibrium behavior, the following IC^P need to be satisfied

$$(IC^P) \quad \begin{aligned} & \frac{\partial u(l_0, l_0)}{\partial \tau} = \delta^C (1 - Pr^*) \frac{\partial Pr^*(l_0)}{\partial l_0} [u(l_0, l_0) - u(l_0, d_0) + \delta^C (V_C^P - V_{D^*}^P)], \\ & \text{and} \\ & \frac{\partial u(l_0, l_0)}{\partial \tau} = \delta^C (1 - Pr^*) \frac{\partial Pr^*(l_0)}{\partial l_0} [u(d_0, l_0) - u(d_0, d_0) + \delta^C (V_D^P - V_{D_s}^P)]. \end{aligned}$$

which implies that $u(l_0, l_0) - u(l_0, d_0) + \delta^C (V_C^P - V_{D^*}^P) = u(d_0, l_0) - u(d_0, d_0) + \delta^C (V_D^P - V_{D_s}^P)$. Using this last equality together with the simplified ICP^P and $V_D^P = V_{D^*}^P$ (by symmetry), one can easily show that $(V_C^P - V_D^P) + (V_C^P - V_{D^*}^P) - (V_C^P - V_{D_s}^P) = 0$, thus $u(d_0, l_0) - u(l_0, l_0) = \delta^C (V_C^P - V_D^P)$. Given these equalities, one can rewrite $V^P(l_0; d_0)$ in (15) into the one in (15') using the following steps:

$$\begin{aligned}
V^P(l_0; d_0) &= u(l_0, l_0) + \delta^C V_C^P + Pr[u(l_0, d_0) - u(l_0, l_0) - \delta^C (V_C^P - V_D^P)] \\
&\quad + Pr^*[u(d_0, l_0) - u(l_0, l_0) - \delta^C (V_C^P - V_{D^*}^P)] \\
(16) \quad &\quad + PrPr^*\{[u(d_0, d_0) - u(l_0, d_0)] - [u(d_0, l_0) - u(l_0, l_0)]\} \\
&\quad + \delta^C PrPr^*[(V_C^P - V_D^P) + (V_C^P - V_{D^*}^P) - (V_C^P - V_{D_s}^P)] \\
&= u(l_0, l_0) + \delta^C V_C^P + Pr[u(l_0, d_0) - u(l_0, l_0) - u(d_0, l_0) + u(l_0, l_0)],
\end{aligned}$$

where the second line through the forth one in (16) all take zero values and the last line in (16) generates (15') using $V^P(l_0; d_0) = V_C^P$. I can summarize this result in the following lemma.

Lemma 5. The payoff of any equilibrium symmetric *PTS* with the initial cooperative protection level being l_0 and the level of explicit tariff that starts the initial punishment phase being d_0 , denoted by $V^P(l_0; d_0)$, is a function of only l_0 and d_0 with

$$V^P(l_0; d_0) = \frac{u(l_0, l_0)}{1 - \delta^C} - Pr \frac{u(d_0, l_0) - u(l_0, d_0)}{1 - \delta^C},$$

where $Pr = Pr^* \equiv Pr^*(l_0) = Pr(\omega_t^* \in \mathcal{Q}^{D^*})$ given $(\tau_t, e_t) = (l_0, 0)$ and $(\tau_t^*, e_t^*) = (l_0, 0)$.

The above lemma establishes that one can fully characterize the payoff of any equilibrium symmetric *PTS* only with the information about l_0 and d_0 that are incentive-compatible. Given $u(d_0, l_0) - u(l_0, l_0) = \delta^C (V_C^P - V_D^P)$, the necessary condition for incentive compatible choice of l_0 is

$$(IC^P) \quad \frac{\partial u(l_0, l_0)}{\partial \tau} = \delta^C (1 - Pr^*) \frac{\partial Pr^*(l_0)}{\partial l_0} [u(d_0, l_0) - u(l_0, d_0)],$$

a condition that is identical to *IC* for simple symmetric *PTS* characterized in (13) if $d_0 = h$, the static optimal protection level. While *Proposition 1* guarantees the existence of incentive-compatible symmetric *PTS* with $d_0 = h$, characterizing the necessary condition for incentive compatible choice of $d_0 \neq h$ is far from being a simple task. Once $d_0 \neq h$, then each country would have an incentive to start an initial punishment by choosing $e = d_0$ but $\tau \neq d_0$, necessitating a punishment scheme against such a deviation incentive. In fact, one can show that the only incentive compatible choice of d_0 is h , if one does not allow each country to start another punishment phase in the second period of an initial punishment phase that it started.

Even when one ignores the issue of finding an incentive-compatible d_0 and pretends that one can choose any value for d_0 , it is not clear whether choosing $d_0 \neq h$ would increase the

expected discounted payoff in comparison with the choice of setting $d_0 = h$. For example, setting $d_0 < h$ would raise $V^P(l_0; d_0) = \{u(l_0, l_0) - \text{Pr}[u(d_0, l_0) - u(l_0, d_0)]\}/(1 - \delta^C)$ by decreasing $u(d_0, l_0) - u(l_0, d_0)$ with $\partial[u(d_0, l_0) - u(l_0, d_0)]/\partial d_0 > 0$. However, lowering d_0 weakens the IC^P by lowering the right hand side value of IC^P shown above, thus decreasing $V^P(l_0; d_0)$ by raising the value for l_0 . The optimality of choosing $d_0 \neq h$, even when it is incentive-compatible, therefore, depends on the trade-off between its direct on the payoff through changing $u(d_0, l_0) - u(l_0, d_0)$ and its indirect effect through changing the incentive-compatible l_0 , which in turn requires further characterization of the private information of concealed trade barriers.

In the following analysis of characterizing the optimal symmetric PTS , I will focus on the optimal symmetric PTS with $d_0 = h$. Note that this constraint of setting $d_0 = h$ allows full flexibility over the choice of strategies that each government can take once an initial punishment phase starts. With regard to the issue of characterizing the efficient frontier among this subset of symmetric PTS with $d_0 = h$, one can focus on the *simple* symmetric PTS characterized in *Proposition 1* as the following corollary clarifies.

Corollary to Lemma 5. The equilibrium payoff of any symmetric PTS with the initial cooperative protection level being l^c and the level of explicit tariff that starts the initial punishment phase being h , is identical to the payoff of the *simple* symmetric PTS characterized in *Proposition 1* with

$$V^P(l^c; h) = V_C = \frac{u(l^c, l^c)}{1 - \delta^C} - \text{Pr} \frac{u(h, l^c) - u(l^c, h)}{1 - \delta^C} \text{ and}$$

$$\frac{\partial u(l^c, l^c)}{\partial \tau} = \delta^C (1 - \text{Pr}^*) \frac{\partial \text{Pr}^*(l^c)}{\partial l^c} [u(h, l^c) - u(l^c, h)],$$

where $\text{Pr} = \text{Pr}^* \equiv \text{Pr}^*(l_0) = \text{Pr}(\omega_t^* \in \mathcal{Q}^{D*})$ given $(\tau_t, e_t) = (l_0, 0)$ and $(\tau_t^*, e_t^*) = (l_0, 0)$.

Note that one can derive H's expected discounted payoff under symmetric PTS , V_C , shown in the above corollary, from (6), using $\delta^C - \delta = \delta^C - \delta^* = [u(h, l^c) - u(l^c, l^c)]/(V_C - V_N)$, $\delta^C + \delta^S = (\delta^* + \delta)$ and $\text{Pr}^*(l^c) = \text{Pr}(l^c)$.

Up to this point, I have assumed that the range of private signals that trigger a tariff war phase (\mathcal{Q}^D for H and \mathcal{Q}^{D*} for F) is fixed. Symmetric countries can change the cooperative protection level l^c under symmetric PTS by changing the range of tariff-war-triggering private

signals, $\Omega^D = \Omega^{D*}$, because it affects the probability of a tariff war being triggered. The following section characterizes optimal symmetric *PTS*, focusing its analysis on the choice of $\Omega^D (= \Omega^{D*})$ that maximizes the expected discounted payoff under *PTS*.

3.2 Optimal Symmetric Private Trigger Strategy

The private signal $\omega \in \Omega$ has two distinctive quality dimensions as a measure that detects the potential use of concealed protection. One is the sensitivity of the signal in detecting possible defections, which links a higher protection to a higher probability of a tariff war. The other is the stability of the signal that rewards the cooperative behavior with a lower probability of a tariff war. I can represent the sensitivity by $Pr'(\tau^*) \equiv \partial Pr(\tau^*)/\partial \tau^* > 0$ and the stability by $1 - Pr(\tau^*)$ at $\tau^* = \ell$ for H's private signal, with corresponding expressions for F.

A change in the range of private signals that trigger a tariff war phase affects these qualities of signals. In particular, H may raise the sensitivity by properly expanding the range of tariff-war-triggering private signals, Ω^D , but at the cost of undermining the stability. By denoting the degree of such expansion with a parameter ω^D (namely, a *trigger control variable*), I can formalize this trade-off that H faces in choosing ω^D by assuming $\partial Pr'(\tau^*)/\partial \omega^D > 0$ and $\partial Pr(\tau^*)/\partial \omega^D > 0$.

The analysis of optimality in this section constrains itself to the symmetric *PTS* identified in *Proposition 1* with the cooperative protection level being determined by a choice over ω^D . Assuming that ω^D uniquely determines l_S^c with $I(l_S^c) = 0$, I can represent l_S^c as a function of ω^D ; $l_S^c = l_S^c(\omega^D)$. Then, as shown in *Corollary to Lemma 5*,

$$(17) \quad V_C \equiv V(\underline{s}, \underline{s}^*) = \frac{u(l_S^c, l_S^c)}{1 - \delta^C} - Pr(l_S^c) \frac{u(h, l_S^c) - u(l_S^c, h^*)}{1 - \delta^C}$$

Note that the expected discounted payoff in (14) is no longer depending on the lengths of the tariff war phases. Therefore, I can describe the optimal choice for $\omega^D (= \omega^{D*})$ as ω^D that satisfies the following first order condition:

$$\begin{aligned}
\frac{\partial V_C}{\partial \omega^D} &= \frac{\partial V_C}{\partial l_S^c} \frac{\partial l_S^c(\omega^D)}{\partial \omega^D} + \frac{\partial V_C}{\partial Pr} \frac{\partial Pr(l_S^c)}{\partial \omega^D} = 0, \text{ with} \\
\frac{\partial V_C}{\partial l_S^c} &= \frac{\partial u(l_S^c, l_S^c)}{\partial l_S^c} \frac{1}{1 - \delta^c} - Pr'(l_S^c) \frac{u(h, l_S^c) - u(l_S^c, h^*)}{1 - \delta^c} \\
(18) \quad & - Pr(l_S^c) \frac{\{\partial u(l_S^c, l_S^c) / \partial \tau^* - \partial u(l_S^c, l_S^c) / \partial \tau\}}{1 - \delta^c} < 0 \text{ for } Pr(l_S^c) \text{ being close to } 0, \\
\frac{\partial l_S^c(\omega^D)}{\partial \omega^D} &= - \frac{\partial I / \partial \omega^D}{\partial I / \partial l_S^c} < 0 \text{ iff } \frac{\partial Pr'(l_S^c)}{\partial \omega^D} [1 - Pr(l_S^c)] - \frac{\partial Pr(l_S^c)}{\partial \omega^D} Pr'(l_S^c) > 0, \text{ and} \\
\frac{\partial V_C}{\partial \omega^D} \frac{\partial Pr(l_S^c)}{\partial \omega^D} &= - \frac{\partial Pr(l_S^c)}{\partial \omega^D} \frac{[u(h, l_S^c) - u(l_S^c, h^*)]}{1 - \delta^c} < 0,
\end{aligned}$$

where $I = I(l_S^c)$ is the implicit function defined in (13). The first order condition is informative about the trade-off that the countries face in choosing an optimal ω^D . Raising the trigger control variable (ω^D) will have a positive effect on the expected discounted payoff (V_C) by lowering the cooperative protection level (l_S^c) if $\partial l_S^c / \partial \omega^D < 0$ and $\partial V_C / \partial l_S^c < 0$, but it also has a negative effect on the expected payoff by increasing the probability of a tariff war phase being invoked, as shown by $\partial V_C / \partial \omega^D < 0$ in (18). Thus, the optimal ω^D should balance the gain from raising the sensitivity of the private signal (thus achieving a lower l_S^c) against the loss from reducing the stability of the cooperative equilibrium with a higher tariff war probability.

When the initial ω^D is at a very low level, then, it is generally possible to lower l_S^c by raising the trigger control variable. For example, if $\Omega^D = \emptyset$, then $l_S^c = h$, $Pr(l_S^c) = Pr'(l_S^c) = 0$, implying $\partial l_S^c / \partial \omega^D < 0$ with $\partial Pr'(l_S^c) / \partial \omega^D > 0$ from (18). If countries continue to raise ω^D , the marginal increase in the sensitivity of private signals in response to an increase in ω^D is likely to get smaller. To formalize this decreasing return to raising the trigger control variable, I assume that $\partial^2 Pr'(l_S^c) / \partial (\omega^D)^2 < 0$ and $\partial^2 Pr(l_S^c) / \partial (\omega^D)^2 = 0$, with the latter assumption making the effect of a higher ω^D on $Pr(l_S^c)$ to be constant. Then, it is possible to have $\partial^2 l_S^c / \partial (\omega^D)^2 > 0$ and $\partial l_S^c / \partial \omega^D = 0$ for a high enough ω^D . While it is possible to raise ω^D to such a point that the countries would no longer be able to lower the cooperative protection level any further ($\partial l_S^c / \partial \omega^D = 0$), note that it is never optimal to do so. If it does, then the first order condition for the optimal ω^D in (18) will be violated as $\partial V_C / \partial \omega^D = (\partial V_C / \partial Pr)(\partial Pr(l_S^c) / \partial \omega^D) < 0$, implying

that countries can increase their payoffs by lowering the trigger control variable. I summarize these characterizations of optimal symmetric *PTS* in the following proposition.

Proposition 2.

The choice over the trigger control variable ω^D is a balancing act between raising the sensitivity of the private signal (thus achieving lower cooperative protection levels) and reducing the stability of the cooperative equilibrium (by a higher tariff war probability). As a result, optimal symmetric *PTS* do not raise the trigger control variable to the level that pushes down the cooperative protection level to its minimum attainable level with $\partial l_s^c / \partial \omega^D = 0$.²⁸

In applying Special Section 301 to protect the U.S. intellectual property rights (IPR) in foreign markets, the USTR specifies not only “Priority Foreign Countries” who are “pursuing the most onerous or egregious policies that have the greatest adverse impact on U.S. right holders or products, and are subject to accelerated investigations and possible sanctions,” but also “Priority Watch List” of countries “who do not provide an adequate level of IPR protection or enforcement, or market access for persons relying on intellectual property protection.”²⁹ This practice indicates the USTR’s willingness to tolerate some level of deviations from agreements, reserving retaliatory sanctions mainly against considerable deviations. Such a practice may not lead to the maximal protection of the U.S. IPR, but may reduce the probability of costly tariff wars invoked by Special Section 301.

4. Limitations of Private Trigger Strategies

In this section, I consider factors that may limit the effectiveness of *PTS* in restraining the use of concealed protection. Section 4.1 analyzes how a reduction in the time lag to adjust protection levels in response to an initiation of an explicit tariff war affects the effectiveness of *PTS*. Section 4.2 discusses how asymmetry among countries may limit the use of *PTS*.

4.1. A Faster Response to an Initiation of an Explicit Tariff War

²⁸ A similar characterization has been drawn for optimal cartel trigger price strategies by Porter (1983).

²⁹ These quoted definitions come from the USTR website (<http://www.ustr.gov>).

The analyses in previous sections assume that H and F simultaneously set their concealed and explicit protection levels at the beginning of each period and cannot adjust those protection levels until that period is over. While this is a standard assumption in the literature that analyzes self-enforcing trade agreements in a repeated game setup, the time lag (one period) for countries to readjust their protection levels plays an important role in determining the effectiveness of *PTS* as shown below. In particular, I analyze how reducing the time lag in adjusting protection levels in response to an initiation of an explicit tariff war affects the level of cooperation attainable under *PTS*.³⁰

To analyze the effect of such a reduction in the time lag in adjusting protection levels without changing the basic structure of the model, I represent one-period payoffs for a period when F initiates a tariff war by imposing its static optimal tariff by $u(l, h^*)/n + (n - 1)u(h, h^*)/n$ for H and $u^*(h^*, l)/n + (n - 1)u^*(h^*, h)/n$ for F, with $n \in [1, \infty)$ denoting how fast each country can readjust its protection level in response to an initiation of a tariff war phase by the other country.³¹ Note that $n = 1$ implies no reduction in the time lag to adjust protection levels and $n \rightarrow \infty$ means instantaneous adjustment (no lag). For a period when H initiates a tariff war, one-period payoffs are $u(h, l^*)/n + (n - 1)u(h, h^*)/n$ for H and $u^*(l^*, h)/n + (n - 1)u^*(h^*, h)/n$ for F.

