

# Firm Heterogeneity, Financial Imperfection, and International Trade

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## Abstract

The paper investigates the role of wealth distributions and financial institutions of an economy on within-industry firm heterogeneity in productivity. If there is no financial imperfection so that entrepreneurs are not constrained in borrowing, all of them make the same, optimal, productivity-enhancing investment. As a result, the industry will be composed of many homogeneous firms. If there exists financial imperfection, on the other hand, borrowing is constrained so the initial wealth of an entrepreneur becomes important. Given that the individuals of the economy are endowed with heterogeneous wealth, entrepreneurs with different wealth levels may choose different investment levels, resulting in the firm heterogeneity in productivity. The paper examines the impacts of the goods and capital trade on the market structure of the differentiated good sector.

Preliminary and incomplete

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# 1 Introduction

Recent empirical studies provide ample evidences that firms are very different in their productivities, size, and export and import activities, even within an industry (see, for example, Bernard, *et al.*, 2007). Inspired by these empirical evidences, Bernard, *et al.* (2003) and Melitz (2003) develop theoretical frameworks, in which heterogeneous firms engage in export activities differently in an industry under monopolistic competition. In their frameworks, productivity of each firm is drawn from a probability distribution, and once drawn it is fixed for the entire lifetime of the firm. As expected, high-productivity firms both export their products and sell them domestically, whereas low-productivity firms only sell domestically.

If the export is profitable for productive firms, however, why do low-productivity firms upgrade production technology to enjoy export profits? To answer this question, Bustos (2005), Atkeson and Burnstein (2007), and Constantini and Melitz (2007) allow firms to upgrade their production technologies with a fixed amount of investment. They show that inherently productive firms have more incentives than others to upgrade their technology and engage in the export. However, their studies do not address questions as to why firms are inherently different in their productivities in the first place.

Indeed, in principle, any firm can hire necessary human capital and purchase the up-to-date technology. Firm heterogeneity in the ability to adopt the up-to-date technology may be minimal in recent years of knowledge spillover. Even if entrepreneurs need to borrow the entire amount of investment when they establish new firms, they can do so under the perfect financial institution. In reality, of course, this is not the case. Not all entrepreneurs can borrow as much as they need to install up-to-date production facilities. This is especially so in less-developed countries which are often financially less developed. In this paper, we analyze how financial imperfection affects the firm heterogeneity and market structure of an industry. We also examine the effects of international trade of capital and goods under financial imperfection.<sup>1</sup>

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<sup>1</sup>Antràs and Caballero (2007) examine the relationship between financial imperfection and substitutability between goods and capital trade. They show that in a Heckscher-Ohlin-type model with financial imperfection, goods trade and capital mobility are complements, rather than substitutes, in less financially developed

We consider countries in which individuals with different wealth levels choose whether or not they become entrepreneurs, producing varieties in a differentiated good sector. If they become entrepreneurs, they choose whether they produce with a more-costly, high-productive production technology or with a less-costly, low-productive technology. Under perfect financial institution, all entrepreneurs invest on the high-productive technology, since they all can borrow the entire amount of investment if necessary. As a consequence, the industry is composed of many homogeneous firms. If there exists financial imperfection, on the other hand, borrowing is constrained so the initial wealth of an entrepreneur becomes important. Given that the individuals of the economy are endowed with heterogeneous wealth, entrepreneurs with different wealth levels may choose different investment levels, resulting in the firm heterogeneity in productivity.<sup>2</sup>

Manova (2008) also develops a model with credit-constrained heterogeneous firms. In her model, firms are faced with credit constraint in financing trade costs. Efficient firms are less financially constrained, so efficient firms in financially developed countries are more likely to engage in the export. Our model is quite different from hers in that financial imperfection leads to firm heterogeneity in not just their attitudes toward exporting but their productivities themselves (which of course affect export activities). We also investigate the impact of international trade of capital and goods on the market structure.

## 2 Model

In the basic model, we consider a country in which the mass  $m$  of individuals resides. Each individual owns one unit of labor and a wealth of  $\omega$  that is uniformly distributed on  $[0, \bar{\omega}]$ ; thus the density of individuals whose wealth is  $\omega \in [0, \bar{\omega}]$  equals  $m/\bar{\omega}$ . All individuals share the same utility function over the two goods, a differentiated good  $X$  and numeraire good

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countries.

<sup>2</sup>Although the interests and aims of the papers are quite different, the model structure of this paper is similar to the ones developed by Matsuyama (2004, 2005). Matsuyama (2004) shows that financial market globalization may widen the inequality of nations in the existence of financial imperfection. Matsuyama (2005) demonstrates that the difference in the financial development between countries may lead to the system of comparative advantage in international trade.

$Y$ , characterized by

$$u = \log u_x + y, \quad (1)$$

where

$$u_x = \left[ \int_{\Omega} x(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}; \quad \sigma > 1. \quad (2)$$

denotes the subutility derived from the consumption of continuum varieties of good  $X$ ,  $\{x(i)\}_{i \in \Omega}$ , and  $y$  denotes the consumption level of good  $Y$ . The numeraire good is competitively produced such that one unit of labor produces one unit of good, so the wage rate equals one.

