# Country Size, Productivity and Trade Share Convergence: An Analysis of Heterogenous Firms and Country Size Dependent Beachhead Costs*

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Second Draft, December 2008

**Abstract**

This paper introduces a market size dependent firm entry cost in the Helpman, Melitz and Yeaple (2004) (HMY) version of the Melitz (2003) model. This is a relatively small generalisation, which preserves the analytical solvability of the model. Nevertheless our model yields several new results that are in line with data. First, the average productivity of firms located in a market increases in the size of the market. Second, the productivity of exporters is U-shaped in the export market size. Exporting to a small market is difficult because the fixed entry cost (e.g. standardization) has to be spread over few units. This problem decreases as the export market grows. However, when the export market is sufficiently large, the market size dependent component of the entry cost (e.g. marketing) starts to dominate, and this again makes it diffiult to enter the market as an exporter. Third, the productivity premium (the difference in average productivity) between exporters and non-exporters decreases with the home country size. Fourth, we derive a set of new results related to trade volume. Contrary to what would be the case in the HMY framework, our model generates the well known fact from the empirical literature that the manufacturing export share decreases in the size of the exporting country. Moreover, it is shown that when the fixed entry cost of exporting declines, for instance as the result of economic integration, export shares converge. This result is shown to be in line with empirical data. Fifth, we use a multicountry version of our model to derive a gravity equation. Our specification yields a gravity equation à la Anderson and van Wincoop (2003), but where GDP per capita enters as an additional explanatory variable. Our specification thus gives a theoretical rationale for the common practise of introducing GDP per capita as an additional explanatory variable in gravity regressions of trade flows.

*We are grateful for comments from Pol Antras, Karolina Ekholm, Thierry Mayer, Marc Melitz, Jim Markusen, Diego Puga, participants at the NOITS conference in Stockholm May 2007, the ERWIT Conference in Appenzell June 2008, as well as from participants at a seminar at the Department of Economics in Stockholm. Financial support from Jan Wallander's and Tom Hedelius' Research Foundation is gratefully acknowledged by Akerman.

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*JEL Classification*: D21, F12, F15
*Keywords: heterogenous firms, market size, beachhead costs*

# 1 Introduction

It is empirically well established that there are systematic productivity differences among firms; see Tybout (2003) for a survey.[1] In particular, exporting firms tend to be more productive, larger, and live longer than domestic firms. There is also evidence of multinational firms tending to be more productive than exporters (Helpman, Melitz and Yeaple 2004). The theoretical literature on trade with heterogenous firms explains these findings by either iceberg trade costs associated with exports, as in Bernard et al. (2003), or higher fixed costs associated with market entry into a foreign market, as in Melitz (2003) and Yeaple (2005). Only the most productive firms will find it profitable to pay the additional cost necessary for exports, and export firms will therefore, on average, be more productive than non-exporters.

Naturally, it is also the case that firm productivity may vary because of country-specific factors. E.g. Bernard et al. (2007) show how comparative advantage may strengthen the productivity gains associated with trade in a Melitz (2003) type model with heterogenous firms. In the present paper, we investigate whether market size dependent entry costs may be one explanation for observed productivity differences between firms in different countries.

This paper modifies a Melitz (2003) type model allowing the market entry cost that firms must pay when entering a new market to increase in country size. We think here e.g. of consumer brands. It is, for instance, considerably more costly to establish a new brand of toothpaste in the U.S. than to establish the same brand in a small country such as Sweden. This is because the cost of television advertising is based on the number of viewers, free sampling based on number samples etc., which makes it more costly to enter a larger market.[2] This notion is also supported by Eaton et al. (2005) who find a strong positive relationship between country size and entry cost when calibrating their model to French firm level data as shown in Figure 1.

An immediate implication of the market entry cost increasing in market size is that firms on average should be more productive in larger markets due to firm selection. This notion is consistent with empirical evidence showing that workers and firms on average are more productive in larger markets (Head and Mayer 2004, Redding and Venables 2004, Syverson 2004, 2006, and Amiti and Cameron 2007).[3]

[1] Other studies include Aw et al. (2000), Bernard and Jensen (1995, 1999a, 1999b, 2001), Clerides et al. (1998) as well as Eaton et al. (2004).

[2] It is clear that our argument is less convincing for smaller niche products such as e.g. swedish "surströmming", which is a particular type of fermented fish. The probability of finding a buyer with a taste for this product probably increases in the number of consumers in a market.