As in Section 3, I continue to focus on symmetric *PTS* for symmetric countries. Then, a change in the speed that each country can readjust its protection level, denoted by n , affects the cooperative protection level that countries can attain under *PTS*, l^c , by affecting the incentive constraint, *IC* in (11) and the lengths of tariff war phases that satisfy *ICP* and *ICP*^{*} in (7).

To analyze how a faster response to an initiation of a tariff war affects the lengths of tariff war phases under symmetric *PTS*, I rewrite *ICP* in (7) as follows:

³⁰ The reduction in the time lag considered in this section does not apply to the time lag in adjusting concealed protection levels, but only to the time lag in adjusting explicit protection levels in response to an initiation of an explicit tariff war. If the speed of adjusting concealed protection levels changes, it may affect countries' optimal protection sequence, thus, the whole optimization problem under *PTS*. While fixing the speed of adjusting concealed protection levels, one may still consider a reduction in the time lag in adjusting protection levels by allowing countries to adjust their explicit tariff levels faster. In comparison with controlling concealed protections, which may require non-explicit and customary arrangements with various players in the market, changing the explicit tariff levels in response to a foreign initiation of a tariff war may take much less time, like issuing an executive order.

³¹ This representation of one-period payoffs for a period when F initiates a tariff war implicitly assumes that H will react to the initiation by adjusting its protection to its static optimal level (h) as soon as possible. Such a reaction is an optimal action for H, thus *PTS* with such reactions would still be a supergame equilibrium as long as *IC*, *IC*^{*}, *ICP*, *ICP*^{*} are satisfied with changes in the speed of such reactions being taken into account.

$$(19) \quad \begin{aligned} & u(l, l^*) + \delta^C V_C - Pr^* \left\{ \frac{u(l, l^*) - u(l, h^*)}{n} + \frac{(n-1)[u(l, l^*) - u(h, h^*)]}{n} + (\delta^C - \delta^*) (V_C - V_N) \right\} = \\ & \frac{u(h, l^*)}{n} + \frac{(n-1)u(h, h^*)}{n} + (\delta^C - \delta) V_N + \delta V_C - Pr^* \left[\frac{u(h, l^*) - u(h, h^*)}{n} + (\delta - \delta^S) (V_C - V_N) \right]. \end{aligned}$$

To have the above *ICP* satisfied for all $Pr^* \in [0, 1]$, $\delta = \delta^*$ and δ^S need to satisfy

$$(20) \quad \begin{aligned} & (\delta^C - \delta) (V_C - V_N) = [u(h, l^*) - u(l, l^*)] - \frac{(n-1)[u(h, l^*) - u(h, h^*)]}{n}, \text{ and} \\ & [(\delta - \delta^S) - (\delta^C - \delta^*)] (V_C - V_N) = \frac{(n-1)[u(l, l^*) - u(h, h^*)]}{n}, \end{aligned}$$

which becomes the same sufficient condition for *ICP* and *ICP*^{*} specified in *Lemma 1(a)* if $n = 1$.

Note that the length of a tariff war phase that H (or F) can initiate, represented by $\delta^C - \delta (= \delta^C - \delta^*$ by symmetry) in (20), decreases in n because the last term on the right side of the first equality in (20), $-(n-1)[u(h, l^*) - u(h, h^*)]/n$ decreases in n . Recall that the length of a tariff war phase that H can initiate under *PTS* increases in its expected gain from imposing its static optimal tariff in the initial period of the tariff war phase, $u(h, l^*) - u(l, l^*)$, as discussed in Section 2.2.1. Because a higher n implies a smaller expected gain in the initial period with $[u(h, l^*)/n + (n-1)u(h, h^*)/n] - u(l, l^*)$ decreasing in n , the length of a tariff war phase H can employ gets shorten with a higher n , weakening the punishment against concealed protection.

Now, using (20) and $F(\tau_d, \tau_{d+1}) = Pr^*(\tau_d)[u(\tau_{d+1}, h^*)/n + (n-1)u(h, h^*)/n + (\delta^C - \delta^*)V_N + \delta^*V_C] + [1 - Pr^*(\tau_d)]u(\tau_{d+1}, l^*)$, I can rewrite *IC* in (11) into:³²

$$(21) \quad \begin{aligned} & \partial u(l^c, l^c) / \partial \tau = \delta^C [\partial Pr^*(l^c) / \partial \tau] [1 - Pr^*(l^c)] [u(h, l^c) - u(l^c, h^*)] \\ & + [(n-1)/n] \{ Pr^*(l^c) [\partial u(l^c, h^*) / \partial \tau] - \delta^C [\partial Pr^*(l^c) / \partial \tau] [1 - Pr^*(l^c)] [u(h, l^c) - u(l^c, h^*)] \}. \end{aligned}$$

Note that *IC* in (18) takes the same form as the one in (13), *IC* under symmetry with $n = 1$, except for the last term on the right-hand side of the equality. This last term on the right-hand side takes a negative value when it is measured at $l^c = l_s^c$ with $I(l_s^c) = 0$ in (13) because $\partial u(l_s^c, h^*) / \partial \tau = \partial u(l_s^c, l_s^c) / \partial \tau = \delta^C [\partial Pr^*(l_s^c) / \partial \tau] [1 - Pr^*(l_s^c)] [u(h, l_s^c) - u(l_s^c, h^*)]$ and $Pr^*(l_s^c) < 1$.

An increase in n raises $[(n-1)/n]$, making the last negative term in (21) to have a larger

³² A change in the speed that each country can readjust its protection level, n , affects V_C by changing H's payoff in the initial period of any unilaterally-triggered tariff war phase and by affecting the lengths of punishment phases. While the derivation of *IC* in (18) does not explicitly consider this effect of a change in n on V_C , *IC* in (18) embodies such effect because the modified sufficient condition for *ICP* and *ICP*^{*} in (17) is assumed to be satisfied with such effect being taken into account.

absolute value. Therefore, the cost associated with raising the concealed protection level, represented by the right-hand side of the equality in (21), decreases in n , implying that each country will have a higher incentive to raise its concealed protection when it can react faster to the other country's initiation of a tariff war phase.

I can dissect the negative effect of a rise in n on the cost associated with raising the concealed protection level, represented by $[(n - 1)/n]\{Pr^*(l^c)[\partial u(l^c, h^*)/\partial \tau] - \delta^C[\partial Pr^*(l^c)/\partial \tau][1 - Pr^*(l^c)][u(h, l^c) - u(l^c, h^*)]\}$ in (21) as follows. For the case where F initiates a tariff war in the current period, H's gain from raising concealed protection decreases as n increases because H would react faster to such a case by setting its static optimal tariff during that period. This is represented by $Pr^*(l^c)[\partial u(l^c, h)/\partial \tau]$, representing an increase in the cost of raising the concealed protection level. This increase in the cost, however, is dominated by the negative term that follows, $-\delta^C[\partial Pr^*(l^c)/\partial \tau][1 - Pr^*(l^c)][u(h, l^c) - u(l^c, h^*)]$, which represents multiple factors that strengthen the incentive to raise concealed protection. With a higher value for n , first, H can react faster to F's initiation of a tariff war in the following period by imposing its static optimal tariff, reducing the cost associated with raising concealed protection. Furthermore, an increase in n reduces the lengths of punishment phases that countries can employ against the possible use of concealed protection under *PTS*, as discussed earlier.

Finally note that it becomes impossible to employ *PTS* to curtail the use of concealed protection if the speed of protection readjustment gets too high. This is because there will be no length of a tariff war phase that satisfies *ICP* if $n > \bar{n} \equiv [u(h, l^*) - u(h, h^*)]/[u(l, l^*) - u(h, h^*)]$ (>1); $(\delta^C - \delta)(V_C - V_N) < 0$ in (20) if $n > \bar{n}$, implying a NEGATIVE length for the tariff war phase that H can initiate ($\delta^C < \delta$) to satisfy *ICP*. The following proposition summarizes the above observation about the effect of a faster reaction to an initiation of a tariff war on the cooperative protection level attainable under symmetric *PTS*.

Proposition 3.

If each country can readjust its protection level faster in response to the other country's initiation of a tariff war phase (a higher n), the cooperative protection level attainable under symmetric *PTS* increases. In particular, if n gets higher than $\bar{n} \equiv [u(h, l^*) - u(h, h^*)]/[u(l, l^*) - u(h, h^*)]$, then it becomes impossible to use *PTS* to sustain any level of cooperative protection because there will be no lengths of tariff war phases that satisfy *ICP* and *ICP**.

This finding on the effect of a faster response to an initiation of a tariff war on *PTS* contrasts with how the same increase in the speed of protection readjustment would affect the cooperative protection level attainable under trigger strategies based on public signals. As such trigger strategies embody simultaneous imposition of explicit tariffs in the initial period of any tariff war phase, a faster response to an initiation of a tariff war would not affect the cooperative protection level attainable under trigger strategies based on public signals. When countries can readjust their protection levels quickly, *Proposition 3* implies that trigger strategies based on public signals are more likely to be successful than *PTS* in restraining the use of concealed protection.

To demonstrate that a change in the time lag in adjusting the protection level matters in enforcing international trade agreements, one may discuss the WTO's amendment of GATT Article XIX; Article XIX, known as the "escape clause," allows a government to temporarily suspend a concession agreed upon in a previous negotiation if its import-competing industry is injured as a consequence of a temporary surge in import volume.³³ To encourage the use of Article XIX as opposed to the use of managed-trade policies (such as voluntary export restraints), the WTO's amendment prohibits retaliatory responses by affected partners for a three-year period following the original imposition of Article XIX-based tariffs. Even though the escape clause is not about imposing tariffs to punish other countries' use of concealed protection, this amendment does reflect the WTO member countries' concern that the lack of time lag in imposing retaliatory tariffs in response to the imposition of Article XIX-motivated tariffs may discourage the use of Article XIX tariffs. One can interpret this motivation behind amending Article XIX as an (indirect) evidence for *Proposition 3*: a faster retaliatory reaction to an imposition of tariffs may undermine the effectiveness of *PTS*.

4.2. Asymmetry among Countries

In this section, I discuss how asymmetry among countries affects the cooperative protection levels sustainable under *PTS*. To save space, this section provides a summary of the analysis, having a more complete analysis in Appendix B. Following Bond and Park (2002), one can introduce asymmetry among countries by analyzing the following partial equilibrium trade model where H exports good 1 and F exports good 2, with $\sigma \in [1, \infty)$ denoting the size of

H's markets relative to F's. Demand for good i in H is $D_i = \sigma(A - Bp_i)$ and supply of good i in H is $X_i = \sigma(\alpha_i + \beta p_i)$, where p_i is the price of good i in H with $i = 1$ or 2 . For F, demand and supply are given by $D_i^* = A - Bp_i^*$ and $X_i^* = \alpha_i^* + \beta p_i^*$. To ensure that H will export good 1 and import good 2 and that the countries will be symmetric when $\sigma = 1$, I assume that $\alpha_1 - \alpha_1^* = \alpha_2^* - \alpha_2 > 0$ and $\alpha_1 = \alpha_2^*$. By varying σ on $[1, \infty)$, one can consider the range of relative country sizes from symmetric countries ($\sigma = 1$) to the case where F is a price taker in the world market ($\sigma \rightarrow \infty$).

To analyze the effect of asymmetry among countries on *PTS*, I focus on how the asymmetry, represented by σ , affects the following *IC* and *IC**:

$$(22) \quad \begin{aligned} \mathbf{IC} : \partial u(l, l^*) / \partial \tau &= \delta^C [\partial Pr^*(l) / \partial \tau] [1 - Pr^*(l)] [u(l, l^*) - u(l, h^*) + (\delta^C - \delta^*) (V_C - V_N)] \\ \mathbf{IC}^* : \partial u^*(l^*, l) / \partial \tau^* &= \delta^C [\partial Pr(l^*) / \partial \tau^*] [1 - Pr(l^*)] [u^*(l^*, l) - u^*(l^*, h) + (\delta^C - \delta) (V_C^* - V_N^*)], \end{aligned}$$

where $(\delta^C - \delta^*) = [u^*(h^*, l) - u^*(l^*, l)] / (V_C^* - V_N^*)$ and $(\delta^C - \delta) = [u(h, l^*) - u(l, l^*)] / (V_C - V_N)$ with $\delta^C + \delta^S = (\delta^* + \delta)$ to satisfy the sufficient condition for *ICP* and *ICP** defined in *Lemma 1 (a)*.³⁴ To satisfy the sufficient condition for *ICP* and *ICP**, note that the lengths of tariff war phases (thus, δ , δ^* , and δ^S) will change in responses to changes in l and l^* , which in turn affects V_C and V_C^* . To make this analysis more tractable, I can rewrite *IC* in (22) into:

$$(23) \quad \begin{aligned} \mathbf{IC} : \partial u(l, l^*) / \partial \tau &= \delta^C [\partial Pr^*(l)] [1 - Pr^*(l)] \left\{ [u(l, l^*) - u(l, h^*)] + [u^*(h^*, l) - u^*(l^*, l)] \frac{V_C - V_N}{V_C^* - V_C} \right\} \\ \text{with } \frac{V_C - V_N}{V_C^* - V_N} &= \frac{[u(l, l^*) - u(h, h^*)] + Pr(l^*) [u(h, l^*) - u(l, l^*)] - Pr^*(l) [u(l, l^*) - u(l, h^*)]}{[u^*(l^*, l) - u^*(h^*, h)] + Pr^*(l) [u^*(h^*, l) - u^*(l^*, l)] - Pr(l^*) [u^*(l^*, l) - u(l^*, h)]}. \end{aligned}$$

I can now illustrate the effect of an increase in the size of H relative to F ($\sigma > 1$) by analyzing its effect on *IC* in (23) and on a corresponding expression for *IC**. First, note that a higher σ leads to a higher value for the left-hand side of the equality in (23) because $\partial^2 u(l, l^*) / \partial l \partial \sigma > 0$, implying a higher incentive for H to raise its protection level above the cooperative one. This reflects H's enhanced ability to change the terms of trade in its favor by

³³ Bagwell and Staiger (Section 6.2.1, 2002) provide a more detailed discussion on Article XIX and its amendment.

³⁴ Given that each country sets its protection level on its imports (equivalent to a specific tariff), denoted by τ for H and τ^* for F, the partial equilibrium model described above yields each country's one-period payoff as a function of σ as well as τ and τ^* . For analytical simplicity, I assume that uncertainties in the economy are such that one-period payoff functions under uncertainties are identical to those under certainty.

raising its protection level as it gets larger relative to F.³⁵ An increase in σ also makes the value of the right hand side of the equality in (23) smaller, representing a decrease in the cost associated with raising the current protection level for H. Two factors contribute to this reduction in the cost of raising the protection level. One of them is F's reduced ability to decrease H's one-period payoff by initiating an explicit tariff war, reflected by $\partial[u(l, l^*) - u(l, h^*)]/\partial\sigma < 0$. As F gets smaller relative to H, its static optimal protection will entail a smaller change in the terms of trade, thus a smaller decrease in H's one-period payoff in the period when F initiates a tariff war phase. The other factor comes from a decrease in the damage that H needs to endure during the tariff war that follows F's imposition of its static optimal tariff. As H gets larger, the damage that H needs to endure during the tariff war that F initiates, represented by $[u^*(h^*, l) - u^*(l^*, l)](V_C - V_N)/(V_C^* - V_N^*)$ on the right-hand side of the equality in (23), decreases because $\partial[u^*(h^*, l) - u^*(l^*, l)]/\partial\sigma < 0$ and $\partial[(V_C - V_N)/(V_C^* - V_N^*)]/\partial\sigma < 0$. The level of such damage depends on both the length of a tariff war phase that F initiates, $\delta^C - \delta^*$, and H's loss of its expected discounted payoff that it could have earned in a cooperative period, $V_C - V_N$. Note that $(\delta^C - \delta^*) = [u^*(h^*, l) - u^*(l^*, l)]/(V_C^* - V_N^*)$ decreases with a lower value for $u^*(h^*, l) - u^*(l^*, l)$ and with a higher value for $(V_C^* - V_N^*)$. As F gets smaller relative to H, F can obtain a less increase in its one-period payoff from imposing its static optimal tariff (a lower value for $u^*(h^*, l) - u^*(l^*, l)$) while it can attain more gains from trading with a bigger H in a cooperative period (a higher value for $V_C^* - V_N^*$). This implies that the length of a tariff war phase that F can employ against H's potential deviation gets shorter with a higher σ . In addition, H's loss of its expected discounted payoff that it could have earned in a cooperative period, $V_C - V_N$, gets smaller as H can obtain smaller gains from trade with a smaller F.³⁶

In summary, an increase in σ induces H to have a higher incentive to raise its protection level in a cooperative period (a larger value for the left-hand side of the equality in (23)) while

³⁵ It is well known in the trade literature that a country's ability to change the terms of trade in its favor by imposing tariffs strengthens as it gets relatively larger than its trading partner. Kennan and Riezman (1988), McLaren (1997), and Park (2000) analyze how such an asymmetry in countries' ability to influence the terms of trade affects the cooperation attainable among asymmetric countries.