Each individual chooses a consumption profile of good  $X$  to maximize  $u_x$  subject to  $\int_{i \in \Omega} p(i)x(i)di \leq E$ , where  $p(i)$  and  $E$  denote the price for variety  $i$  and the total expenditure on all varieties of good  $X$ , respectively. It is immediate to obtain  $x(i) = p(i)^{-\sigma} E/P^{1-\sigma}$ , where  $P \equiv [\int_{i \in \Omega} p(i)^{1-\sigma} di]^{\frac{1}{1-\sigma}}$  denotes the price index of good  $X$ . We substitute this result into (2) to obtain  $u_x = E/P$ . Therefore, an individual's utility function can be written as  $u = \log E - \log P + y$ . Maximizing this with the constraint  $E + y \leq I$ , where  $I$  denote the individual's income (which is at least as large as one, the labor income of hers), we obtain  $E = 1$ . That is, each individual spends  $E = 1$  on good  $X$ , so the country's aggregate expenditure on good  $X$  is  $m$ .

Good  $X$  industry is characterized by a monopolistic competition with free-entry and free-exit. When a firm enters, however, it incurs an R&D (or setup) cost. There are two types of production technology (or facility). The higher the investment, the lower is the marginal cost of production. More specifically, if a firm invests  $g_h$  ( $g_l$ ) units of the numeraire good, its marginal cost becomes  $c/\varphi_h$  ( $c/\varphi_l$ ). We assume that  $\varphi_l < \varphi_h$  and  $g_l < g_h < \bar{\omega}$ . To obtain each firm  $i$ 's profits, we define the ‘‘aggregate’’ productivity

$$\tilde{\varphi} \equiv \left[ \int_{i \in \Omega} \varphi(i)^{\sigma-1} di \right]^{\frac{1}{\sigma-1}}. \quad (3)$$

We make a standard assumption that each firm ignores the impact of its pricing on the price index, so that firms select prices that are  $\sigma/(\sigma - 1)$  times their individual marginal costs. It

is easy to see that firm  $k$ 's profits equal

$$\pi(\varphi(k)) = \frac{m}{\sigma} \left( \frac{\varphi(i)}{\tilde{\varphi}} \right)^{\sigma-1}. \quad (4)$$

Individuals decide whether or not they become entrepreneurs who can borrow money at a gross interest rate of  $R$  to finance their investments if necessary. However, entrepreneurs are faced with a financial constraint: entrepreneur  $i$  can borrow only up to  $\theta\pi(\varphi(i))$ , the fraction  $\theta \in (0, 1]$  of the profits that her firm will earn.<sup>3</sup>

### 3 Equilibrium under perfect financial institution

This section shows that if there is no financial constraint, all entrepreneurs choose the same production technology and hence all firms in the differentiated good sector are homogeneous.

Each individual chooses one of the following three options: (i) she becomes an entrepreneur and invests  $g_h$ , (ii) she becomes an entrepreneur and invests  $g_l$ , and (iii) she lends her money to entrepreneurs. Consider an individual with the wealth  $\omega$ . If she invests  $g_h$ , she would obtain  $\pi(\varphi_h) - R(g_h - \omega)$ . If  $\omega < g_h$ , she borrows  $g_h - \omega$  to earn  $\pi(\varphi_h)$  and pay  $R(g_h - \omega)$  back to the lenders. If  $\omega \geq g_h$ , she obtains  $\pi(\varphi_h)$  from the production of good  $X$  (by investing  $g_h$ ) and  $-R(g_h - \omega)$  from lending out. Similarly, if she invests  $g_l$ , she would obtain  $\pi(\varphi_l) - R(g_l - \omega)$ . Finally, if she lends out her wealth, she would get  $R\omega$ .

An entrepreneur chooses the high-productivity technology rather than the low-productivity technology if

$$\pi(\varphi_h) - R(g_h - \omega) > \pi(\varphi_l) - R(g_l - \omega),$$

which can be written as

$$\pi(\varphi_h) \left[ 1 - \left( \frac{\varphi_l}{\varphi_h} \right)^{\sigma-1} \right] > R(g_h - g_l). \quad (5)$$

Note that this inequality does not depend on  $\omega$ , so all entrepreneurs choose the same technology. Whether or not the inequality (5) holds depends on the productivity and investment cost parameters. In this paper, we focus on the natural case in which entrepreneurs choose

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<sup>3</sup>Matsuyama (2005, footnote 1) lists various possible causes for financial imperfection of this kind.

the high-productivity technology if they are not financially constrained, so the inequality (5) holds. In equilibrium, some individuals lend money to entrepreneurs, and hence the net benefit of being an entrepreneur and that of lending money must be the same. That is,

$$\pi(\varphi_h) - R(g_h - \omega) = R\omega,$$

which is reduced to

$$\pi(\varphi_h) = Rg_h. \quad (6)$$

Substituting this equality into (5) and rearranging terms, we obtain  $g_l/g_h > (\varphi_l/\varphi_h)^{\sigma-1}$ .

**Assumption 1**

$$\frac{g_l}{g_h} > \left( \frac{\varphi_l}{\varphi_h} \right)^{\sigma-1}.$$

Under this assumption, all entrepreneurs choose the high-productivity technology while some individuals lend their wealth to those entrepreneurs. Moreover, it is easy to check that under this assumption, there does not exist equilibrium in which entrepreneurs choose the low-productivity technology.