[3] An alternative explanation for these productivity differences is externalities associated with agglomeration. Some preliminary evidence using plant level data for French cities indicate, however, that both firm seletion and agglomeration may play a role (see Puga et al 2008).

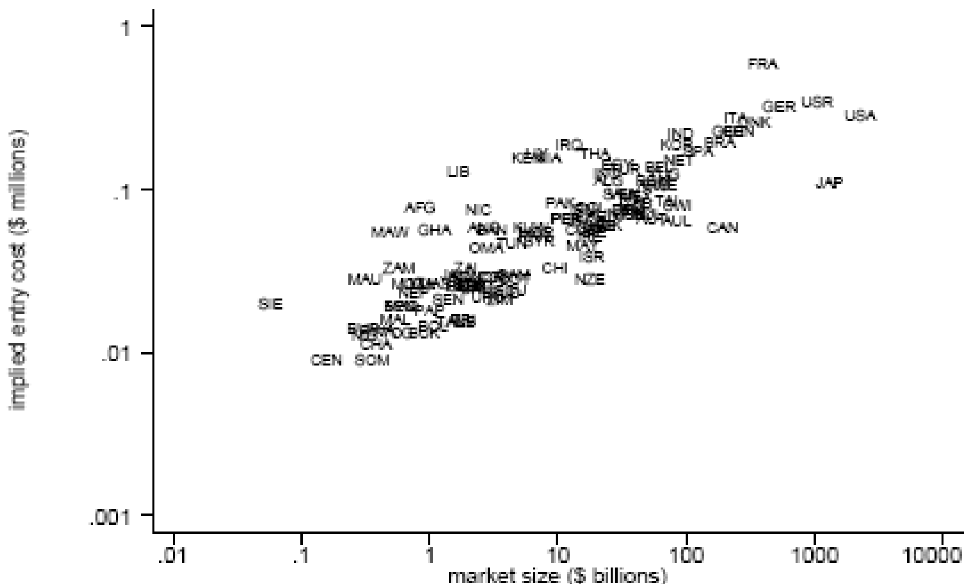


Figure 3: Implied Entry Cost and Market Size

We model the market entry cost as having a fixed and a market size dependent component. The fixed part reflects costs such as standardization of the product for a particular market or to creating a marketing message for the market. The market size dependent component of the entry cost is the marketing cost of introducing a new variety in a market (e.g. product sampling or television ads). As argued above, it is quite natural that this cost depends on the size of the market, and the fact that part of the entry cost depends on market size is normally taken for granted in the marketing literature, where the marketing cost over sales ratio is a key variable[4].

This paper makes a relatively small generalization of the Helpman, Melitz and Yeaple (2004) (HMY) version of the Melitz (2003) model, which preserves the analytical solvability of the model. Nevertheless our analysis yields several new results that are supported by data. First, the average productivity of firms located in a market increases in the size of the market. Second, the productivity of exporters is U-shaped in the export market size. Exporting to a small market is difficult because the fixed entry cost (e.g. standardization) has to be spread over few units. This problem decreases as the export market grows. However, when the export market is sufficiently large, the market size dependent component of the entry cost (e.g. marketing) starts to dominate, and this again makes it difficult to enter the market as an exporter. Third, the productivity premium between exporters and non-exporters decreases with the home country size. Fourth, we derive a set of new results related to trade volume. Contrary to what would be the case in the HMY framework, our model generates the well known property of data that

[4] See e.g. Buzzell et al. (1975).

the manufacturing export share decreases in the size of the exporting country. Moreover, it is shown that, as the fixed entry cost of exporters declines, for instance as the result of economic integration, export shares converge. Fifth, we use a multicountry version of our model to derive a gravity equation. Our specification yields a gravity equation à la Anderson and van Wincoop (2003), but where GDP per capita enters as an additional explanatory variable. Our specification thus gives a theoretical rationale for the common practise of introducing GDP per capita as an additional explanatory variable in gravity regressions of trade flows.

We confront our main theoretical results with data in various ways. A limiting factor is that our model produces cross-country predictions on the firm level, and such a data set is not yet available. The first result is, as mentioned, in line with existing empirical evidence of firms being more productive in larger markets, and we do not perform any independent test of this. The second result that the productivity of exporters is U-shaped in the export market size, implies that our model in principle is consistent with exporter productivity decreasing in market size as in Arkolakis (2006) as well as with the opposite case in Melitz and Ottaviano (2005). Again we do not perform any independent test of this. For the third result that the exporter productivity premium is larger in a small market, we present some cross-country evidence on wage premia in the export sector in the empirical section of this paper. The empirical section explicitly tests result four, showing that there is evidence of manufacturing export shares converging over time. The section also confirms the well known property of data that large markets have smaller manufacturing export shares. Finally, a large number of empirical papers on the gravity model supports result five that GDP per capita should enter the gravity regression.