³⁶ In a bilateral trade relationship, the terms of trade under free trade gets closer to a larger country's autarky price ratio as the larger country become larger relative to the smaller one. This implies that the larger country can realize smaller gains from the bilateral trade as it gets relatively larger and the reverse is true for the smaller country. This discussion about the size of gains from free trade does not directly translate into a higher value for $V_C^* - V_N^*$ and a lower value for $V_C - V_N$ under *PTS* because the changes in the lengths of tariff war phases may

the cost associated with such a deviatory behavior gets smaller for H (a smaller value for the right-hand side of the equality in (23)). This leads to an overall increase in H's incentive to raise its protection level under *PTS*. One can apply the same type of logic to explain how a higher σ lowers F's incentive to raise its protection level under *PTS*. These change in *IC* and IC^* together imply that a larger country (H) ends up using more concealed protection than a smaller country under *PTS* and *Proposition 4* summarizes these findings as follows:

Proposition 4.

If there is an increase in H's market size relative to F's with their trigger control variables, ω^D and ω^{D*} , being fixed and identical with each other, then the cooperative protection levels that H and F sustain under *PTS* change into a direction where l gets higher and l^* gets lower. A decrease (increase) in the length of a tariff war phase that F (H) can initiate against H's (F's) potential deviations under *PTS*, as well as an increase (decrease) in H's (F's) ability to change the terms of trade through protection, contribute to this change. (See Appendix B for Proof)

According to *Proposition 4*, the asymmetry among countries may severely limit the small country's ability to restrain the large country's use of concealed protection under *PTS*. In fact, the U.S. is the only country in the world that has the legislation, Section 301, authorizing its government to invoke a tariff war based on its own unilateral judgment of potential deviations from trade agreements. In the above analysis, I assume that H and F do not change their trigger control variables ($\omega^D = \omega^{D*}$ being fixed) when the asymmetry in their size affects the cooperative protection levels they sustain under *PTS*. This implies that the smaller country F will initiate tariff wars more often than the larger country H as $Pr^*(l) > Pr(l^*)$ with $l > l^*$ and $\omega^D = \omega^{D*}$. However, this result may change if H and F change their trigger control variables in response to an increase in the size of H relative to F. If H raises ω^D and F lowers ω^{D*} in response to a higher σ , it is possible to have a case where F initiates tariff wars less often than H even when H uses concealed protection more intensely than F.³⁷ In the presence of an

also affect V_C^* and V_C . However, $\partial\{[u^*(h^*, l) - u^*(l^*, l)](V_C - V_N)/(V_C^* - V_N^*)\}/\partial\sigma < 0$ ensures the damage that H needs to endure during the tariff war that F initiates gets smaller under *PTS* as H gets larger relative to F.

³⁷ When one considers the case with asymmetric countries, defining "optimal" *PTS* regarding the choice over the trigger control variables becomes a less obvious matter because maximizing the simple sum of the two countries' expected discounted payoffs under *PTS* may entail a gain for one country at the expense of the other. This opens up the question of how each country will change its trigger control variable in response to an increase in the

alternative way to enforce international trade agreements, such as dispute settlement procedure of the WTO, small countries may choose not to rely on trigger strategies based on unilateral (and private) judgments of deviations.

5. A Possible Role for the WTO: Optimal Third Party Trigger Strategies

Regarding the issue of enforcing international trade agreements, this paper focuses on a phenomenon that the trade literature has not fully explored; countries may form different opinions about potential violations of trade agreements. As shown by the analysis of *PTS* in section 4, the effectiveness of enforcement mechanism based on unilateral judgments of potential deviations can be severely limited because the *private* nature of such judgments may undermine the credibility of strong punitive actions. This understanding provides a new perspective in interpreting the role that the WTO may play in enforcing international trade agreements: it generates (possibly imperfect) *public* signals about potential deviations from agreements of which countries may have different opinions, which in turn enhances the credibility of punitive actions against such deviations.³⁸ For example, the WTO mandates a regular review on its members under the Trade Policy Review Mechanism (TPRM), generating “*public*” reports which consist of detailed chapters examining the trade policies and practices of the members. According to the WTO’s website, “Surveillance of national trade policies is a fundamentally important activity running throughout the work of the WTO. At the centre of this work is the TPRM.”

To illustrate how and by what degree that the existence of a third-party opinion of potential deviations, such as the opinion of the WTO, may help countries to enforce international trade agreements, this section analyzes *third-party trigger strategies* under which a third-party, such as the WTO, decides upon whether a (potential) deviation has occurred and tells each country to initiate a punishment phase based on its decision. To make the direct comparison between the *third-party trigger strategies* with (optimal) symmetric *PTS* characterized in section 3, I make the following assumptions in this section.

asymmetry among countries, possibly to maximize its expected discounted payoff under *PTS*. While this potential game between H and F of setting trigger control variables is an interesting issue, it remains to be analyzed, possibly in a future work.

³⁸ Note that countries can employ a tariff war phase of any length under trigger strategies where a public signal (which is correlated with concealed protection levels) triggers tariff wars.

The stage-game payoffs and action variables of H and F are the same as those described in Section 2. In addition to these two players, there exists a third-party, namely the WTO, who receives its own (private) signals of each country's potential deviations at the end of period t , denoting it by $\omega_t \in \Omega$ regarding potential deviations of F and $\omega_t^* \in \Omega^*$ regarding potential deviations of H with $Pr(\omega \in \Omega^D)$ and $Pr(\omega^* \in \Omega^{D*})$ taking the same values as those in section 2. This assumption implies that the WTO has no informational advantage over H and F with regard to detecting potential deviations.³⁹ Once again Ω^D (Ω^{D*}) denote the range of private signals that triggers H (F) to initiate a punishment phase by imposing an explicit tariff, but it is the WTO that tell each country to initiate such a punishment phase in third-party trigger strategies. The infinitely repeated protection-setting game between H and F stays the same as before, except that now the WTO tells or does not tell each country to initiate a punishment phase by imposing an explicit tariff based on its own private signals of potential deviations. For simplicity, I denote the WTO's decision to tell H to initiate a punishment phase in period t based on its signals in period t by $\mu_{t-1} \in M \equiv \{1, 0\}$, with μ_{t-1} being 1 iff $\omega_t \in \Omega^D$, denoting its similar decision for F by $\mu_{t-1}^* \in M^* \equiv \{1, 0\}$. Then, a strategy for each country is defined by $s^{W^i} = (s^{W^i}(t))_{t=1}^\infty$, similarly to the ones in section 2, with

$$(24) \quad s^W(t): A^{t-1} \times M^{t-1} \times M^{*t-2} \times E^{*t-1} \rightarrow A \text{ and } s^{W^*}(t): A^{*t-1} \times M^{*t-1} \times M^{t-2} \times E^{t-1} \rightarrow A^*$$

where M^{t-1} and M^{*t-1} , respectively denote the history of WTO's decision of telling H and F to initiate a punishment phase up to period t , having $i = *$ or none. Note that the strategies defined in (24) allow each country to observe the WTO's decision for the other country to initiate a punishment phase only afterwards. This strategy specification under which each country choose its current action without knowing the WTO's current decision on the other country's initiation of a punishment phase may seem unnatural. This specification, however, enables a direct comparison between the third-party-trigger strategies and the private trigger

³⁹ This assumption on the WTO's information of each government's potential deviations does not necessarily imply that its signals are perfectly correlated with those of each government's private signals of the other government's potential deviations. What is crucial is that the quality of information that the WTO uses in detecting potential deviations is the same as that of each government's so that any improvement of third-party trigger strategies over the *PTS* does not come from the WTO's informational advantage. I indicate that the WTO's information can be potentially private by putting "(private)" in front of "signals" in this sentence. The WTO can make these private signals (or at least its decision on whether any deviation has occurs based on such signals) to be public by announcing its signals or its decision. Different from governments, I assume that the WTO does not have any incentive to lie about its information.

strategies by making these two types of strategies only differ in their ability in selecting the lengths of punishment phases. The following analysis will focus on the symmetric *third-party trigger strategies* for symmetric countries defined in *Definition 4* below.

- (i) Given that period $t - 1$ was a “*cooperative*” period with $(e_{t-1}, e_{t-1}^*) = (0, 0)$, each country keeps cooperating by setting $(\tau_t^i, e_t^i) = (l^c, 0)$ as long as the WTO does not tell it to initiate to a punishment phase by having $\mu_{t-1}^i = 0$ with $i = *$ or none.
- (ii) Given that period $t - 1$ was a “*cooperative*” period with $(e_{t-1}, e_{t-1}^*) = (0, 0)$, the WTO tells H to initiate a punishment phase by setting $(\tau_t, e_t) = (h, 0)$ iff $\omega_{t-1} \in \Omega^D = \Omega^{D*}$ and it tells F to initiate a punishment phase by setting $(\tau_t^*, e_t^*) = (h, 0)$ iff $\omega_{t-1}^* \in \Omega^{D*} = \Omega^D$.
- (iii) Given that a “*tariff war phase*” was initiated in period $t - 1$ by only one country, the countries set $(\tau, e) = (h, h)$ and $(\tau^*, e^*) = (h, h)$ for the following $(T - 2)$ periods and it continues to do so one more period with probability λ . Given that a “*tariff war phase*” was initiated in period $t - 1$ simultaneously by both countries, the countries set $(\tau, e) = (h, h)$ and $(\tau^*, e^*) = (h, h)$ for the following $(T^S - 2)$ periods and they continues to do so one more period with probability λ^S . T and T^S are integer numbers that are greater than or equal to 2 with λ and λ^S belonging to $[0, 1]$.
- (iv) In period 1 and other “*initial*” periods right after the end of any tariff war phase, each country sets $(\tau^i, e^i) = (l^c, 0)$ with probability $(1 - Pr^i)$ but initiates a tariff war by setting $(\tau^i, e^i) = (h^i, h^i)$ with probability Pr^i , where $Pr^i = Pr(\omega_t^i \in \Omega^{D^i})$ with $(\tau_t, e_t) = (l^c, 0)$, $(\tau_t^*, e_t^*) = (l^c, 0)$, and $i = *$ or none.

Definition 4. If (i), (ii), (iii), (iv) describe $(\underline{s}^W, \underline{s}^{W*})$, then, $(\underline{s}^W, \underline{s}^{W*})$ are symmetric *third-party trigger strategies (TTS)* with $(l^c, \Omega^D, T, T^S, \lambda, \lambda^S)$ as characterizing parameters.

Given this definition, it is easy to check that the expected discounted payoff under $(\underline{s}^W, \underline{s}^{W*})$ with $(l^c, \Omega^D, T, T^S, \lambda, \lambda^S)$, denoted by $V^W(\underline{s}^W, \underline{s}^{W*})$, is identical $V(\underline{s}, \underline{s}^*)$ in (6) as follows:

$$(25) \quad V^W(\underline{s}^W, \underline{s}^{W*}) = \frac{(1 - PrPr^*)[u(l^c, l^c) - u(h, h)]}{1 - \delta^C + Pr(1 - Pr^*)(\delta^C - \delta) + Pr^*(1 - Pr)(\delta^C - \delta) + PrPr^*(\delta^C - \delta^S)} \\ + \frac{Pr^*(1 - Pr)[u(l^c, h) - u(l^c, l^c)] + Pr(1 - Pr^*)[u(h, l^c) - u(l^c, l^c)]}{1 - \delta^C + Pr(1 - Pr^*)(\delta^C - \delta) + Pr^*(1 - Pr)(\delta^C - \delta) + PrPr^*(\delta^C - \delta^S)} + \frac{u(h, h)}{1 - \delta^C},$$

where $\delta^k = \lambda^k (\delta^C)^{T^k} + (1 - \lambda^k)(\delta^C)^{T^k - 1}$ with $k = s$, or none, having $(\delta^C - \delta)$ and $(\delta^C - \delta^S)$ respectively represent the relative length of the tariff war phase initiated by one country and by both countries simultaneously.

While the expression for the expected discounted payoff is same under the symmetric *TTS* defined above and the symmetric *PPT* defined earlier in *Definition 2* (with $l = l^* = l^c$, $\Omega^D = \Omega^{D*}$, $\delta = \delta^*$), there exists an important distinction between these two types of trigger strategies: the WTO does not have any incentive to lie about its private signals so that the *TTS* is not subject to the *ICP*. This implies that one can choose any values for the lengths of punishment phases with $(\delta^C - \delta)$ and $(\delta^C - \delta^S) \in [0, \delta^C]$. Note that the sufficient condition for satisfying the *ICP* is $\delta^C - \delta = [u(h, l^c) - u(l^c, l^c)]/(V_C - V_N)$ and $\delta^C + \delta^S = 2\delta$. To make the comparison between the *TTS* and the *PTS* even simpler, I continue to assume that $\delta^C + \delta^S = 2\delta$ (which eliminates the incentive to use the initiation of punishment phases as a part of a deviation path) holds under the symmetric *TTS*, thus allowing full flexibility only over the choice of (T, λ) , the length of the single-country initiated punishment phase. This simplifying assumption enables one to tell whether the length of the punishment phase under the optimal symmetric *PTS* is too short or too long (compared with the optimal symmetric *TTS*) by comparing the endogenously determined value for (T, λ) under the *PTS* with the optimal choice of (T, λ) under the *TTS*. Given this assumption of $\delta^C - \delta^S = 2\delta$ and $Pr = Pr^*$ by symmetry, one can further simplify $V^W(\underline{s}^W, \underline{s}^{W*})$ in (25) into $V_C^W = V^W(\underline{s}^W, \underline{s}^{W*}) = (1 - Pr)[u(l^c, l^c) - u(h, h)]/[1 - \delta^C + 2Pr(\delta^C - \delta)] + V_N$ with $V_N = u(h, h)/(1 - \delta^C)$.

To be able to support the symmetric *TTS* as an equilibrium of the repeated protection-setting game between H and F, the symmetric *TTS* need to satisfy the following incentive constraint, denoted by IC^W :

$$(IC^W) \quad I^W(l^c) \equiv \partial u(l^c, l^c)/\partial \tau - \{ \delta^C [\partial Pr^*(l^c)/\partial \tau] [1 - Pr^*(l^c)] [u(l^c, l^c) - u(l^c, h)] + (\delta^C - \delta)(V_C^W - V_N) \} = 0.$$

Note that the IC^W is identical to the IC in (12) under the symmetric *PTS* as long as δ under the *TTS* is the same as the one under the *PTS*. This equivalence results from constructing the

symmetric *TTS* in the way that it only differs from the symmetric *PTS* in its flexibility to have the single-country-initiated punishment phase to last in any length. The intuition behind this equivalence between the incentive constraint for each country to follow the specified strategies under the *PTS* and the one under the *TTS* is quite simple: each country chooses its cooperative-period protection level, knowing that raising the protection level increases the probability of a punishment phase being triggered in the same manner under both trigger strategies.⁴⁰

In addition to the IC^W , there is one more incentive constraint that the *TTS* needs to satisfy: each country does not have any incentive not to follow the WTO's decision on initiating a punishment phase of a given length. Because the WTO's decision becomes public information in a period lag, one may construct a (off-equilibrium-path) punishment strategy, such as a permanent Nash tariff war, against the behavior of not following the WTO's decision on triggering a punishment phase. Given that the expected discounted payoff under symmetric *TTS* is strictly greater than the discounted payoff of playing the static Nash tariff war forever, its is easy (and quite standard in the literature) to show that no country has any incentive not to follow the WTO's decision as long as the discount factor (δ^C) is high enough. I assume that this standard result is valid for the following analysis with a high enough value for δ^C .

For the analytical simplicity (and without loss of generality), one can represent a choice of (T, λ) by a real number $T^W \in [1, \infty)$ with $\delta^C - (\delta^C)^{T^W} = (\delta^C - \delta)$: $T^W = 1$ (equivalent to the case of $T = 2$ and $\lambda = 0$) represents the case where no-tariff period is followed by an initiation of a punishment phase (the shortest possible punishment phase) and $T^W \rightarrow \infty$ represents the case where a permanent Nash tariff war is followed by an initiation of a punishment phase (the longest possible punishment phase). Then, the problem of finding the optimal symmetric *TTS* is solving the following maximization problem:

$$(26) \quad \omega^D \text{ and } T^W \in [1, \infty) \left\{ \frac{[1 - Pr(l^c)][u(l^c, l^c) - u(h, h)]}{1 - \delta^C + 2Pr(l^c)[\delta^C - (\delta^C)^{T^W}]} + V_N \right\} \text{ subject to}$$

$$\frac{\partial u(l^c, l^c)}{\partial \tau} = \delta^C \frac{\partial Pr(l^c)}{\partial \tau} [1 - Pr(l^c)] \{u(l^c, l^c) - u(h, h) + [\delta^C - (\delta^C)^{T^W}](V_C^W - V_N)\},$$

⁴⁰ Formally proving this equivalence, however, is not totally trivial because each government needs to follow the decision of the WTO on triggering a punishment phase after any cooperative period under the *TTS*, which in turn complicates the expression for the incentive constraint under the *TTS*. Despite this complication, one can show that the resulting incentive constraint is the IC^W above. I can provide a formal proof of this equivalence result upon request.

where ω^D represents a *trigger control variable*, defined in the same way as in section 3.2.