To find the details of the equilibrium, we first investigate the credit market. Letting  $n$  denote the mass of firms (or equivalently the number of entrepreneurs) in equilibrium, the total investment demands equal  $ng_h$  under Assumption 1. The total supply of assets, on the other hand, equals

$$\frac{m}{\bar{\omega}} \int_0^{\bar{\omega}} \omega d\omega = \frac{m\bar{\omega}}{2}.$$

By equating the asset demands and supplies, we find that the mass of firms is given by

$$n = \frac{m\bar{\omega}}{2g_h}. \quad (7)$$

Recall that the decision as to whether or not an individual becomes an entrepreneur does not depend on her wealth. This means that despite that the number of entrepreneurs is unambiguously determined, who become entrepreneurs is indeterminate. Now, we calculate  $\tilde{\varphi}$  from (3) using (7) to obtain

$$\tilde{\varphi} = \left( \frac{m\bar{\omega}}{2g_h} \right)^{\frac{1}{\sigma-1}} \varphi_h.$$

Substituting this into (4), we have

$$\pi(\varphi) = \frac{2g_h}{\sigma\bar{\omega}}. \quad (8)$$

Then, it follows from (6) that

$$R = \frac{2}{\sigma\bar{\omega}}.$$

We record our findings in the following proposition.

**Proposition 1** *Under perfect financial institution, all entrepreneurs in the differentiated good sector choose the same production technology upon entry, and hence firm heterogeneity will not arise.*

## 4 Financial Imperfection and Firm Heterogeneity

We have shown that under perfect financial institution, an individual's wealth is irrelevant in her decision as to whether or not she becomes an entrepreneur. As expected, individual's wealth will be an important factor under financial imperfection. Moreover, since individuals are heterogeneous in their wealth, their choice of technology may be heterogeneous leading to the firm heterogeneity in productivity.

Under financial imperfection, in addition to the profitability constraint

$$\pi(\varphi_h) - Rg_h \geq 0,$$

$$\pi(\varphi_l) - Rg_l \geq 0,$$

the following borrowing constraints must be satisfied for both types of firm selecting the high-productivity technology and low-productivity technology:

$$\theta\pi(\varphi_h) \geq R(g_h - \omega), \quad (9)$$

$$\theta\pi(\varphi_l) \geq R(g_l - \omega). \quad (10)$$

It is easy to see that for each type of firm, the profitability constraint is tighter than the borrowing constraint if  $\theta$  is large, whereas the borrowing constraint is tighter than the profitability constraint if  $\theta$  is small.

If  $\theta$  is less than one but is large enough, the borrowing constraint does not bind for any entrepreneur. All of them still choose the high-productivity technology earning zero profits. To find the threshold value of  $\theta$  at which the borrowing constraint starts binding, we substitute (6) into (9) to obtain

$$\omega \geq (1 - \theta)g_h. \quad (11)$$

In the case where all entrepreneurs choose the high-productivity technology, the borrowing constraint binds for all individuals with the wealth less than or equal to  $(1 - \theta)g_h$ . Suppose that all individuals with the wealth higher than or equal to  $\omega_h$  become entrepreneurs choosing the high-productivity technology. Then, it follows from (7) that

$$\frac{m}{\bar{\omega}}(\bar{\omega} - \omega_h) = \frac{m\bar{\omega}}{2g_h},$$

which can be rewritten as

$$\omega_h = \bar{\omega} - \frac{\bar{\omega}^2}{2g_h}. \quad (12)$$

Therefore, the equilibrium derived in the previous section continues to exist as long as this critical entrepreneur derived in (12) is free from the borrowing constraint, i.e.,

$$\begin{aligned} \bar{\omega} - \frac{\bar{\omega}^2}{2g_h} &\geq (1 - \theta)g_h \\ \theta &\geq 1 - \frac{\bar{\omega}}{g_h} \left(1 - \frac{\bar{\omega}}{2g_h}\right). \end{aligned} \quad (13)$$

Notice that all firms adopt the high-productivity technology and earn zero profits in this case (i.e.,  $\pi(\varphi_h) - Rg_h = 0$ ), and as a consequence no firm chooses the low-productivity technology (i.e.,  $\pi(\varphi_l) - Rg_l < 0$ ) under Assumption 1.

If  $\theta$  is so small that the inequality (13) is violated, the borrowing constraint binds for entrepreneurs whose wealth is the smallest among them. Only wealthy individuals become entrepreneurs and relatively poor individuals lend their money. The borrowing constraint is binding for the critical individuals whose wealth is  $\omega_h$ , i.e.,

$$\theta\pi(\varphi_h) = R(g_h - \omega_h). \quad (14)$$



In addition, the credit market clearing condition can be written as

$$\frac{m}{\bar{\omega}}(\bar{\omega} - \omega_h)g_h = \frac{m\bar{\omega}}{2}.$$

Since the number of firms,  $n$ , equals  $m(\bar{\omega} - \omega_h)/\bar{\omega}$ , this gives us  $n = m\bar{\omega}/(2g_h)$ . Somewhat surprisingly, the number of firms is the same as that under the perfect financial institution (see (7)). Total wealth of the economy completely determines the number of firms. Since the environment surrounding the differentiated good sector is the same as before, every firm earns the same profits (shown in (8)) as in the case of perfect financial institution. Furthermore, we substitute (8) into (14) to obtain

$$R = \frac{4\theta g_h^2}{\sigma\bar{\omega}[2g_h(g_h - \bar{\omega}) + \bar{\omega}^2]}. \quad (15)$$

The gross interest rate  $R$  is linearly increasing in  $\theta$ . As  $\theta$  decreases, it becomes more difficult to borrow money for the individuals with relatively small  $\omega$ . The gross interest rate  $R$  decreases to completely offset this negative impact of  $\theta$  for the critical individuals so that they continue to be critical entrepreneurs (recall that the number of entrepreneurs are completely determined by the availability of credit).