Our analysis is related to that of Melitz and Ottaviano (2005) who introduce firm heterogeneity in the model by Ottaviano et al. (2002) with a linear demand system and where the endogenous mark-ups of monopolistically competitive firms depend on market size. Melitz and Ottaviano (2005) find that firms selling in large markets are larger and more productive, since higher competition forces the mark-ups in a large market downwards. The same holds in our model for domestic producers, but the mechanism leading to higher productivity in a large market is instead that firms need to be more productive to afford the higher market entry cost associated with a larger market. For exporters, our model instead yields a U-shaped relationship between foreign market size and exporter productivity as discussed above. A further difference as compared to Melitz and Ottaviano (2005) is that the productivity of firms in a market also depends on the size of other markets in our model. E.g. a larger foreign market implies more competition from imports, which forces up the productivity of domestic firms. One consequence of this dependence of foreign market size is that export shares will vary with the market size. Finally, our result that trade shares converge as the entry cost into foreign markets falls is naturally not present in Melitz and Ottaviano (2005), since they do not employ any market entry costs.

Arkolakis (2006) uses a formal model of advertising in a model of heterogenous firms where the market penetration cost of each firm is endogenous. The probability of a marketing message reaching at least one consumer increases with the population in this model. Thus, a firm gets a relatively larger payoff for a small investment in marketing when exporting to a large country as compared to a small one. The marginal exporter, which is just sufficiently productive to export, will therefore export to a large rather than to a small market. This is consistent with the feature of the firm level data e.g. in Eaton et al. (2005) that many small firms export a small amount to large markets. Our model generates the same result for not too large export markets, since the fixed market entry cost (e.g. standardization) can be spread over more units when the market is large. For large enough export markets the variable entry cost (advertising) dominates in our model, implying that exporter productivity instead has to increase in the foreign market size. We believe that our set-up complements Arkolakis (2006). Clearly for some products it is more likely to find a buyer in a large market as modeled by Arkolakis (2006). Other more standardized products, such as consumers brands, are certainly more costly to establish in large markets. Except for the results on exporter productivity our specification differs by the results on trade share convergence, that are supported by data in the empirical section. Finally, because our set-up is simpler than that of Arkolakis (2006), we can solve our model for the general equilibrium with free entry of firms.

The paper is organized as follows: Section 2 contains the model and section 3 presents the theoretical results. Section 4 contains empirical tests of our prediction that trade shares converge. Finally, section 5 concludes the paper.

# 2 The Model

This paper employs a modified Helpman et al. (2004) version of Melitz' (2003) monopolistic competition trade model with heterogeneous firms.

## 2.1 Basics

There are $m$ countries. Each country $j$ has a single primary factor of production labour, $L_j$, used in the A-sector and the M-sector. The A-sector is a Walrasian, homogenous-goods sector with costless trade. The M-sector (manufactures) is characterized by increasing returns, Dixit-Stiglitz monopolistic competition and iceberg trade costs. M-sector firms face constant marginal production costs and three types of fixed costs. The first fixed cost, $F_E$, is the standard Dixit-Stiglitz cost of developing a new variety. The second and third fixed costs are 'beachhead' costs reflecting the one-time expense of introducing a new variety into a market. These costs are here assumed to depend on the size of the market.

There is heterogeneity with respect to firms' marginal costs. Each Dixit-Stiglitz firm/variety is associated with a particular labour input coefficient – denoted as $a_i$ for firm $i$. After sinking $F_E$ units of labour in the product innovation process, the firm is randomly assigned an '$a_i$' from a probability distribution G(a).

Our analysis exclusively focuses on steady-state equilibria and intertemporal discounting is ignored; the present value of firms is kept finite by assuming that firms face a constant Poisson hazard rate $\delta$ of 'death'.

Consumers in each nation have two-tier utility functions with the upper tier (Cobb-Douglas) determining the consumer's division of expenditure among the sectors and the second tier (CES), dictating the consumer's preferences over the various differentiated varieties within the M-sector.