Because the problem of finding the optimal symmetric *PTS* in section 3.2 is to choose only ω^D to maximize the same payoff function as in (26) subject to the same incentive compatibility condition, but with T^W (or equivalently the corresponding T and λ) being determined by $\delta^C - (\delta^C)^{T^W} = [u(h, l^c) - u(l^c, l^c)]/(V_C^W - V_N)$, it is obvious that the optimal symmetric *TTS* of solving the maximization problem in (26) will yield an expected discounted payoff that is greater (or at least equal) than the one under the optimal symmetric *PTS*. The question is how and by what degree the less-constrained optimal symmetric *TTS* will outperform the optimal symmetric *PTS*. Because providing a full analytical answer to this question is difficult (if not impossible) as I explain later, I will start with a numerical analysis that shows some interesting characteristics of the optimal symmetric *TTS*.

To do a numerical analysis of comparing the optimal *TTS* with the optimal *PTS*, I use the same partial equilibrium model of trade between H and F with $\sigma = 1$ (for symmetry of countries). In addition, I assume that $Pr(l^c)$ takes the following functional form, an adjusted Rayleigh distribution:

$$(27) \quad Pr(l^c) = Pr(l^c | \omega^D; \rho, \chi) = 1 - \exp[-(\omega^D l^c)^2 / (2\rho^2) - \chi\omega^D],$$

where $1/\rho \in (0, \infty)$ represents the sensitivity of the signal in detecting an increase in the level of concealed trade barriers and $\chi \in [0, \infty)$ is a parameter that determines the likelihood of a punishment phase being triggered (even in the absence of any protection) with $\omega^D \in [0, \infty)$.

Given $\chi = 8$, $\alpha_1 - \alpha_1^* = 3$, $\beta + B = 1$ and $\delta^C = 0.9$, then Figure 4 shows the outcome of the numerical analysis, demonstrating how (a) the expected percentage payoff gains compared with playing the static Nash tariff war forever, $(V_C^W - V_N)/V_N$ and $(V_C - V_N)/V_N$; (b) the cooperative protection level, l^c ; (c) the length of a punishment phase, T^W ; (d) the probability of a punishment phase being triggered, $Pr(l^c)$, changes in response to a decrease in the sensitivity of the signal (a higher value for ρ). The bold lines represent variables for the optimal symmetric *TTS* and the thin lines depict variables for the optimal symmetric *PTS*. Some outcomes of the analysis are as expected (not surprising): (i) the expected payoff gains decrease as the sensitivity of the signal decreases and the optimal *TTS* always provides a higher gain than the optimal *PTS*, having the bold line is always located higher than the thin line in Figure 4 (a),

except when $\rho = 0.0439$ where the *TTS* and the *PTS* coincides; (ii) the cooperative protection levels rise as the sensitivity of the signal decreases.

The numerical analysis shown in Figure 4 also reveals some interesting (at first, even surprising) characteristics of the optimal symmetric *TTS*. The most notable aspect is that the optimal symmetric *TTS* may require $T^W = 1$, thus no tariff-war period being followed by an initiation of a punishment phase, when the signals are sensitive enough against concealed protection with $\rho < 0.033$. Having $T^W = 1$ (or equivalently, $T = 2$ and $\lambda = 0$) never occur under the *PTS* because it will surely violate the *ICP* because each country would always prefer initiating a punishment phase to not initiating one with no costly tariff-war period(s) following the action of setting the protection level to be equal to the static optimal protection level. When the signals are relatively sensitive to an increase in concealed trade barriers, the optimal symmetric *PTS* can be inferior to the optimal symmetric *TTS* because it requires a punishment that is too strong!⁴¹

Analyzing the first order conditions of the maximization problem for the optimal symmetric *TTS* in (26) can provide some insight of the factors that drive this result of no-tariff-war punishment:

$$\frac{dV_C^W}{d\omega^D} = \frac{\partial V_C^W}{\partial l^c} \frac{\partial l^c}{\partial \omega^D} + \frac{\partial V_C^W}{\partial Pr} \frac{\partial Pr(l^c)}{\partial \omega^D} = \left(-\frac{\partial V_C^W / \partial l^c}{\partial I^W(l^c) / \partial l^c} \right) \frac{\partial I^W(l^c)}{\partial \omega^D} + \frac{\partial V_C^W}{\partial Pr} \frac{\partial Pr(l^c)}{\partial \omega^D} = 0,$$

$$\frac{dV_C^W}{dT^W} = \frac{\partial V_C^W}{\partial l^c} \frac{\partial l^c}{\partial T^W} + \frac{\partial V_C^W}{\partial T^W} = \left(-\frac{\partial V_C^W / \partial l^c}{\partial I^W(l^c) / \partial l^c} \right) \frac{\partial I^W(l^c)}{\partial T^W} + \frac{\partial V_C^W}{\partial T^W} = 0$$

with

$$(28) \quad \frac{\partial V_C^W}{\partial Pr} = -\frac{\{[1 - \delta^C + 2[\delta^C - (\delta^C)^{T^W}]] [u(l^c, l^c) - u(h, h)]\}}{\{1 - \delta^C + 2Pr[\delta^C - (\delta^C)^{T^W}]\}^2} < 0 \text{ and}$$

$$\frac{\partial V_C^W}{\partial T^W} = \frac{2 \ln(\delta^C) (\delta^C)^{T^W} Pr(1 - Pr) [u(l^c, l^c) - u(h, h)]}{\{1 - \delta^C + 2Pr[\delta^C - (\delta^C)^{T^W}]\}^2}$$

$$= \frac{-2 \ln(\delta^C) (\delta^C)^{T^W} Pr(1 - Pr)}{1 - \delta^C + 2[\delta^C - (\delta^C)^{T^W}]} \frac{\partial V_C^W}{\partial Pr} < 0,$$

where $I^W(l^c)$ represents the implicit function defining the *IC*^W above, $\partial l^c / \partial \omega^D = -(\partial I^W / \partial \omega^D) / (\partial I^W / \partial l^c)$ and $\partial l^c / \partial T^W = -(\partial I^W / \partial T^W) / (\partial I^W / \partial l^c)$ generate the second equality for $dV_C^W / d\omega^D$ and

⁴¹ This result also contrasts to the limitations of the *PTS* discussed in section 4 where the limitations of the *PTS* largely come from a weak punishment rather than a strong one.

dV_C^W/dT^W , respectively, and the expression after the second equality for $\partial V_C^W/\partial T^W$ is obtained using the expression for $\partial V_C^W/\partial Pr$ in (28). As explained in section 3.2, the optimal choice of ω^D involves the balance between its positive effect of lowering the cooperative protection and its negative effect of increasing the probability of costly punishment phases. Similarly, increasing the length of a punishment phase has a positive effect of lowering the cooperative protection by strengthening the punishment but also entails a negative effect of increasing the cost of punishment with costly punishment period being longer, implying that the optimal choice of T^W also involves balancing between these counter acting forces.

Then, how one can explain the corner solution of choosing $T^W = 1$ in Figure 4 (a) for the case where the signals are sensitive with $\rho < 0.033$. Using the fact that $dV_C^W/d\omega^D = 0$ holds under the optimal symmetric *TTS* together with the second expression for $\partial V_C^W/\partial T^W$ in (28), I can rewrite dV_C^W/dT^W as follows:

$$(29) \quad \frac{dV_C^W}{dT^W} = \left(-\frac{\frac{\partial V_C^W}{\partial l^c}}{\frac{\partial I^W}{\partial l^c}} \right) \left[\frac{\partial I^W}{\partial T^W} + \frac{2 \ln(\delta^C)(\delta^C)^{T^W} Pr(1-Pr)}{1-\delta^C + 2[\delta^C - (\delta^C)^{T^W}]} \left(\frac{\partial Pr(l^c)}{\partial \omega^D} \right)^{-1} \frac{\partial I^W}{\partial \omega^D} \right] \text{ with}$$

$$\left(-\frac{\frac{\partial V_C^W}{\partial l^c}}{\frac{\partial I^W}{\partial l^c}} \right) < 0, \frac{\partial I^W}{\partial T^W} < 0, \frac{\partial I^W}{\partial \omega^D} < 0, \text{ and } \frac{2 \ln(\delta^C)(\delta^C)^{T^W} Pr(1-Pr)}{1-\delta^C + 2[\delta^C - (\delta^C)^{T^W}]} \left(\frac{\partial Pr(l^c)}{\partial \omega^D} \right)^{-1} \frac{\partial I^W}{\partial \omega^D} > 0.$$

The above first order condition for the choice of T^W , which also embodies the first order condition for the choice of ω^D , reveals potentially a competing nature of these two choice variables in restraining the use of concealed trade barriers. $\partial I^W/\partial T^W < 0$ and $\partial I^W/\partial \omega^D < 0$ represent the nature of these choice variables in relaxing the IC^W , which in turn enable countries to lower the cooperative protection level, l^c . If the effectiveness of ω^D in relaxing the IC^W dominates that of T^W , then it is possible that the optimal symmetric *TTS* may only involve using ω^D to support cooperation, having T^W take its minimum value, 1. In fact, the second term in the second bracket of the expression on the right side of the equality in (29) is greater than the first term ($\partial I^W/\partial T^W$) in absolute values, having $dV_C^W/dT^W < 0$ for $T^W = 1$ when $\rho < 0.033$. Thus, one possible explanation for $T^W = 1$ when $\rho < 0.033$ is that the relatively accurate signals of concealed trade barriers makes raising the trigger control variable (ω^D) a more effective

means to curtail the incentive to use the concealed trade barrier than strengthening the punishment phase by making it longer, which in turn leads to $T^W = 1$.

One can also use the same expression and logic in explaining the other corner solution of setting $T^W \rightarrow \infty$ when $\rho > 0.04656$: the relatively inaccurate signals of concealed trade barriers makes strengthening the punishment phase by raising T^W a more effective means to curtail the incentive to use the concealed trade barrier than raising the trigger control variable (ω^D), which in turn leads to setting $T^W \rightarrow \infty$, having $dV_C^W/dT^W > 0$ for $T^W \rightarrow \infty$ when $\rho > 0.04656$. This understanding of the corner solution with regard to the choice of the punishment length is also helpful in explaining other parts of Figure 4. For example, the probability of a punishment phase decreases in response to a rising ρ in Figure 4 (d) when $\rho \in (0.033, 0.04656)$. Because the cooperative protection level rises in response to a higher value for ρ as shown in Figure 4, it must be the case that ω^D decreases in response to an increase in ρ . When ω^D no longer dominates T^W in its effectiveness in restraining the use of concealed barriers with $\rho > 0.033$, then the WTO should strengthen the punishment phase by raising T^W and down-playing the role of ω^D by lowering its value in response to an increase in ρ , if such an increase in ρ makes T^W relatively more effective means to curtailing the use of concealed trade barriers than ω^D .

If one can theoretically establish that the variables on the right side of the equality in (29) change as the above discussion implies in response to a reduction in the sensitivity of signals (higher values for ρ), it would worth putting such a result into a proposition. Unfortunately, establishing such a result has not been possible (even with the specific functional form assumed for Pr in the numerical simulation). This is because the variables on the right side of the equality in (29) are highly non-linear in ρ and the optimal choices of ω^D and T^W (as well as the resulting l^c) also vary in response to a change in ρ , further complicating the theoretical analysis. Therefore, I summarize the above findings into the following observation

Observation (based on the numerical analysis of the optimal *TTS* and the optimal *PTS*).

(a) When the sensitivity of signals detecting the use of concealed trade barriers is high enough, then T^W can be lower under the optimal symmetric *TTS* than under the optimal symmetric *PTS*, possibly having $T^W = 1$.

(b) When the sensitivity of signals detecting the use of concealed trade barriers is low enough, then T^W can be higher under the optimal symmetric *TTS* than under the optimal symmetric *PTS*, possibly having $T^W \rightarrow \infty$.

Note that $T^W = 1$ in Observation (a) implies that the enforcement involves highly asymmetric actions: only the government under valid suspicion of using concealed trade barriers will be penalized and the other government is rewarded by having the chance of imposing punitive tariffs. This kind of asymmetric actions is often one of important characteristics of optimal strategies of repeated games under various applications, such as in Kandori of Matsushima (1998) and Compte (1998), because such asymmetry allow player to avoid actions with (at least heavy) dead-weight losses. Thus, $T^W = 1$ under the optimal *TTS* is no longer a surprising result after having a good understanding of the reason behind it.

Observation (i) provides a potentially useful new insight regarding the role that the WTO may play in enforcing international trade agreement: the presence of an impartial third party that can judge (possibly with some errors) whether deviation has occurred, such as the WTO, may enable countries to shorten and possibly avoid a costly tariff war, that might have been necessary to curtail the incentive to wrongly punish other countries in the absence of the WTO.

While Observation (ii) seemingly confirms another role that the WTO may play in enforcing the international trade agreement that the analysis in Section 4 already emphasized: the WTO may enable governments to employ a longer punishment phase than they could employ in the absence of the WTO, thereby strengthening cooperation by a stronger punishment scheme. However, T^W being higher under the optimal symmetric *TTS* than under the optimal symmetric *PTS* in Observation (ii) is driven by a reason that is slightly different from those in Section 4 where the introduction of the WTO would have strengthened the cooperation by lowering the cooperative protection levels. Figure 4 shows that l^c is higher under the optimal *TTS* than under the optimal *PTS*, a result that contracts the prediction based on the notion of using stronger punishments for lower levels of cooperative protection. As emphasized in the earlier discussion using (29), the presence of the WTO may enable countries to substitute the less effective means of curtaining the use of concealed trade barriers, namely ω^D with a more effective one, T^W . If one reduces ω^D enough in response to an increase T^W , then

it is possible to have l^c being raised and Pr being lowered at the same time (compared with those under the *PTS*), as shown in Figure 4 for $\rho > 0.0439$.

6. Concluding Remarks

The analysis in this paper establishes that symmetric countries may restrain the use of concealed trade barriers with *simple symmetric PTS (Private Trigger Strategies)* if the sensitivity of their private signals rises in response to an increase in such barriers. This paper shows that the payoff of any equilibrium *symmetric PTS* will be the same as the one under *simple symmetric PTS* as long as the initial punishment is triggered by a one-shot Nash tariff, justifying its focus on *simple PTS*. The analysis reveals that it is not optimal to push down the cooperative protection level to its minimum level attainable under symmetric *PTS* due to the cost associated with increasing the probability of costly tariff wars. This paper identifies two factors that may limit the effectiveness of *PTS*. One is a reduction in each country's time lag in adjust its tariff protection level in response to the other country's initiation of an explicit tariff war and the other is asymmetry among countries. Both of these factors may limit the level of cooperation attainable under *PTS* by reducing the lengths of tariff war phases that countries can employ against potential deviations.

To illustrate how and by what degree that the existence of a third-party opinion of potential deviations, such as the opinion of the WTO, may help countries to enforce international trade agreements, this paper analyzes *TTS (Third-party Trigger Strategies)* under which a third-party, such as the WTO, decides upon whether a (potential) deviation has occurred and tells each country to initiate a punishment phase based on its decision. A numerical analysis of symmetric *TTS* demonstrates that the optimal *TTS* may entail a punishment phase only with an initial tariff (no tariff-war period following it) as well as the one with a permanent Nash tariff war, depending on the sensitivity of the WTO's signals of potential deviations. On the one hand, the WTO may enable countries to shorten and possibly avoid a costly tariff war, which might have been necessary to curtail the incentive to wrongly punish other countries in the absence of the WTO. On the other hand, the WTO may also enable countries to adopt a longer punishment phase than the one under *PTS*, which in turn helps countries to attain higher levels

of expected payoffs. Publicizing the WTO's judgment of potential deviations, as in the case of TPRM (Trade Policy Review Mechanism), may enable countries to adopt punishment phases of optimal lengths, which in turn raises countries' expected payoffs under international trade agreements.