**Proposition 2** *Under mild financial imperfection, the same number of individuals as in the case of perfect financial institution become entrepreneurs. As the degree of financial imperfection becomes severe, the gross interest rate falls proportionately so the firms' profits and the number of firms do not change.*

As  $\theta$  falls further and  $R$  falls accordingly, the net profit from entering the good  $X$  sector (when all incumbent firms adopt the high-productivity technology),  $\pi(\varphi_l) - Rg_l$ , becomes non-negative at some point. The zero-profit condition for a firm with the low-productivity technology,  $\pi(\varphi_l) = Rg_l$ , gives us the critical  $\theta$ . Using (8) and (15), we can reduce this equality to obtain

$$\theta = \frac{2g_h(g_h - \bar{\omega}) + \bar{\omega}^2}{2g_h g_l} \left( \frac{\varphi_l}{\varphi_h} \right)^{\sigma-1}. \quad (16)$$

Thus, if  $\theta$  is smaller than this critical value, an entrepreneur has an incentive to enter the differentiated good sector with the low-productivity technology. If  $g_l$  is so small that it is

less than  $\omega_h$  given in (12), an individual whose wealth is  $g_l$  indeed enters the sector. In this case, therefore, the equilibrium described above will not exist. Henceforth, we focus on such a case in which the gap between  $g_h$  and  $g_l$  is so large that firms with different productivities coexist when  $\theta$  is smaller than the critical value given in (16).

If  $\theta$  is smaller than the one given in (16) but close enough to that critical value, the profitability constraint is binding while the borrowing constraint is slack. Indeed, we can rewrite the borrowing constraint (10) using  $\pi(\varphi_l) = Rg_l$  to obtain  $\omega \geq (1 - \theta)g_l$ , which is similar to (11). This inequality is satisfied with strict inequality if  $\omega (< g_l)$  is close enough to  $g_l$ .

As  $\theta$  falls further, the borrowing constraint for the low-productivity firms starts binding, as well as that for the high-productivity firms. In the rest of the paper, we assume that  $\theta$  is small enough that for either type of firm, it is the borrowing constraint that is binding. Before proceeding, we record one of our main findings.

**Proposition 3** *Firm heterogeneity within the differentiated good sector arises if and only if the economy is faced with relatively severe financial imperfection.*

Now, we derive equilibrium in which  $\theta$  is small enough that the borrowing constraint is binding for either type of firm. We define  $\omega_l$  such that all individuals with  $\omega \in [\omega_l, \omega_h]$  become entrepreneurs choosing the low-productivity technology while all individuals with  $\omega \in [\omega_h, \bar{\omega}]$  become entrepreneurs choosing the high-productivity technology. Defining  $\beta > 1$  by  $\varphi_h = \beta\varphi_l$ , we have

$$\begin{aligned}\tilde{\varphi} &= \left[ (\beta\varphi_l)^{\sigma-1} \frac{m}{\bar{\omega}} (\bar{\omega} - \omega_h) + \varphi_l^{\sigma-1} \frac{m}{\bar{\omega}} (\omega_h - \omega_l) \right]^{\frac{1}{\sigma-1}} \\ &= \varphi_l \left( \frac{m}{\bar{\omega}} \right)^{\frac{1}{\sigma-1}} [\beta^{\sigma-1} (\bar{\omega} - \omega_h) + \omega_h - \omega_l]^{\frac{1}{\sigma-1}}.\end{aligned}\tag{17}$$

We first derive from the two borrowing constraints a condition for  $\omega_h$  and  $\omega_l$  to satisfy. It follows from (9) and (10) that

$$\frac{g_h - \omega_h}{g_l - \omega_l} = \left( \frac{\varphi_h}{\varphi_l} \right)^{\sigma-1},$$

which can be rewritten as

$$\omega_l = g_l - \beta^{1-\sigma}(g_h - \omega_h). \quad (18)$$

We call this condition the market structure (MS) condition, since this condition determines the relative size of the groups of firms (the high-productivity firms and low-productivity firms) which simultaneously satisfies the two borrowing constraints.

The next condition for  $\omega_h$  and  $\omega_l$  to satisfy is the credit market (CM) clearing condition. It is written as

$$\frac{m}{\bar{\omega}}(\bar{\omega} - \omega_h)g_h + \frac{m}{\bar{\omega}}(\omega_h - \omega_l)g_l = \frac{m\bar{\omega}}{2}, \quad (19)$$

which can be reduced to

$$\omega_l = \frac{2g_h\bar{\omega} - \bar{\omega}^2}{2g_l} - \frac{g_h - g_l}{g_l}\omega_h. \quad (20)$$

The lower panel of Figure 1 illustrates the market structure and credit market clearing conditions. It is easy to verify that the market structure schedule is upward-sloping while credit market clearing schedule is downward-sloping, as the figure shows. We immediately find from the figure (or from (18) and (20)) that the critical asset levels,  $\omega_h$  and  $\omega_l$ , are solely determined from these conditions.