Another crucial role that the WTO plays in enforcing trade agreements is settling disputes through its Dispute Settlement Procedure (DSP).⁴² While enforcing trade agreements ultimately relies on the threat of invoking trade sanctions against violations, the DSP of the WTO contributes to the enforcement mechanism by generating third-party rulings on disputed cases. If countries have perfect information about violations, such third-party rulings would play no role in enforcing the agreements. If countries may form different opinions of potential deviations, then publicizing the opinion of a third-party on disputed cases plays the role of generating *public* signals about potential deviations. As shown in this paper, this publicizing a supposedly impartial opinion may enhance the credibility of punitive actions against such deviations. However, the DSP of the WTO encourages settlements through consultations among disputing parties as a preferred way to settle trade disputes. According to the official website of the WTO, "The priority is to settle disputes, through consultations if possible. By July 2005, only 130 of the nearly 332 WTO's dispute cases had reached the full panel process. Most of the rest have either been notified as settled "out of court" or remain in a prolonged consultation phase — some since 1995."⁴³ This indicates that the DSP plays a role that goes beyond simply generating public signals of potential deviations. Carefully analyzing the role that the DSP plays in the WTO regime, therefore would be a meaningful extension of this paper.

⁴² When countries bring a dispute case to the WTO presenting different opinions about potential deviations, the DSP first encourage them to solve disputes through a consultation stage prior to initiating a panel stage where a third-party panel provides a ruling on the disputed case. Countries can appeal the panel's ruling to have the case examined by an Appellate Body. Once the case has been determined by the Appellate Body, then the losing "defendant" must comply with the ruling or face the possibility of trade sanctions by the complaining side.

⁴³ This quote comes from the following website: http://www.wto.org/english/thewto_e/whatis_e/tif_e/disp1_e.htm.

References

- Abreu, D., P. Milgrom, and D. Pearce, 1991, Information and Timing in Repeated Partnerships, *Econometrica* 59 (6), 1713-1733.
- Abreu, D. D. Pearce, and E. Stacchetti, 1986, Optimal Cartel Equilibria with Imperfect Monitoring, *Journal of Economic Theory* 39, 251-269.
- Athey, S. and K. Bagwell, 2001, Optimal Collusion with Private Information, *Rand Journal of Economics* 32 (3), 428-465.
- Bagwell, K, Self-enforcing Trade Agreements and Private Information, mimeo.
- Bagwell, K. and Staiger, R., 2002, The Economics of The World Trading System, MIT Press.
- Bagwell, K. and Staiger, R., 2005, Enforcement, Private Political Pressure and the GATT/WTO Escape Clause, *Journal of Legal Studies* 34 (2), 471-514.
- Bayard, T. O. and Elliot K.A., 1992, "Aggressive Unilateralism" and Section 301: Market Opening or Market Closing?, *The World Economy* 15, 685-706.
- Blonigen, B. and Park, J-H., 2004, Dynamic Pricing in the Presence of Antidumping Policy: Theory and Evidence, *American Economic Review* 94 (1), 134-154
- Bond, E. and Park, J-H., 2002, Gradualism in Trade Agreements with Asymmetric Countries, *Review of Economic Studies* 69 (2), 379-406.
- Compte, O., 1998, Communication in Repeated Games with Imperfect Private Monitoring, *Econometrica* 66, 597-626.
- Cotter, K. and Park, J-H., 2006, Non-concave Dynamic Programming, *Economics Letters* 90(1), 141-146.
- Dixit, A., 1987, Strategic Aspects of Trade Policy, in Truman F. Bewley, ed., *Advances in Economic Theory: Fifth World Congress*, Cambridge University Press.
- Fudenberg, D. and Tirole, J., 1991, *Game Theory*, MIT Press.
- Green, E. and Porter, R., 1984, Noncooperative Collusion under Imperfect Price Information, *Econometrica* 52, 87-100.
- Kandori, M., 2002, Introduction to Repeated Games with Private Monitoring, *Journal of Economic Theory* 102, 1-15.
- Kandori, M. and Matsushima, H., 1998, Private Observation, Communication and Collusion, *Econometrica* 66, 627-652.

- Kennan, John and Riezman, R., 1988, Do Big Countries Win Tariff Wars?, *International Economic Review* 29 (1), 88-85.
- Ludema, R., 2001, Optimal International Trade Agreements and Dispute Settlement Procedures, *European Journal of Political Economy* 72 (2), 355-376.
- Maggi, G., 1999, The Role of Multilateral Institutions in International Trade Cooperation, *American Economic Review* 89 (1), 190-214.
- Maggi, G., and Staiger, R., On the Role and Design of Dispute Settlement Procedures in International Trade Agreements, mimeo.
- Matsushima, H., 1991, On the Theory of Repeated Games with Private Information: Part I: Anti-Folk Theorem without Communication, *Economics Letters* 35 (3), 253-256.
- McLaren, J. 1997, Size, Sunk Costs, and Judge Bowker's Objection to Free Trade, *American Economic Review* 87, 400-420.
- Milgrom, P., and Segal, I., 2002, Envelope Theorems for Arbitrary Choice Sets, *Econometrica* 70 (2), 583-601.
- Park, J-H., 2000, International Trade Agreements between Countries of Asymmetric Size, *Journal of International Economics* 50, 473-95.
- Porter, R., 1983, Optimal Cartel Trigger Price Strategies, *Journal of Economic Theory* 29, 313-338
- Riezman, R., 1991, Dynamic Tariffs with Asymmetric Information, *Journal of International Economics* 30, 267-283.
- Sannikov, Y, and A. Skrzypacz, 2008, Impossibility of Collusion under Imperfect Monitoring with Flexible Production, *American Economic Review* 97 (5), 1794-1823.
- Stocky, Nancy L., and Lucas, Robert E. with Prescott, Edward C., 1989, *Recursive Methods in Economic Dynamics*, Harvard University Press.

Appendix A

Proof for Lemma 1 (b)

If $Pr = Pr^* = 0$, then $\delta = \delta^C - (1 - \delta^C) [u(h, l^*) - u(l, l^*)]/[u(l, l^*) - u(h, h^*)]$ and $\delta^* = \delta^C - (1 - \delta^C) [u^*(h^*, l) - u^*(l^*, l)]/[u^*(l^*, l) - u^*(h, h^*)]$ with such δ , δ^* , and $\delta^S = \delta + \delta^* - \delta^C$ all belonging to $(0, \delta^C)$ if δ^C is close enough to 1. This proves (b) for the case of $Pr = Pr^* = 0$ with δ^C being close enough to 1.

Now, it remains to prove (b) for the case that Pr and Pr^* are close to 0 with δ^C being close enough to 1. To prove this, I first rewrite the sufficient condition in (a) as $\delta = \delta^C - (1 - \delta^C)k$, $\delta^* = \delta^C - (1 - \delta^C)k^*$, and $\delta^S = (\delta^* + \delta) - \delta^C$, where $k = [u(h, l^*) - u(l, l^*)]/[(1 - \delta^C)V_C - u(h, h^*)]$ and $k^* = [u^*(h^*, l) - u^*(l^*, l)]/[(1 - \delta^C)V_C^* - u^*(h^*, h)]$. Note that $\partial k/\partial \delta = -Pr(1 - Pr^*)C$, $\partial k/\partial \delta^* = -Pr^*(1 - Pr)C$, $\partial k/\partial \delta^S = -PrPr^*C$, $\partial k^*/\partial \delta = -Pr(1 - Pr^*)C^*$, $\partial k^*/\partial \delta^* = -Pr^*(1 - Pr)C^*$, and $\partial k^*/\partial \delta^S = -PrPr^*C^*$ with $C = (1 - \delta^C)[u(h, l^*) - u(l, l^*)]\{(1 - PrPr^*)[u(l, l^*) - u(h, h^*)] + Pr(1 - Pr^*)[u(h, l^*) - u(l, l^*)] + Pr^*(1 - Pr)[u(l, h^*) - u(l, l^*)]\}/\{(1 - \delta^C)V - u(h, h^*)\}[1 - \delta^C + Pr(1 - Pr^*)(\delta^C - \delta) + Pr^*(1 - Pr)(\delta^C - \delta^*) + PrPr^*(\delta^C - \delta^S)]^2 > 0$, and $C^* = (1 - \delta^C)[u^*(h^*, l) - u^*(l^*, l)]\{(1 - PrPr^*)[u^*(l^*, l) - u^*(h^*, h)] + Pr(1 - Pr^*)[u^*(l^*, h) - u^*(l^*, l)] + Pr^*(1 - Pr)[u^*(h^*, l) - u^*(l^*, l)]\}/\{(1 - \delta^C)V^* - u^*(h^*, h)\}[1 - \delta^C + Pr(1 - Pr^*)(\delta^C - \delta) + Pr^*(1 - Pr)(\delta^C - \delta^*) + PrPr^*(\delta^C - \delta^S)]^2 > 0$. Let $\delta(\delta^*)$ to be the implicit function from $\delta - \delta^C + (1 - \delta^C)k = 0$ and $\delta^*(\delta)$ to be the implicit function from $\delta^* - \delta^C + (1 - \delta^C)k^* = 0$ with $\delta^S = \delta + \delta^* - \delta^C$. Then, $\partial \delta(\delta^*)/\partial \delta^* = [Pr^*(1 - Pr)(1 - \delta^C)C]/[1 - Pr(1 - Pr^*)(1 - \delta^C)C]$ and $\partial \delta^*(\delta)/\partial \delta = [Pr(1 - Pr^*)(1 - \delta^C)C^*]/[1 - Pr^*(1 - Pr)(1 - \delta^C)C^*]$. Note that $\partial \delta(\delta^*)/\partial \delta^* \in (0, 1)$ and $\partial \delta^*(\delta)/\partial \delta \in (0, 1)$ with $\delta(\delta^* = 0) \approx \delta^C - (1 - \delta^C) [u(h, l^*) - u(l, l^*)]/[u(l, l^*) - u(h, h^*)] > 0$ and $\delta^*(\delta = 0) \approx \delta^C - (1 - \delta^C)[u^*(h^*, l) - u^*(l^*, l)]/[u^*(l^*, l) - u^*(h, h^*)] > 0$ for Pr and Pr^* being close enough to 0 and δ^C is close enough to 1. Because $\partial \delta(\delta^*)/\partial \delta^* \rightarrow 0$ and $\partial \delta^*(\delta)/\partial \delta \rightarrow 0$ as $\delta^C \rightarrow 1$ with $\delta(\delta^* = 0) \approx (<) \delta^C$ and $\delta^*(\delta = 0) \approx (<) \delta^C$, this implies that there exists a unique $(\delta, \delta^*, \delta^S)$ that satisfies $\delta = \delta^C - (1 - \delta^C)k$, $\delta^* = \delta^C - (1 - \delta^C)k^*$, and $\delta^S = (\delta^* + \delta) - \delta^C$, with such δ , δ^* , and δ^S all belonging to $(0, \delta^C)$ as δ^C is close enough to 1.

Proof for Lemma 2

Proofs for the results in Lemma 2 follow the same logics as the proofs for the corresponding results in Stokey and Lucas (1989). More specifically, Theorem 4.2, 4.3, 4.4, and 4.5 in Stokey and Lucas correspond to (i), (ii), (iii), and (iv) of Lemma 2 (a), respectively. One may also find corresponding proofs for Lemma 2 (b) and Lemma 2 (c) in Theorem 4.6 in Stokey and Lucas. To save the space, I discuss how one can adjust the corresponding proofs in Stokey and Lucas to prove the results in Lemma 2. A complete proof for Lemma 2 is available upon request.

For Lemma 2 (a):

Let $F: X \rightarrow X$ denote the correspondence describing the feasibility constraints with $X=[0, h]$. Given $x_0 \in X$, let $\Pi(x_0) = \{ \{x_t\}_{t=0}^{\infty} : x_{t+1} \in I(x_t), t = 0, 1, \dots \}$ be the set of plan that are feasible from x_0 . Define $F(x_t, x_{t+1})$ as $F(\cdot)$ in (8). Then, Assumption 4.1 in Stokey and Lucas is satisfied. I modify Assumption 4.2 with $\lim_{n \rightarrow \infty} \sum_{t=0}^n (\delta^C)^t [\prod_{i=0}^{t-1} (1 - Pr^*(x_i))] F(x_t, x_{t+1})$ existing for all $x_0 \in X$ and $\underline{x} \in \Pi(x_0)$, then it is also satisfied. For each $n = 0, 1, \dots$, define $u_n: \Pi(x_0) \rightarrow R$ by $u_n(\underline{x}) = \sum_{t=0}^n (\delta^C)^t [\prod_{i=0}^{t-1} (1 - Pr^*(x_i))] F(x_t, x_{t+1})$. Define $u: \Pi(x_0) \rightarrow \bar{R}$ by $u(\underline{x}) = \lim_{n \rightarrow \infty} u_n(\underline{x})$. Then, it is easy to show that Lemma 4.1 in Stocky and Lucas holds when one replaces $u(\underline{x}) = F(x_0, x_1) + \delta^C u(\underline{x}')$ with $u(\underline{x}) = F(x_0, x_1) + \delta^C (1 - Pr^*(x_0)) u(\underline{x}')$. Having v^* and v in Stocky and Lucas representing V_S and V in Lemma 2, I can also show that Theorem 4.2, 4.3, 4.4, and 4.5 hold for these newly defined variables, replacing $v^*(x_t^*) = F(x_t^*, x_{t+1}^*) + \delta^C v(x_{t+1}^*)$ of (9) in Stocky and Lucas with $v^*(x_t^*) = F(x_t^*, x_{t+1}^*) + \delta^C (1 - Pr^*(x_t^*)) v(x_{t+1}^*)$. While one needs to modify some lines of proofs in Stocky and Lucas, it is a pretty straightforward extension of the logics of their proofs, as mentioned earlier.

For Lemma 2 (b) and (c):

First note that Lemma 2 (b) and Lemma 2 (c) correspond to Theorem 4.6 of Stocky and Lucas. Also note that Theorem 4.6 basically uses the Contraction Mapping Theorem (Theorem 3.2) and the Theorem of Maximum (Theorem 3.6) to prove the results. To show that the proof in Theorem 4.6 works for proving Lemma 2 (b) and Lemma 2 (c), I establish the following result. Define an operator T by $(Tv)(x) = \text{Max}_{y \in [0, h]} \{F(x, y) + \delta^C [1 - Pr^*(x)]v(y)\}$. T satisfies Blackwell's sufficient condition for contraction mapping as it satisfies both "Monotonicity" and "Discounting" criteria:

(Monotonicity)

If $v(y) \leq w(y)$ for all values of y , then $Tv(y) \leq Tw(y)$ because $[1 - Pr^*(x)] \geq 0$ by definition.

(Discounting)

$$T(v + a)(x) = \text{Max}_{y \in [0, h]} \{F(x, y) + \delta^C [1 - Pr^*(x)][v(y) + a]\} = \text{Max}_{y \in [0, h]} \{F(x, y) + \delta^C [1 - Pr^*(x)]v(y) + \delta^C [1 - Pr^*(x)]a\} = (Tv)(x) + \delta^C [1 - Pr^*(x)]a \leq (Tv)(x) + \delta^C a$$
 because $[1 - Pr^*(x)] \in [0, 1]$.

In addition, $T: C(X) \rightarrow C(X)$ from the Theorem of Maximum with $C(X)$ denoting the set of bounded continuous functions $f: X \rightarrow R$. Thus, $T: C(X) \rightarrow C(X)$ is a contraction mapping with modulus δ^C ,

implying that I can apply the Contraction Mapping Theorem to T . Thus, I can show that *Lemma 2* (b) and (c) hold using the Theorem of Maximum as in Theorem 4.6.

Proof for Lemma 3

For Lemma 3 (a):

Define $f(\tau_{-1}, \tau) \equiv F(\tau_{-1}, \tau) + \delta^C [1 - Pr^*(\tau_{-1})]V(\tau)$. Note that $f(\tau_{-1}, \tau)$ is everywhere differentiable w.r.t. τ_{-1} for all $\tau \in [0, h]$ and $\partial f(\tau_{-1}, \tau)/\partial \tau_{-1} = -[\partial Pr^*(\tau_{-1})/\partial \tau_{-1}]\{u(\tau, l^*) + \delta^C V(\tau) - u(\tau, h^*) - (\delta^C - \delta^*)V_N - \delta^* V_C\}$ is bounded for all $\tau \in [0, h]$. This implies that $f(\tau_{-1}, \tau)$ is absolutely continuous w.r.t. τ_{-1} for all $\tau \in [0, h]$. Therefore, I can use Theorem 2 of Milgrom and Segal (2002) in deriving the following expression

$$(A1) \quad V(\tau_{-1}) = V(0) + \int_0^{\tau_{-1}} [\partial f(m, g(m))/\partial m] dm,$$

where $g(m) \in G(m)$ and $\partial f(m, g(m))/\partial m = -[\partial Pr^*(m)/\partial m]\{u(g(m), l^*) + \delta^C V(g(m)) - u(g(m), h^*) - (\delta^C - \delta^*)V_N - \delta^* V_C\}$.

(A1) implies that $V(\tau_{-1})$ will be strictly decreasing in $\tau_{-1} \in [0, h]$, if $u[g(m), l^*) + \delta^C V(g(m)) - u(g(m), h^*) - (\delta^C - \delta^*)V_N - \delta^* V_C > 0$ for all $m \in [0, h]$, because $\partial Pr^*(m)/\partial m > 0$ by assumption. To show that $u(g(m), l^*) + \delta^C V(g(m)) - u(g(m), h^*) - (\delta^C - \delta^*)V_N - \delta^* V_C > 0$ for all $m \in [0, h] > 0$, I first establish that the inequality holds for any $g(m) \leq l$, and then show that the inequality holds for any $g(m) > l$.

First, assume that $g(m) \leq l$. To have $u(g(m), l^*) + \delta^C V(g(m)) \leq u(g(m), h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C$, $V_C > V(g(m))$ because $u(g(m), l^*) > u(g(m), h^*)$ with $l^* < h^*$ and $V(g(m)) \geq V_N$. The last inequality is obvious because the strategy of always setting $\tau = h$ will generate a discounted expected payoff at least as good as V_N , regardless of $g(m)$ taking any feasible values. $V(g(m)) \geq [1 - Pr^*(g(m))][u(l, l^*) + \delta^C V_C] + Pr^*(g(m))[u(l, h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C] \geq [1 - Pr^*(l)][u(l, l^*) + \delta^C V_C] + Pr^*(l)[u(l, h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C]$, where the last inequality comes from $g(m) \leq l$ and $[u(l, l^*) + \delta^C V_C] \geq [u(l, h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C]$, and the first inequality comes from the fact that $[1 - Pr^*(g(m))][u(l, l^*) + \delta^C V_C] + Pr^*(g(m))[u(l, h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C]$ represents a discounted expected payoff of playing a potentially suboptimal strategy of setting $\tau = l$ with $\tau_{-1} = g(m)$. From *ICP*, $V_C = [1 - Pr^*(l)][u(l, l^*) + \delta^C V_C] + Pr^*(l)[u(l, h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C]$, which implies that $V_C \leq V(g(m))$, thus a contradiction. Therefore, $u(g(m), l^*) + \delta^C V(g(m)) > u(g(m), h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C$ if $g(m) \leq l$.

Now, I will show that $u(g(m), l^*) + \delta^C V(g(m)) > u(g(m), h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C$ if $g(m) > l$. Define $K \equiv u(g(m), h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C$. Then, $V(g(m)) \geq [1 - Pr^*(g(m))]u(g(m), l^*)/\{1 - \delta^C [1 - Pr^*(g(m))]\} + Pr^*(g(m))K/\{1 - \delta^C [1 - Pr^*(g(m))]\}$ because the right-hand side of the inequality

represents a discounted expected payoff from playing a potentially suboptimal strategy of setting the current and all the future protection level at $g(m)$ with $\tau_{-1} = g(m)$. This implies that $u(g(m), l^*) + \delta^C V(g(m)) - K \geq u(g(m), l^*) + \delta^C [1 - Pr^*(g(m))]u(g(m), l^*) / \{1 - \delta^C [1 - Pr^*(g(m))]\} + \delta^C Pr^*(g(m))K / \{1 - \delta^C [1 - Pr^*(g(m))]\} - K = (1 - \delta^C) \{u(g(m), l^*) / (1 - \delta^C) - [u(g(m), h^*) + (\delta^C - \delta^*)V_N + \delta^*V_C]\} / \{1 - \delta^C [1 - Pr^*(g(m))]\}$. Note that the last term has a positive sign because $u(g(m), l^*) / (1 - \delta^C) > [u(g(m), h^*) + (\delta^C - \delta^*)V_N + \delta^*V_C]$ with $u(g(m), l^*) / (1 - \delta^C) > V_C$ as $g(m) > l$. This implies that $u(g(m), l^*) + \delta^C V(g(m)) > K$.

For Lemma 3 (b):

To prove that $G(\tau_{-1})$ is strictly increasing in τ_{-1} , I first show that $\tau'' \geq \tau'$ for all $\tau''_1 > \tau'_1 \in [0, h]$ with $\tau'' \in G(\tau''_1)$ and $\tau' \in G(\tau'_1)$. Then, I show that $\tau'' = \tau'$ will lead to a contradiction using a result in Cotter and Park (2006). Consider $\tau''_1 > \tau'_1$, having $V(\tau'_1) = F(\tau'_1, \tau') + \delta^C [1 - Pr^*(\tau'_1)]V(\tau')$ and $V(\tau''_1) = F(\tau''_1, \tau'') + \delta^C [1 - Pr^*(\tau''_1)]V(\tau'')$. Then, $F(\tau'_1, \tau') + \delta^C [1 - Pr^*(\tau'_1)]V(\tau') \geq F(\tau'_1, \tau'') + \delta^C [1 - Pr^*(\tau'_1)]V(\tau'')$ and $F(\tau''_1, \tau'') + \delta^C [1 - Pr^*(\tau''_1)]V(\tau'') \geq F(\tau''_1, \tau') + \delta^C [1 - Pr^*(\tau''_1)]V(\tau')$ because the terms of the right-hand sides of these inequalities represent discounted expected payoffs from playing potentially suboptimal strategies. These two inequalities together imply that

(A2) $[F(\tau'_1, \tau') - F(\tau''_1, \tau')] - [F(\tau'_1, \tau'') - F(\tau''_1, \tau'')] \geq \delta^C [Pr^*(\tau''_1) - Pr^*(\tau'_1)][V(\tau'') - V(\tau')]$. Define $E(\tau; \tau'_1, \tau''_1) = F(\tau'_1, \tau) - F(\tau''_1, \tau)$. According to the mean value theorem (using the fact that $E(\tau; \tau'_1, \tau''_1)$ is continuous and differentiable w.r.t. τ , then $\exists \bar{\tau} \in [\text{Min}(\tau', \tau''), \text{Max}(\tau', \tau'')]$ such that

$$(A3) \quad \begin{aligned} E(\tau'; \tau'_1, \tau''_1) - E(\tau''; \tau'_1, \tau''_1) &= (\tau' - \tau'') [\partial E(\bar{\tau}; \tau'_1, \tau''_1) / \partial \tau] \\ &\geq \delta^C [Pr^*(\tau''_1) - Pr^*(\tau'_1)][V(\tau'') - V(\tau')] \end{aligned}$$

with the inequality coming from (A2). Note that $[\partial E(\bar{\tau}; \tau'_1, \tau''_1) / \partial \tau] = [\partial u(\bar{\tau}, l^*) / \partial \tau - \partial u(\bar{\tau}, l^*) / \partial \tau] = 0$ as $\partial^2 u(\tau, \tau^*) / \partial \tau \partial \tau^* = 0$. Now, I will show that $\tau'' < \tau'$ leads to a contradiction. If $\tau'' < \tau'$, $\delta^C [Pr^*(\tau''_1) - Pr^*(\tau'_1)][V(\tau'') - V(\tau')] > 0$ because $Pr^*(\tau''_1) - Pr^*(\tau'_1) > 0$ and $[V(\tau'') - V(\tau')] > 0$ from Lemma 3 (a). This contradicts $\delta^C [Pr^*(\tau''_1) - Pr^*(\tau'_1)][V(\tau'') - V(\tau')] \leq 0$ in (A3), thus $\tau'' \geq \tau'$ for all $\tau''_1 > \tau'_1 \in [0, h]$.

Now, it remains to prove that $\tau'' = \tau'$ leads to a contraction. From Theorem 2 of Cotter and Park (2006), $V(\tau)$ is differentiable for $\tau \in G(\tau_{-1})$ for all $\tau_{-1} \in [0, h]$. Therefore,

$$(A4) \quad \begin{aligned} \partial F(\tau'_{-1}, \tau') / \partial \tau + \delta^C [1 - Pr^*(\tau'_{-1})][\partial V(\tau') / \partial \tau] &= 0 \text{ and} \\ \partial F(\tau''_{-1}, \tau'') / \partial \tau + \delta^C [1 - Pr^*(\tau''_{-1})][\partial V(\tau'') / \partial \tau] &= 0. \end{aligned}$$

If $\tau'' = \tau'$, $\partial F(\tau'_{-1}, \tau') / \partial \tau - \partial F(\tau''_{-1}, \tau'') / \partial \tau = -\delta^C [Pr^*(\tau''_{-1}) - Pr^*(\tau'_{-1})][\partial V(\tau') / \partial \tau]$ from (A4), contradicting $\partial [F(\tau'_{-1}, \tau') - F(\tau''_{-1}, \tau'')] / \partial \tau = 0$, $[Pr^*(\tau''_{-1}) - Pr^*(\tau'_{-1})] > 0$, and $\partial V(\tau') / \partial \tau > 0$.

Proof for Lemma 4

For Lemma 4 (a):

In proving *Lemma 4 (a)*, I use Theorem 4 in Cotter and Park (2006). According to the theorem, if there exists a unique $\tau_S \in (0, h)$ that satisfies *IC* defined in (11): $\partial F(\tau_S, \tau_S) / \partial \tau + \delta^C [1 - Pr^*(\tau_S)][\partial V(\tau_S) / \partial \tau] = 0$ and $\tau \in (0, h)$ for every $\tau_{-1} \in [0, h]$ and $\tau \in G(\tau_{-1})$, then $G(\tau_S) = \{\tau_S\}$ and τ_S is a strongly stable protection level in the sense that for every $\tau_{-1} > \tau_S$ and $\tau \in G(\tau_{-1})$, $\tau < \tau_{-1}$, and for every $\tau_{-1} < \tau_S$ and $\tau \in G(\tau_{-1})$, $\tau > \tau_{-1}$. To prove *Lemma 4 (a)*, therefore, I first show that there exists a unique $\tau_S \in (0, h)$ such that $\partial F(\tau_S, \tau_S) / \partial \tau + \delta^C [1 - Pr^*(\tau_S)][\partial V(\tau_S) / \partial \tau] = 0$ if $[\partial^2 Pr^*(\tau) / (\partial \tau)^2][1 - Pr^*(\tau)] - \{1 + \delta^C [1 - Pr^*(\tau)]\}[\partial Pr^*(\tau) / \partial \tau]^2 > 0$ for all $\tau \in [0, h]$ and $\partial Pr^*(\tau) / \partial \tau \approx 0$ at $\tau = 0$, then establish that $\tau \in (0, h)$ for every $\tau_{-1} \in [0, h]$ and $\tau \in G(\tau_{-1})$.

First note that $\partial F(\tau_S, \tau_S) / \partial \tau = \partial u(\tau_S, l^*) / \partial \tau > 0$ at $\tau_S = 0$ and $\partial^2 F(\tau_S, \tau_S) / \partial \tau^2 < 0$ with $\partial F(\tau_S, \tau_S) / \partial \tau = \partial u(\tau_S, l^*) / \partial \tau = 0$ at $\tau_S = h$ from the assumptions on the derivatives of $u(\tau, \tau^*)$ w.r.t. τ . Because $\partial V(\tau_S) / \partial \tau = -[\partial Pr^*(\tau_S) / \partial \tau] \{u(\tau_S, l^*) + \delta^C V(\tau_S) - [u(\tau_S, h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C]\} \approx 0$ at $\tau_S = 0$ from the assumption of $\partial Pr^*(\tau) / \partial \tau \approx 0$ at $\tau = 0$, $F(\tau_S, \tau_S) / \partial \tau > 0$ at $\tau_S = 0$ implies that *IC* in (11) will not be satisfied at $\tau_S = 0$. Now, define $A(\tau_S) \equiv u(\tau_S, l^*) + \delta^C V(\tau_S) - [u(\tau_S, h^*) + (\delta^C - \delta^*)V_N + \delta^* V_C]$ and $B(\tau_S) \equiv \delta^C [1 - Pr^*(\tau_S)][\partial Pr^*(\tau_S) / \partial \tau] A(\tau_S)$, thus $\delta^C [1 - Pr^*(\tau_S)][\partial V(\tau_S) / \partial \tau] = -B(\tau_S)$. Then, $\partial B(\tau_S) / \tau_S = \delta^C A(\tau_S) \{[\partial^2 Pr^*(\tau) / (\partial \tau)^2][1 - Pr^*(\tau)] - \{1 + \delta^C [1 - Pr^*(\tau)]\}[\partial Pr^*(\tau) / \partial \tau]^2\} > 0$ for all $\tau_S \in [0, h]$ because $[\partial^2 Pr^*(\tau) / (\partial \tau)^2][1 - Pr^*(\tau)] - \{1 + \delta^C [1 - Pr^*(\tau)]\}[\partial Pr^*(\tau) / \partial \tau]^2 > 0$ for all $\tau_S \in [0, h]$ by assumption and $A(\tau_S) > 0$ as shown in the proof for *Lemma 3 (a)*. This implies that there exists a unique $\tau_S \in (0, h)$ such that $\partial F(\tau_S, \tau_S) / \partial \tau + \delta^C [1 - Pr^*(\tau_S)][\partial V(\tau_S) / \partial \tau] = 0$.

Now, I only need to prove that $\tau \in (0, h)$ for every $\tau_{-1} \in [0, h]$ and $\tau \in G(\tau_{-1})$. Because $G(\tau_{-1})$ is strictly increasing in τ_{-1} as proved in *Lemma 3 (b)*, it suffices to prove that $0 \notin G(0)$ and $h \notin G(h)$. Note that $0 \notin G(0)$ is already proven above: “*IC* in (11) will not be satisfied at $\tau_S = 0$.” Because *IC* in (11) is a necessary condition for any stationary protection level, *IC* in (11) being not satisfied at $\tau_S = 0$ implies that $0 \notin G(0)$.

I can show that $h \notin G(h)$ by contradiction. First, assume that $h = G(h)$, implying that $V(h) = \sum_{d=0}^{\infty} \left\{ (\delta^C)^d \cdot \left[\prod_{i=0}^{d-1} [1 - Pr^*(\tau_i)] \right] \cdot F(\tau_d, \tau_{d+1}) \right\}$ with $\{\tau_d = h\}_{d=0}^{\infty}$. Consider an alternative protection sequence with $\tau_0 = h$, $\tau_1 = h - \varepsilon$, and $\{\tau_d = h\}_{d=2}^{\infty}$, which defines a corresponding discounted expected payoff, denoted by $V_A(h)$. Then, I can show that $V_A(h) - V(h) = \{Pr^*(h)u(h - \varepsilon, h^*) + [1 - Pr^*(h)]u(h - \varepsilon, l^*) + Pr^*(h)[(\delta^C - \delta^*)V_N + \delta^*V_C]\} - \{Pr^*(h)u(h, h^*) + [1 - Pr^*(h)]u(h, l^*) + Pr^*(h)[(\delta^C - \delta^*)V_N + \delta^*V_C]\} + \delta^C[1 - Pr^*(h)][Pr^*(h - \varepsilon) - Pr^*(h)]\{u(h, h^*) - u(h, l^*) + [(\delta^C - \delta^*)V_N + \delta^*V_C]\} - \delta^C[Pr^*(h) - Pr^*(h - \varepsilon)]F(h, h)\delta^C[1 - Pr^*(h)]/[1 - \delta^C[1 - Pr^*(h)]]$. $\lim_{\varepsilon \rightarrow 0} [V_A(h) - V(h)]/(\varepsilon - 0) = -\delta^C[\partial Pr^*(h)/\partial \tau][1 - Pr^*(h)]\{(1 - \delta^C)[u(h, h^*) + (\delta^C - \delta^*)V_N + \delta^*V_C] - u(h, l^*)\}/[1 - \delta^C[1 - Pr^*(h)]] > 0$ where the last inequality comes from $\partial Pr^*(h)/\partial \tau > 0$ and $u(h, l^*)/(1 - \delta^C) > u(h, h^*) + (\delta^C - \delta^*)V_N + \delta^*V_C$ as shown in Lemma 3 (a). This implies that $h \notin G(h)$.

For Lemma 4 (b):

To prove Lemma 4 (b), I will show that H cannot strictly increase its discounted payoff by initiating an explicit tariff war in a period that follows a cooperative period during which H set its protection level at $l' \neq l = \tau_S$, as long as the lengths of tariff war phases satisfy the sufficient condition for ICP and ICP* in Lemma 1 (a). Once I prove this result, this implies that H cannot increase its discounted expected payoff by initiating explicit tariff wars along any (deviatory) protection sequence, thus Lemma 4 (b).