To determine the gross interest rate, we consider the high-productivity firms' borrowing constraint ( $BC_h$ ). To derive the condition, we use (17) to rewrite the high-productivity firm's profits as

$$\begin{aligned} \pi(\varphi_h) &= \frac{m}{\sigma} \left( \frac{\varphi_h}{\tilde{\varphi}} \right)^{\sigma-1} \\ &= \frac{\bar{\omega}\beta^{\sigma-1}}{\sigma[\beta^{\sigma-1}(\bar{\omega} - \omega_h) + \omega_h - \omega_l]}. \end{aligned} \quad (21)$$

It follows from (18) that

$$\omega_h - \omega_l = \beta^{1-\sigma}g_h - g_l + (1 - \beta^{1-\sigma})\omega_h.$$

Substituting this into (21) gives us

$$\pi(\varphi_h) = \frac{\bar{\omega}\beta^{\sigma-1}}{\sigma[\beta^{\sigma-1}\bar{\omega} + \beta^{1-\sigma}g_h - g_l - (\beta^{\sigma-1} + \beta^{1-\sigma} - 1)\omega_h]}.$$

Thus, we can rewrite  $\theta\pi(\varphi_h) = R(g_h - \omega_h)$  to obtain the condition that determines  $R$ :

$$R = \frac{\theta\bar{\omega}\beta^{\sigma-1}}{\sigma(g_h - \omega_h)[\beta^{\sigma-1}\bar{\omega} + \beta^{1-\sigma}g_h - g_l - (\beta^{\sigma-1} + \beta^{1-\sigma} - 1)\omega_h]}. \quad (22)$$

As the upper panel of Figure 1 shows, the right-hand side of (22) increases in  $\omega_h$ . The autarkic equilibrium rate of  $R$  is determined so that firms' borrowing constraints are binding, once  $\omega_h$  and  $\omega_l$  are determined by the credit and product markets.

Having derived the equilibrium in which the borrowing constraints are binding for both types of firm, we investigate the effect of an increase in  $\theta$  on the equilibrium. Since  $\theta$  does not appear in (18) and (20), an increase in  $\theta$  only shifts up the  $BC_h$  schedule in the upper panel of Figure 1. As a result,  $R$  increases proportionately as (22) suggests. An increase in  $\theta$  would not affect the market structure at all. As  $\theta$  increases, credit demands increase accordingly, which is completely offset by an increase in  $R$  to meet the fixed supply of credit.

**Proposition 4** *When financial imperfection is severe so that the borrowing constraints are binding for both high-productivity and low-productivity firms, an improvement of financial institution would only increase the interest rate leaving the market structure of the differentiated good sector unchanged.*

## 5 Opening to International Trade and Capital Mobility

This section extends the model to a two-country setting in which countries trade goods and/or capital (credit) to each other.

There are two countries, North and South, which differ only in the degree of financial imperfection such that  $\theta^N > \theta^S$ . There is no trade barriers in goods and capital trade, if they trade them at all. Thus, if the goods are traded, the worldwide ‘‘aggregate’’ productivity in sector  $X$  becomes

$$\begin{aligned} \tilde{\varphi}^T &= \left[ (\beta\varphi_l)^{\sigma-1} \sum_{i=N,S} \frac{m}{\bar{\omega}} (\bar{\omega} - \omega_h^i) + \varphi_l^{\sigma-1} \sum_{i=N,S} \frac{m}{\bar{\omega}} (\omega_h^i - \omega_l^i) \right]^{\frac{1}{\sigma-1}} \\ &= \frac{m\varphi_l}{\bar{\omega}} \sum_{i=N,S} \left[ \beta^{\sigma-1} (\bar{\omega} - \omega_h^i) + \omega_h^i - \omega_l^i \right]^{\frac{1}{\sigma-1}}. \end{aligned}$$

Then, it follows from the fact that the total expenditure on good  $X$  in the world equals  $2m$  that firm  $k$ 's profit, whether it is Northern or Southern, is given by

$$\pi^T(\varphi(k)) = \frac{2m}{\sigma} \left( \frac{\varphi(k)}{\tilde{\varphi}^T} \right)^{\sigma-1}.$$

Whereas the profit for firm  $k$  in country  $i = N, S$  when goods are not traded can be written, similarly to (4) as

$$\pi^i(\varphi(k)) = \frac{m}{\sigma} \left( \frac{\varphi(k)}{\tilde{\varphi}^i} \right)^{\sigma-1},$$

where  $\tilde{\varphi}^i$  is defined as country  $i$ 's  $\varphi$  expressed in (17).