Suppose that H sets its protection level at l in a period that follows a cooperative period during which H sets its protection level at $l' \neq l = \tau_S$, then chooses its optimal protection sequence from the next period on. Denote the discounted expected payoff from taking this potentially suboptimal action by $C(l')$, then

$$(A5) \quad C(l') = Pr^*(l')[u(l, h^*) + (\delta^C - \delta^*)V_N + \delta^*V_C] + [1 - Pr^*(l')][u(l, l^*) + \delta^C V_C].$$

Now suppose that H initiates a tariff war phase by setting tariff level at h in a period that follows a cooperative period where H set its protection level $l' \neq l = \tau_S$, then follows its specified strategy once the tariff war phase is over. Denote the discounted expected payoff from taking this potentially suboptimal action by $D(l')$, then

$$(A6) \quad D(l') = Pr^*(l')[u(h, h^*) + (\delta^C - \delta^S)V_N + \delta^S V_C] + [1 - Pr^*(l')][u(h, l^*) + (\delta^C - \delta)V_N + \delta V_C].$$

I can rewrite $C(l')$ and $D(l')$ into

$$(A7) \quad \begin{aligned} C(l') &= u(l, l^*) + \delta^C V_C - Pr^*(l') [u(l, l^*) - u(l, h^*) + (\delta^C - \delta^*)(V_C - V_N)] \\ D(l') &= u(h, l^*) + (\delta^C - \delta) V_N + \delta V_C - Pr^*(l') [u(h, l^*) - u(h, h^*) + (\delta - \delta^S)(V_C - V_N)]. \end{aligned}$$

Now, note that $C(l') - D(l') = [u(l, l^*) - u(h, l^*)] + (\delta^C - \delta)(V_C - V_N) - Pr^*(l') \{ [u(l, l^*) - u(l, h^*)] - [u(h, l^*) - u(h, h^*)] + [(\delta^C - \delta^*) - (\delta - \delta^S)](V_C - V_N) \} = 0$ from $[u(l, l^*) - u(l, h^*)] = [u(h, l^*) - u(h, h^*)]$ and the sufficient condition for ICP and ICP^* in *Lemma 1 (a)*: $\delta^C - \delta = [u(h, l^*) - u(l, l^*)]/(V_C - V_N)$ and $\delta^C + \delta^S = (\delta^* + \delta)$. Because $C(l')$ is equal or possibly lower than a discounted expected payoff from choosing an optimal protection sequence of not involving an initiation of a tariff war phase, this implies that H cannot strictly increase its discounted payoff by initiating an explicit tariff war in a period that follows a cooperative period during which H sets its protection level at $l' \neq l = \tau_S$.

Proof for Proposition 1

With $\delta = \delta^* = \delta^C - [u(h, l_S^c) - u(l_S^c, l_S^c)]/(V_C - V_N)$, and $\delta^S = \delta + \delta^* - \delta$, note that setting $\tau_S = l_S^c$ satisfies IC in (11), thus l_S^c is the unique stationary protection level from which H does not have any incentive to deviate from, as described in *Lemma 4*. By symmetry, l_S^c is also such a protection level for F. If $l = l^* = l_S^c$, then PTS satisfy ICP and ICP^* as well as IC and IC^* , thus becoming a supergame equilibrium of the protection setting game between H and F from which no country has any unilateral incentive to change its specified strategy.

What is the relationship between the condition for *Lemma 4 (a)* and the existence of $l^c (< h)$ that satisfies $I(l^c) = 0$ in (13)? For example, does the condition for *Lemma 4 (a)* guarantee the existence of such l^c ? To address this issue, I show that the second term of $I(l^c)$ in (13), $\delta^C [\partial Pr^*(l^c)/\partial \tau] [1 - Pr^*(l^c)] [u(h, l^c) - u(l^c, h^*)]$, representing H's dynamic incentive to avoid a tariff war, may not necessarily increase in l^c when the condition for *Lemma 4(a)* is satisfied. $\partial \{ [\partial Pr^*(l)/\partial l] [1 - Pr^*(l)] [u(h, l) - u(l, h^*)] \} / \partial l = \langle [\partial^2 Pr^*(l)/(\partial l)^2] [1 - Pr^*(l)] - [\partial Pr^*(l)/\partial l]^2 \rangle [u(h, l) - u(l, h^*)] + [\partial Pr^*(l)/\partial l] [1 - Pr^*(l)] \{ \partial [u(h, l) - u(l, h^*)] / \partial l \} = \langle [\partial^2 Pr^*(l)/(\partial l)^2] [1 - Pr^*(l)] - \{ 1 + \delta^C [1 - Pr^*(l)] \} [\partial Pr^*(l)/\partial l]^2 \rangle [u(h, l) - u(l, h^*)] + \langle \delta^C [1 - Pr^*(l)] \{ \partial Pr^*(l)/\partial l \}^2 [u(h, l) - u(l, h^*)] + [\partial Pr^*(l)/\partial l] [1 - Pr^*(l)] \{ \partial [u(h, l) - u(l, h^*)] / \partial l \} \rangle$. Because $[\partial Pr^*(l)/\partial l] [1 - Pr^*(l)] \{ \partial [u(h, l) - u(l, h^*)] / \partial l \} < 0$, once cannot rule out the possibility of having $\{ \delta^C [1 - Pr^*(l)] \} [\partial Pr^*(l)/\partial l]^2 [u(h, l) - u(l, h^*)] + [\partial Pr^*(l)/\partial l] [1 - Pr^*(l)] \{ \partial [u(h, l) - u(l, h^*)] / \partial l \} < 0$, thus $\partial \{ [\partial Pr^*(l)/\partial l] [1 - Pr^*(l)] [u(h, l) - u(l, h^*)] \} / \partial l < 0$ even when $[\partial^2 Pr^*(l)/(\partial l)^2] [1 - Pr^*(l)] - \{ 1 + \delta^C [1 - Pr^*(l)] \} [\partial Pr^*(l)/\partial l]^2 > 0$. Therefore, the condition for *Lemma 4 (a)* does not necessarily guarantee the existence of $l^c (< h)$ that satisfies $I(l^c) = 0$, validating the insertion of an additional condition to guarantee the existence of such l^c in *Proposition 1*.

Appendix B: the Analysis of the Effect of Asymmetry among Countries on PTS

Given that each country sets its protection level on its imports (equivalent to a specific tariff), denoted by τ for H and τ^* for F, domestic prices are $p_2 = p_2^* + \tau$ and $p_1^* = p_1 + \tau^*$, and each country's one-period payoff function can be expressed as

$$(B1) \quad w^j(\tau^j, \tau^k) = \sum_{i=1,2} \left[\int_{p_i^j}^{A/B} D_i^j(u) du + \int_{-\alpha_i^j/\beta}^{p_i^j} X_i^j(u) du \right] + \tau^j (D_m^j(p_m^j) - X_m^j(p_m^j))$$

where $j, k = *$ or none with $j \neq k$ and $m = 1$ (2) when $j = *$ (none). The above trade model does not incorporate uncertainties in the economy. For analytical simplicity, however, I assume that uncertainties in the economy are such that one-period payoff functions under uncertainties defined by (3) in Section 2 are the same as those in (B1); $u(\tau, \tau^*) = w(\tau, \tau^*)$ and $u^*(\tau^*, \tau) = w^*(\tau^*, \tau)$. Then, I can show that all the assumptions about derivatives of u and u^* with respect to τ and τ^* in Section 2 are satisfied; $\partial u / \partial \tau > 0$ at $\tau = 0$ and $\partial u^* / \partial \tau^* > 0$ at $\tau^* = 0$; $\partial u / \partial \tau^* < 0$, $\partial u^* / \partial \tau < 0$, $\partial u / \partial \tau + \partial u^* / \partial \tau < 0$, and $\partial u / \partial \tau^* + \partial u^* / \partial \tau^* < 0$ for (τ, τ^*) that are not trade-prohibitive; $\partial^2 u / \partial \tau^2 < 0$ and $\partial^2 u^* / \partial \tau^{*2} < 0$; $\partial^2 u / \partial \tau \partial \tau^* = 0$ and $\partial^2 u^* / \partial \tau \partial \tau^* = 0$.

To analyze the effect of asymmetry on PTS, I use Figure B, which shows the combinations of (l, l^*) that satisfy IC and IC^* , denoted by IC and IC^* . First, I establish that IC and IC^* are both positively sloped as shown in Figure B with IC being flatter than 45 degree line and IC^* being steeper than 45 degree line for the case where H and F are symmetric ($\sigma = 1$; $IC_{\sigma=1}$ and $IC^*_{\sigma=1}$ in Figure B), then I explain how an increase in the relative size of H affects IC and IC^* ($\sigma > 1$; $IC_{\sigma>1}$ and $IC^*_{\sigma>1}$ in Figure B), thus the cooperative protection levels sustainable under PTS.

Proof for Proposition 4

Proof for Proposition 4 is composed of two parts: (i) proving that IC and IC^* in Figure B are positively sloped with IC being flatter than 45 degree line and IC^* being steeper than 45 degree line; (ii) proving that a higher σ shifts IC upwards and IC^* to the left in Figure B. To save the space, the following proof will only provide the derived values of the variables of which I need to know signs of their derivatives to prove *Proposition 4* and the resulting signs of those derivatives, without showing the corresponding derivation processes. The work showing those derivations is available upon request.

Part (i):

To prove that IC is positively sloped in Figure B, I use IC in (20). First, I sketch the process of deriving

$$(20) \quad \begin{aligned} \mathbf{IC}: \quad & \partial u(l, l^*) / \partial \tau = \delta^C [\partial \Pr^*(l)] [1 - \Pr^*(l)] \{ [u(l, l^*) - u(l, h^*)] + [u^*(h^*, l) - u^*(l^*, l)] \frac{V_C - V_N}{V_C^* - V_N^*} \} \\ \text{with} \quad & \frac{V_C - V_N}{V_C^* - V_N^*} = \frac{[u(l, l^*) - u(h, h^*)] + \Pr(l^*) [u(h, l^*) - u(l, l^*)] - \Pr^*(l) [u(l, l^*) - u(l, h^*)]}{[u^*(l^*, l) - u^*(h^*, h)] + \Pr^*(l) [u^*(h^*, l) - u^*(l^*, l)] - \Pr(l^*) [u^*(l^*, l) - u(l^*, h)]}, \end{aligned}$$

from ICP and ICP^* in (7) together with the sufficient condition for them to hold in *Lemma 1 (a)*. From $(\delta^C - \delta^*) = [u^*(h^*, l) - u^*(l^*, l)] / (V_C^* - V_N^*)$, $(\delta^C - \delta^*)(V_C - V_N) = [u^*(h^*, l) - u^*(l^*, l)] (V_C - V_N) / (V_C^* - V_N^*)$ as shown in (20). To derive the expression for $(V_C - V_N) / (V_C^* - V_N^*)$ in (20), I starts from $(V_C - V_N) / (V_C^* - V_N^*) = [(\delta^C - \delta^*) / (\delta^C - \delta)] \{ [u(h, l^*) - u(l, l^*)] / [u^*(h^*, l) - u^*(l^*, l)] \}$, implied by the sufficient condition for them to hold in *Lemma 1 (a)*. ICP in (7) implies that $\{ [u(l, l^*) - u(h, h^*) - \Pr^*(l) [u(l, l^*) - u(l, h^*)]] / \{ u(h, l^*) - u(h, h^*) - \Pr^*(l) [u(h, l^*) - u(h, h^*)] \} = [(1 - \delta^C) + \Pr^*(l) (\delta^C - \delta^*)] / [(1 - \delta) + \Pr^*(l) (\delta^C - \delta^*)]$ and ICP^* in (7) implies that $\{ [u^*(l^*, l) - u^*(h^*, l) - \Pr(l^*) [u^*(l^*, l) - u^*(l^*, h)]] / \{ u^*(h^*, l) - u^*(h^*, h) - \Pr(l^*) [u^*(h^*, l) - u^*(h^*, h)] \} = [(1 - \delta^C) + \Pr(l^*) (\delta^C - \delta)] / [(1 - \delta^*) + \Pr(l^*) (\delta^C - \delta)]$. From these two equalities, I can derive an expression for $[(\delta^C - \delta^*) / (\delta^C - \delta)]$, which in turn being plugged into $(V_C - V_N) / (V_C^* - V_N^*) = [(\delta^C - \delta^*) / (\delta^C - \delta)] \{ [u(h, l^*) - u(l, l^*)] / [u^*(h^*, l) - u^*(l^*, l)] \}$ to generate the corresponding expression in (20). While I omit the complete derivation process to save the space, it is available upon request.

Having derived (20), now I provide the closed-form solutions (in terms of parameters of the model) for the variables in (20), from which one can derive the signs of derivatives of variables that are relevant for the analysis to follow:

$$(B2) \quad \begin{aligned} \partial u(l, l^*) / \partial \tau &= \frac{\sigma}{(1 + \sigma)^2} [\sigma(\alpha_2^* - \alpha_2) - (2\sigma + 1)(\beta + B)l], \\ u(l, l^*) - u(l, h^*) &= \frac{\sigma}{2(1 + \sigma)^2} [(\beta + B)(l^* + h^*) + 2(\alpha_1^* - \alpha_1)](l^* - h^*), \\ u^*(h^*, l) - u^*(l^*, l) &= -\frac{\sigma}{2(1 + \sigma)^2} [2(\alpha_1 - \alpha_1^*) - (2 + \sigma)(\beta + B)(l^* - h^*)](h^* - l^*) \\ & [u(l, l^*) - u(h, h^*)] + \Pr(l^*) [u(h, l^*) - u(l, l^*)] - \Pr^*(l) [u(l, l^*) - u(l, h^*)] \\ &= [1 - \Pr^*(l)] \frac{\sigma}{2(1 + \sigma)^2} [(\beta + B)(l^* + h^*) + 2(\alpha_1^* - \alpha_1)](l^* - h^*) \\ & + [1 - \Pr(l^*)] \frac{\sigma}{2(1 + \sigma)^2} [\sigma(\alpha_2^* - \alpha_2) - (2\sigma + 1)(\beta + B)l](l - h), \end{aligned}$$

$$\begin{aligned}
& [u^*(l^*, l) - u^*(h^*, h)] + Pr^*(l)[u^*(h^*, l) - u^*(l^*, l)] - Pr(l^*)[u^*(l^*, l) - u(l^*, h)] \\
&= -[1 - Pr^*(l)] \frac{\sigma}{2(1+\sigma)^2} [2(\alpha_1 - \alpha_1^*) - (2+\sigma)(\beta+B)(l^* + h^*)](h^* - l^*) \\
&\quad + [1 - Pr(l^*)] \frac{\sigma}{2(1+\sigma)^2} [-2(\alpha_2^* - \alpha_2) + (\beta+B)(l+h)](l-h), \\
&\partial u^*(l^*, l) / \partial \tau^* = \frac{\sigma}{(1+\sigma)^2} [(\alpha_1 - \alpha_1^*) - (\sigma+2)(\beta+B)l^*], \\
&u^*(l^*, l) - u^*(l^*, h) = \frac{\sigma}{2(1+\sigma)^2} [-2(\alpha_2^* - \alpha_2) + (\beta+B)(l+h)](l-h), \text{ and} \\
&u(h, l^*) - u(l, l^*) = -\frac{\sigma}{2(1+\sigma)^2} [\sigma(\alpha_2^* - \alpha_2) - (2\sigma+1)(\beta+B)l](l-h),
\end{aligned}$$

where $h = \sigma(\alpha_2^* - \alpha_2) / [(2\sigma+1)(\beta+B)]$, $h^* = (\alpha_1 - \alpha_1^*) / [(2+\sigma)(\beta+B)]$, and the last three equations provide closed-form solutions for the variables in IC^* defined in the similar way as in (20).

Given the above expressions in (B2), it is easy to show that IC is positively sloped. Note that an increase in l^* will reduce the term on the right-hand side of the equality in (20) because $\partial u(l, l^*) / \partial l^* < 0$, $-\partial u^*(l^*, l) / \partial l^* < 0$, and $\partial[(V_C - V_N) / (V_C^* - V_N^*)] / \partial l^* < 0$ with the last inequality can be checked by differentiating the corresponding expression in (B2), implying a reduced incentive for H keep its protection level down. Also note that an increase in l will reduce the left-hand side of the equality in (20) as $\partial^2 u(l, l^*) / (\partial l)^2 < 0$ and will increase the right-hand side of the equality in (20) because $\partial\{\delta^C[\partial Pr^*(l) / \partial l][1 - Pr^*(l)]\} / \partial l > 0$, $\partial[(V_C - V_N) / (V_C^* - V_N^*)] / \partial l > 0$. Therefore, $dl / dl^* \Big|_{IC} > 0$, and I can prove that $dl / dl^* \Big|_{IC^*} > 0$ in a similar way.