The market structure condition under free trade of goods is derived from the set of borrowing conditions:

$$\begin{aligned} \theta^i \pi^T(\varphi_h) - R^i(g_h - \omega_h^i) &= 0, \\ \theta^i \pi^T(\varphi_l) - R^i(g_l - \omega_l^i) &= 0, \end{aligned}$$

for  $i = N, S$ . Consequently, we have

$$\beta^{\sigma-1} = \frac{g_h - \omega_h^i}{g_l - \omega_l^i}.$$

Note that this market structure condition is the same as that in the closed-economy model expressed in (18). This condition also serves as the market structure condition when goods are not traded, since the same condition obtains even if we replace  $\pi^T$  with  $\pi^i$ , for  $i = N, S$  in the above set of equations. Also note that the market structure condition is not affected by capital mobility. The market structure condition is depicted as the MS schedule in Figure 2.

In contrast, the credit market clearing condition critically depends on capital mobility between the countries. When capital is immobile between the countries, it can be written as

$$(\bar{\omega} - \omega_h^i)g_h + (\omega_h^i - \omega_l^i)g_l = \frac{\bar{\omega}^2}{2},$$

for  $i = N, S$ , similarly to (18). The CM schedule, common to both countries, in Figure 2 illustrates this condition. Without capital mobility, goods trade does not affect the condition.

Under perfect capital mobility, the credit market is integrated, so the worldwide credit market clearing condition together with the interest parity, i.e.,  $R^N = R^S$ , must hold. The worldwide credit market clearing condition can be written as

$$\sum_{i=N,S} \frac{m}{\bar{\omega}} [(\bar{\omega} - \omega_h^i)g_h + (\omega_h^i - \omega_l^i)g_l] = \sum_{i=N,S} \frac{m\bar{\omega}}{2},$$

which is reduced to

$$\left(\bar{\omega} - \frac{\omega_h^N + \omega_h^S}{2}\right)g_h + \left(\frac{\omega_h^N + \omega_h^S}{2} - \frac{\omega_l^N + \omega_l^S}{2}\right)g_l = \frac{\bar{\omega}^2}{2}. \quad (23)$$

The condition under which  $R^N = R^S$  can be expressed differently depending on whether or not goods are traded. If goods are not traded, the borrowing constraint for the high-productivity firms in country  $i$  can be rewritten as  $R^i = \theta^i \pi^i(\varphi_h)/(g_h - \omega_h^i)$  for  $i = N, S$ . Consequently,  $R^N = R^S$  can be reduced to

$$\omega_h^S = \left[1 - \frac{\theta^S}{\theta^N} \left(\frac{\tilde{\varphi}^N}{\tilde{\varphi}^S}\right)^{\sigma-1}\right]g_h + \frac{\theta^S}{\theta^N} \left(\frac{\tilde{\varphi}^N}{\tilde{\varphi}^S}\right)^{\sigma-1} \omega_h^N. \quad (24)$$

Similarly, we have

$$\omega_l^S = \left[1 - \frac{\theta^S}{\theta^N} \left(\frac{\tilde{\varphi}^N}{\tilde{\varphi}^S}\right)^{\sigma-1}\right]g_l + \frac{\theta^S}{\theta^N} \left(\frac{\tilde{\varphi}^N}{\tilde{\varphi}^S}\right)^{\sigma-1} \omega_l^N,$$

for the low-productivity firms. Substituting these relationships into (23) gives us the condition for  $\omega_h^N$  and  $\omega_l^N$  to be satisfied in order to clear the worldwide credit market. This condition is depicted as the  $CM^T$  schedule in Figure 2. As the figure shows, it is located to the left of the CM schedule if

$$\frac{\theta^S}{\theta^N} \left(\frac{\tilde{\varphi}^N}{\tilde{\varphi}^S}\right)^{\sigma-1} \equiv \frac{\theta^S \beta^{\sigma-1}(\bar{\omega} - \omega_h^N) + \omega_h^N - \omega_l^N}{\theta^N \beta^{\sigma-1}(\bar{\omega} - \omega_h^S) + \omega_h^S - \omega_l^S} < 1.$$

If goods are traded, on the other hand,  $R^N = R^S$  is reduced to

$$\frac{\theta^N \pi^T(\varphi_h)}{g_h - \omega_h^N} = \frac{\theta^S \pi^T(\varphi_h)}{g_h - \omega_h^S},$$

so we have

$$\omega_h^S = \left(1 - \frac{\theta^S}{\theta^N}\right)g_h + \frac{\theta^S}{\theta^N}\omega_h^N. \quad (25)$$

The weight on  $\omega_h^N$  is smaller in (25) than in (24), which implies that the  $\tilde{\text{CM}}^T$  schedule that depicts (23) with (25) is located to the left of the  $\text{CM}^T$  schedule.

Turning to the  $\text{BC}_h$  schedule, it is readily verified that the borrowing constraint for the high-productivity firms in country  $i$  can be written as

$$R^i = \frac{\theta^i \bar{\omega} \beta^{\sigma-1}}{\sigma(g_h - \omega_h^i) [\beta^{\sigma-1} \bar{\omega} + \beta^{1-\sigma} g_h - g_l - (\beta^{\sigma-1} + \beta^{1-\sigma} - 1) \omega_h^i]} \quad (26)$$

if goods are not traded, while it is written as

$$R^i = \frac{\theta^i \bar{\omega} \beta^{\sigma-1}}{\sigma(g_h - \omega_h^i) \left[ \beta^{\sigma-1} \bar{\omega} + \beta^{1-\sigma} g_h - g_l - (\beta^{\sigma-1} + \beta^{1-\sigma} - 1) \frac{\omega_h^N + \omega_h^S}{2} \right]} \quad (27)$$

if goods are not traded.

Without capital mobility, both  $(\omega_h^N, \omega_l^N)$  and  $(\omega_h^S, \omega_l^S)$  are determined as a common intersection between the MS and CM schedules in the lower panel of Figure 2. Threshold values of wealth are the same between the countries because the market structure is constrained by the limited wealth endowment, so conditions (26) and (27) are equivalent. Even if capital is mobile between the countries, the  $\text{BC}_h^i$  schedules are the same as before when goods are not traded. But as goods trade is allowed, the  $\text{BC}_h^N$  schedule shifts up while the  $\text{BC}_h^S$  schedule shifts down. To see this, we first note that  $\omega_h^N < (\omega_h^N + \omega_h^S)/2 < \omega_h^S$  as we can write  $(\omega_h^N + \omega_h^S)/2$  in two different ways:

$$\frac{\omega_h^N + \omega_h^S}{2} = \frac{1}{2} \left[ \left( 1 - \frac{\theta^S}{\theta^N} \right) g_h + \left( 1 + \frac{\theta^S}{\theta^N} \right) \omega_h^N \right] = \frac{1}{2} \left[ \left( 1 + \frac{\theta^N}{\theta^S} \right) \omega_h^S - \left( \frac{\theta^N}{\theta^S} - 1 \right) g_h \right].$$

Consequently, the  $\tilde{\text{BC}}_h^N$  schedule is located above the  $\text{BC}_h^N$  schedule, while the  $\tilde{\text{BC}}_h^S$  schedule is located below the  $\text{BC}_h^S$  schedule.

Figure 2 shows the equilibrium points for four different scenarios. Points 1 show the autarkic equilibrium. Point 2 shows the equilibrium when only goods are traded, while point 3 shows the one when only capital moves between the countries. Finally, point 4 shows the equilibrium when both goods and capital are internationally traded.

As the figure shows, the market structures and interest rates will not change if countries start engaging in goods trade. Consumers benefit from trade because they become able

to consume more varieties of good  $X$ . But the market structure does not change since the individual countries' credit markets are constrained by their own wealth endowments. Capital mobility changes the environment surrounding sector  $X$ . Capital moves from North (with a higher autarkic interest rate) to South, expanding sector  $X$  in North (by both lowering  $\omega_h^N$  and  $\omega_l^N$ ) and shrinking sector  $X$  in South. This change in the market structures is amplified if goods trade is also allowed. Interestingly, goods trade affect the market structure only when it is accompanied by international capital mobility. Capital mobility leads to heterogeneity in market structure between the two countries. Sector  $X$  in North enjoys trade benefits only when this heterogeneity exist.

**Proposition 5** *The equilibrium market structures in both countries will not change if only goods trade is opened. Capital mobility, on the other hand, will expand the differentiated good sector in North and shrink the sector in South. International trade of goods amplifies this impact of capital mobility. Indeed, goods trade affect the market structures only when it is accompanied by international capital mobility.*

## 6 Concluding Remarks

In the model where entrepreneurs with different wealth levels choose technology levels when they enter a differentiated good sector, we have shown that the firm heterogeneity in productivity arises only if there exists financial imperfection. We have examined the impact of international trade of goods and capital between two countries with different degree of financial imperfection. We have found that (i) goods trade alone will not affect the market structure, (ii) capital movement induces the financially constrained sector to expand in North with a smaller degree of financial imperfection, (iii) goods trade will amplify this impact of capital movement, and (iv) goods trade will affect the market structures only when it is accompanied by capital mobility.

Capital would move between two countries if the countries are asymmetric also in their population size, average wealth level, and so forth. It would be interesting to examine the complementarity between goods trade and capital trade in such contexts.