Now, I need to prove that IC is sloped flatter than the 45 degree line and IC^* is sloped steeper than 45 degree line as shown in Figure B. For $\sigma = 1$, I prove that IC is sloped flatter than the 45 degree line once again using IC in (20). First note that IC cannot be sloped as the 45 degree line because there exists a unique $l^c (= l = l^*)$ that satisfies IC in (13), or equivalently IC in (20) with $\sigma = 1$, given the assumption of *Proposition 1*. For $l^c (= l = l^*) > l_{\sigma=1} = l_{\sigma=1}^*$, one can easily check that IC in (20) is violated because the right-hand side of the equality in (20) gets greater than the left-hand side. Note also that a decrease in l will increase the left-hand side of the equality in (20) and will decrease the right-hand side of it as shown above. Therefore, the equality in (20) can be restored for $l^* (= l_c = l) > l_{\sigma=1} = l_{\sigma=1}^*$, only by lowering l from $l = l^*$, implying that IC should be sloped less than 45 degree line. By symmetry, I can use the same argument for proving that IC^* is sloped steeper than 45 degree line for $\sigma = 1$. One can link this fact to the stability of the cooperative equilibrium under *PTS* in the following

way. Even when the cooperative protection combination, $E_{\sigma=1}$ is perturbed by some shocks (possibly random errors in setting protection levels), the relative slopes of IC and IC^* ensure that countries' self-correction incentives (to change its protection level back to its payoff-maximizing level) move the protection combination back to $E_{\sigma=1}$.

Part (ii):

To prove that a higher σ shifts IC upwards in Figure B, I analyze how an increase in σ will affect IC in (20). Using (B2), I can show that $\partial^2 u(l, l^*)/\partial l \partial \sigma > 0$, $\partial[u(l, l^*) - u(l, h^*)]/\partial \sigma < 0$, $\partial[u^*(h^*, l) - u^*(l^*, l)]/\partial \sigma < 0$, and $\partial[(V_C - V_N)/(V_C^* - V_N^*)]/\partial \sigma < 0$, implying that an increase in σ will increase the left-hand side of the equality in (20) and will decrease the right-hand side of it. To satisfy IC in (20), therefore, an increase in σ requires an increase in l (which will decrease the left-hand side of the equality in (20) and will increase the right-hand side of it, as shown above) for any given level of l^* . This implies that a higher σ shifts IC upward in Figure B. I can prove that a higher σ shifts IC^* to the left in Figure B in a similar way.

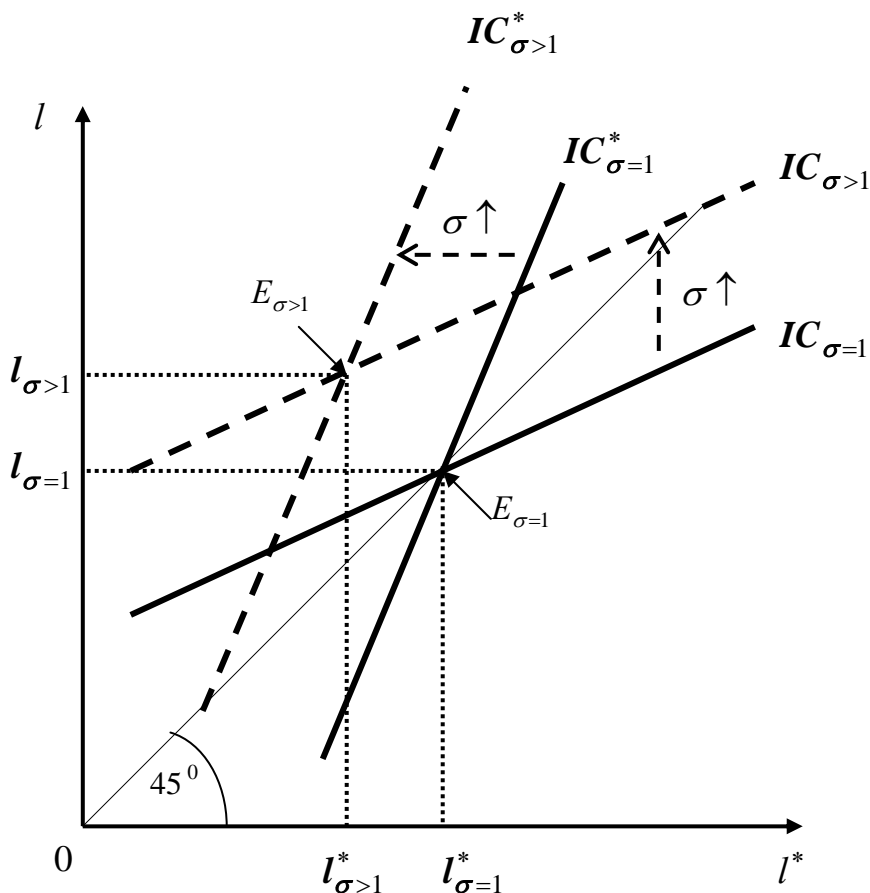


Figure B. The Effect of Asymmetry among Countries on IC and IC^*

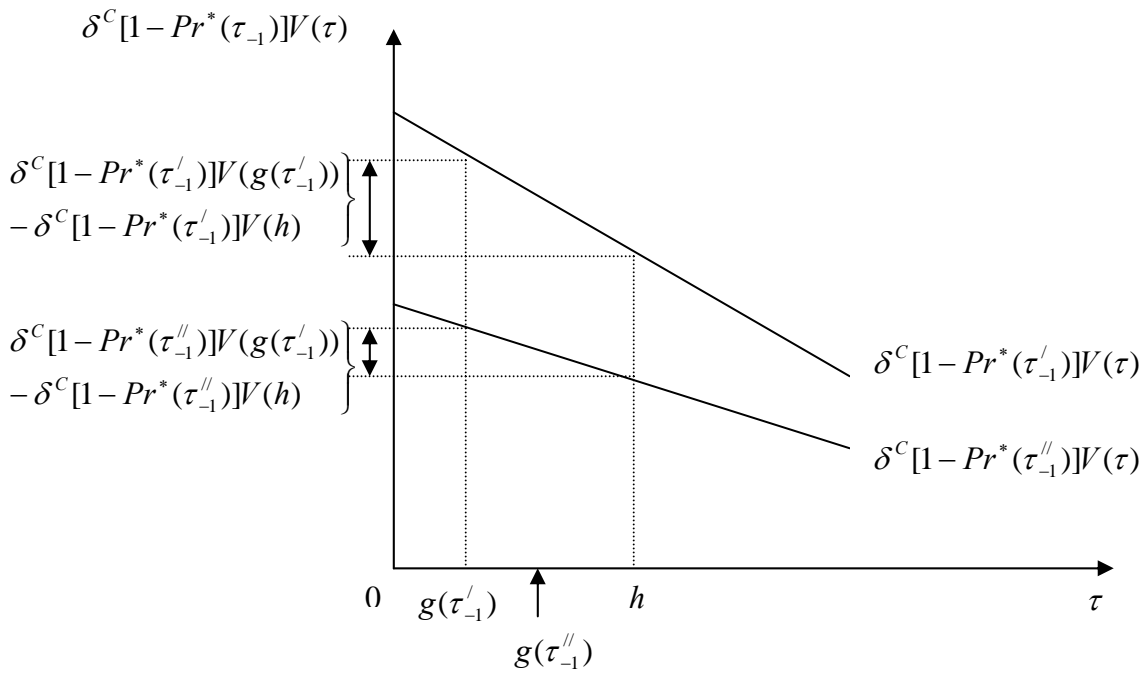
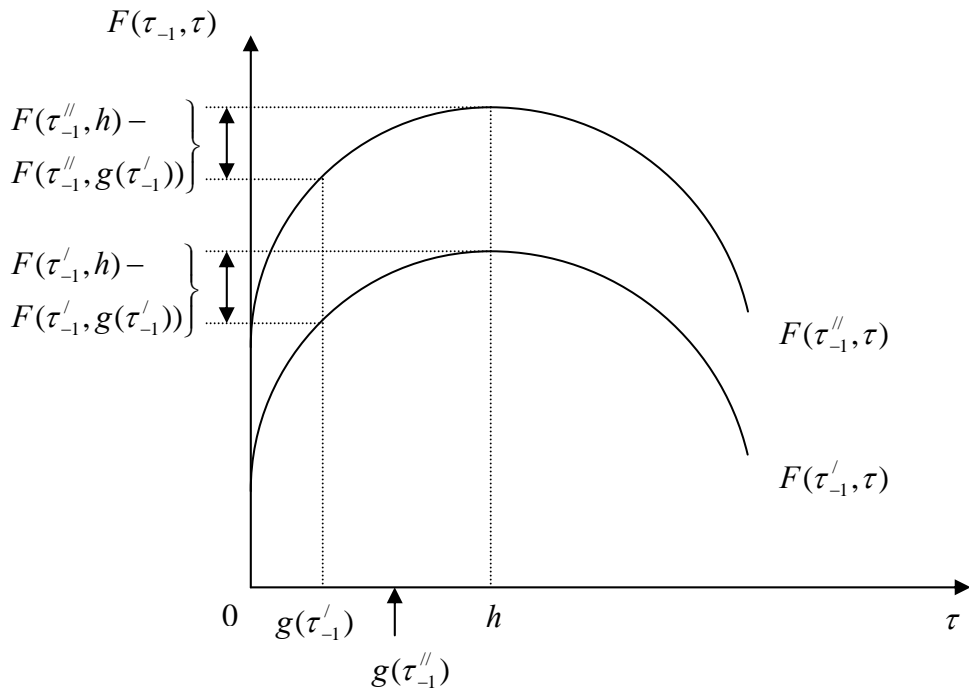


Figure 1. The Effect of a Higher τ_{-1} on the Optimal Choice of τ

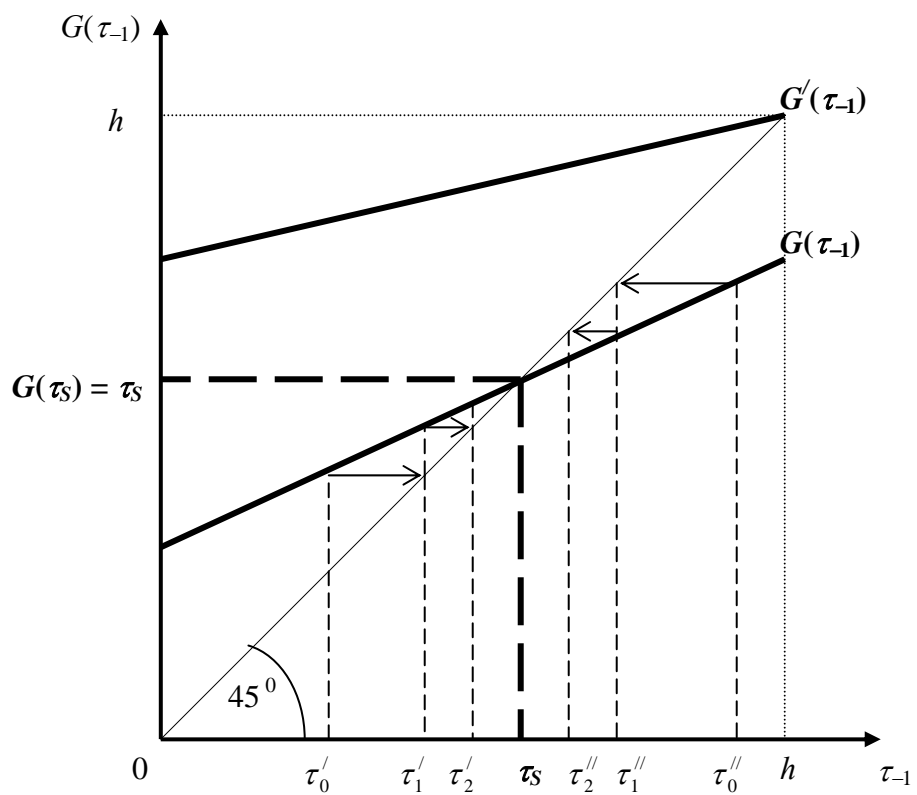


Figure 2. The Existence of a Stationary Protection Sequence at τ_s

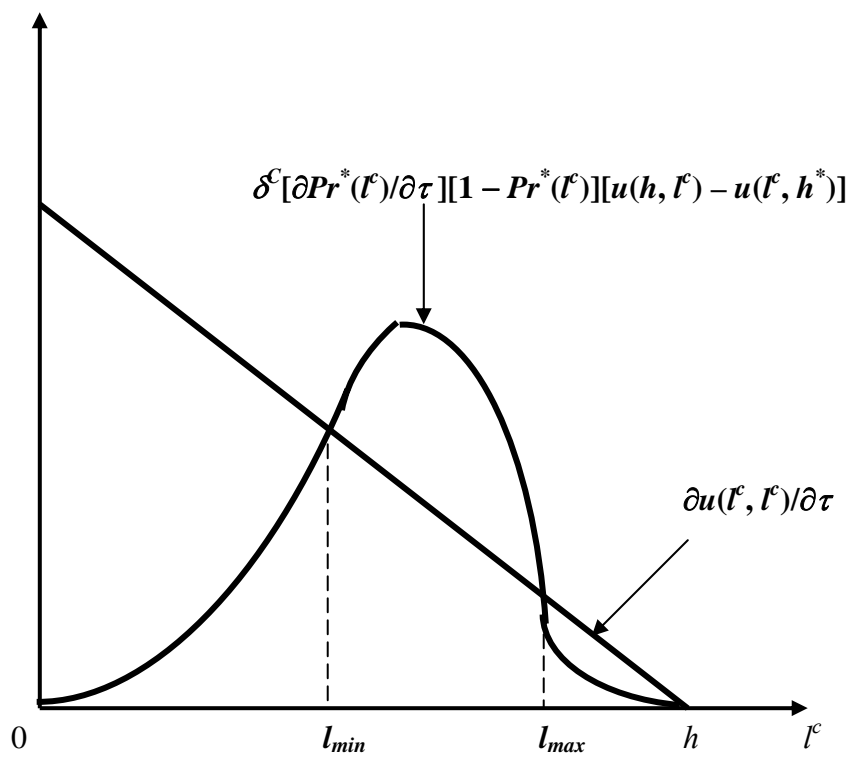


Figure 3. Multiple l^c satisfying $I(l^c) = 0$

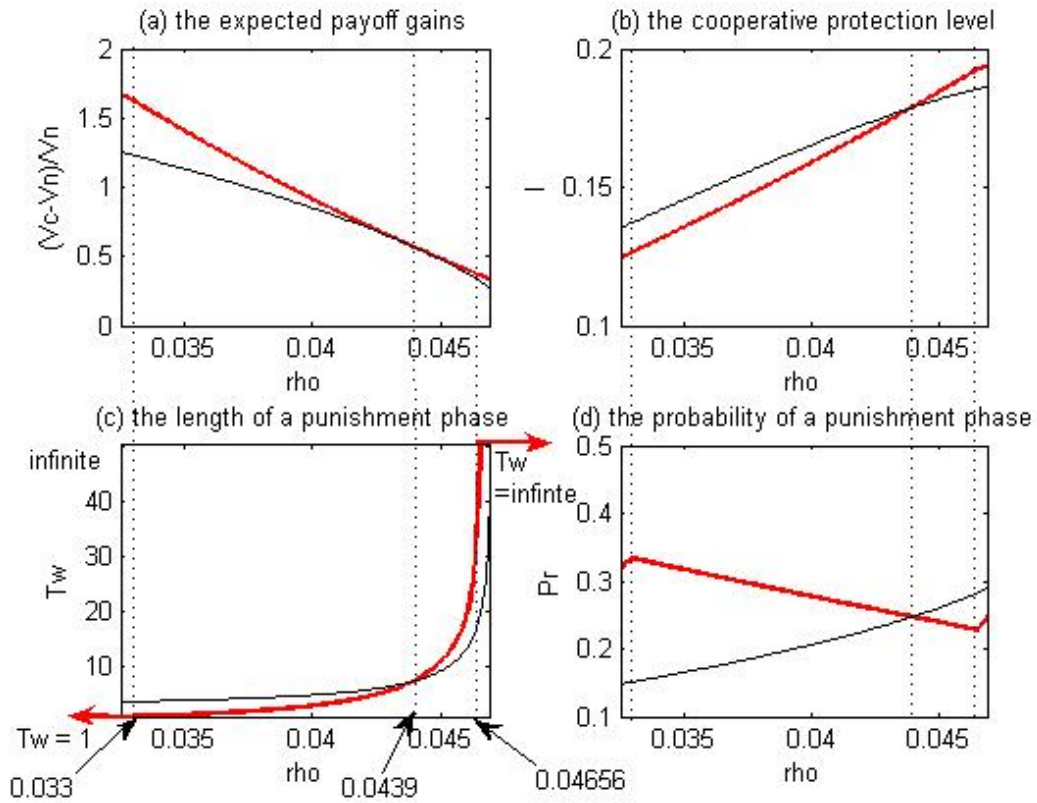


Figure 4. A numerical analysis of optimal symmetric *TTS* and optimal symmetric *PTS* for different values of ρ (ρ) with $\chi = 8$, $\alpha_1 - \alpha_1^* = 3$, $\beta + B = 1$ (so, $h = 1$), and $\delta^C = 0.9$