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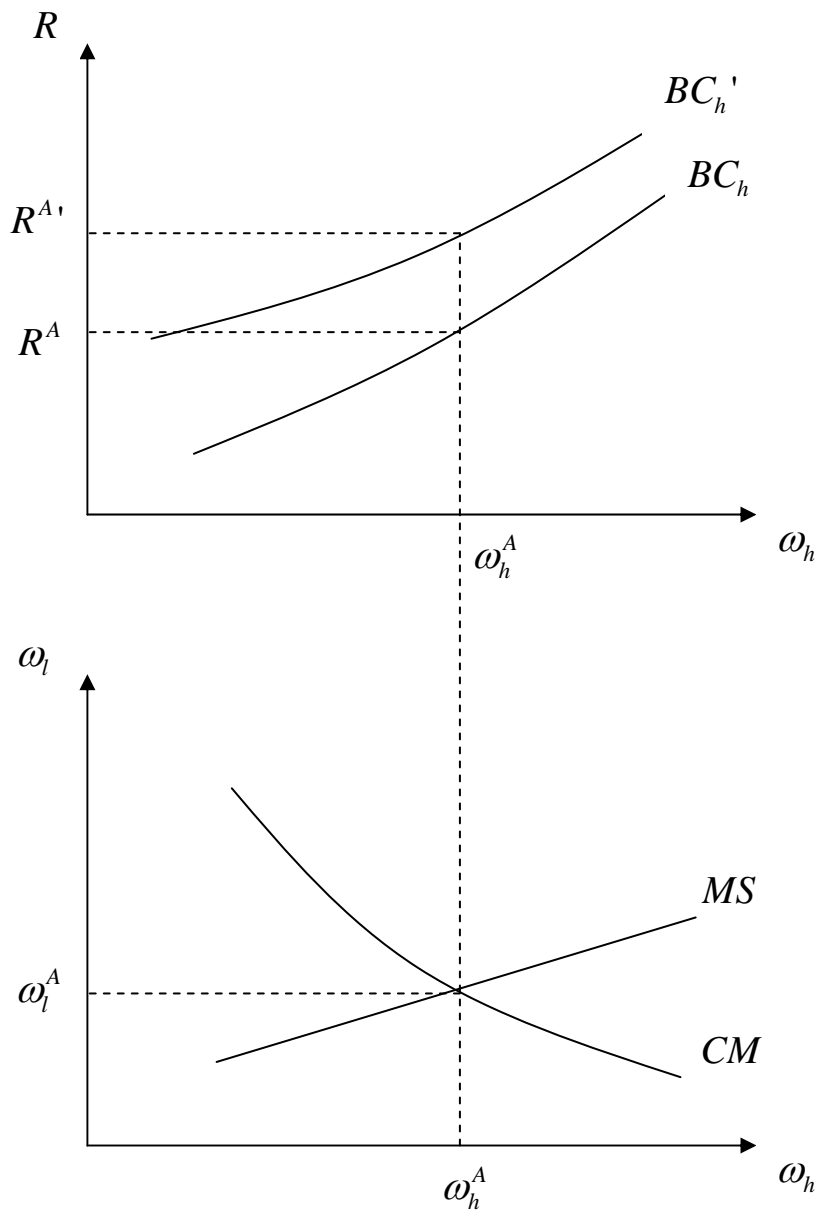


Figure 1. Autarkic Equilibrium

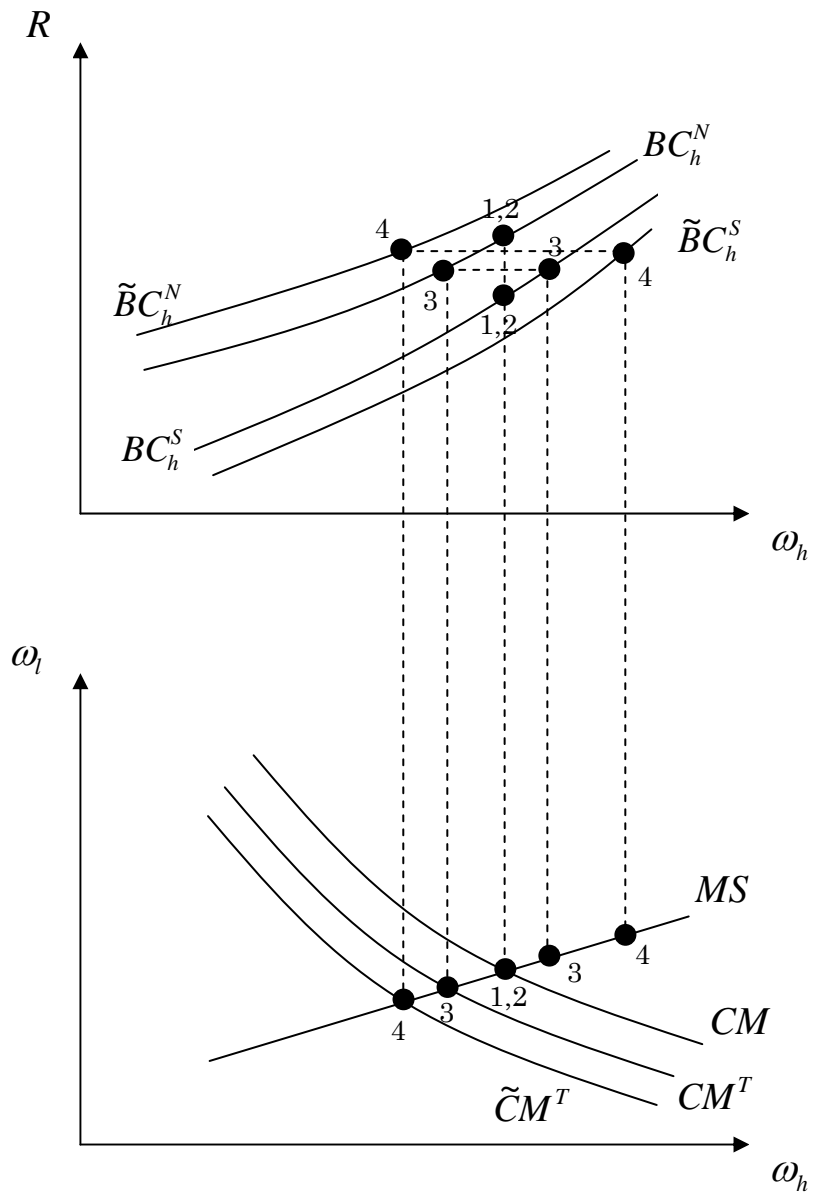


Figure 2. Trade Equilibrium