All individuals in country $j$ have the utility function

$$U_j = C_{Mj}^{\mu} C_{Aj}^{1-\mu}, \tag{1}$$

where $\mu \in (0,1)$, and $C_{Aj}$ is consumption of the homogenous good. Manufactures enter the utility function through the index $C_{Mj}$, defined by

$$C_{Mj} = \left[ \int_0^{N_j} c_{ij}^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \tag{2}$$

$N_j$ being the mass of varieties consumed in country $j$, $c_{ij}$ the amount of variety $i$ consumed in country $j$, and $\sigma > 1$ the elasticity of substitution.

Each consumer spends a share $\mu$ of his income on manufactures, and demand for a variety $i$ in country $j$ is therefore

$$x_{ij} = \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} \mu Y_j, \tag{3}$$

where $p_{ij}$ is the consumer price of variety $i$ in country $j$, $Y_j$ is income, and $P_j \equiv \left( \int_0^{N_j} p_{ij}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$ the price index of manufacturing goods in country $j$.

The unit factor requirement of the homogeneous good is one unit of labour. This good is freely traded and since it is chosen as the numeraire

$$p_A = w = 1; \tag{4}$$

$w$ being the nominal wage of workers in all countries.

Shipping the manufactured good involves a frictional trade cost of the "iceberg" form: for one unit of a good from country $j$ to arrive in country $k$, $\tau_{jk} > 1$ units must be shipped. It is assumed that trade costs are equal in both directions and that $\tau_{jj} = 1$. Profit maximization by a manufacturing firm $i$ located in country $j$ leads to consumer price

$$p_{ijk} = \frac{\sigma}{\sigma - 1} \tau_{jk} a_i \tag{5}$$

in country $k$.

Manufacturing firms draw their marginal cost, $a$, from the probability distribution $G(a)$ after having sunk $F_E$ units of labour to develop a new variety. Having learned their productivity, firms decide on entry in the domestic and foreign market, respectively. Firms will enter a market as long as the operating profit in this market is sufficiently large to cover the beachhead (market entry) cost associated with the market. Because of the constant mark-up pricing, it is easily shown that operating profits equal sales divided by $\sigma$. Using this and (3), the critical 'cut-off' levels of the marginal costs are given by:

$$a_{Dj}^{1-\sigma} B_j = F_D(L_j), \tag{6}$$

$$a_{Xjk}^{1-\sigma} \phi_{jk} B_k = F_X(L_k), \tag{7}$$

where $F_D(L_j) \equiv \delta\sigma\widetilde{F}_D(L_j)$, $F_X(L_j) \equiv \delta\sigma\widetilde{F}_X(L_j)$, $B_j \equiv \frac{\mu L_j}{P_j^{1-\sigma}}$, and $\phi_{jk} \equiv \tau_{jk}^{1-\sigma} \in [0,1]$ represents trade freeness. The market entry cost (beachhead cost) is assumed to increase in the size of the market $\frac{dF_D(L_j)}{dL_j}, \frac{dF_X(L_j)}{dL_j} > 0$. We will parametrize how the beachhead cost depends on market size below. However, note that it is natural that $F$ depends on $L$, since the marketing costs of establishing a new brand in a large market, such as e.g. the US, are typically much higher than in a small country.

Finally, free entry ensures that the ex-ante expected profit of developing a new variety in country $j$ equals the investment cost:

$$\int_0^{a_{Dj}} \left(a^{1-\sigma} B_j - F_D(L_j)\right) dG(a) + \sum_{k,k\neq j} \int_0^{a_{Xjk}} \left(\phi_{jk} a^{1-\sigma} B_k - F_X(L_k)\right) dG(a) = F_E. \tag{8}$$

## 2.2 Solving for the Long-run Equilibrium

In this section, we apply two simplifying assumptions. First, the model is solved with two countries, $j$ and $k$ (appendix 6.1 indicates how the multicountry case is solved). We refer to $j$ as 'Home' and $k$ as 'Foreign'. Second, we follow HMY in assuming the probability density function to be Pareto[5]:

$$G(a) = a^{\theta}. \tag{9}$$

Substituting the cut-off conditions (6) and (7) into the free-entry condition (8) gives $B_j$,

$$B_j = \left(\frac{F_E F_D^{\beta-1}(L_j) \cdot (\beta-1)\left(1-\Omega(L_k)\right)}{1-\Omega(L_j)\Omega(L_k)}\right)^{\frac{1}{\beta}}, \tag{10}$$

where $\beta \equiv \frac{\theta}{\sigma-1} > 1$, and $\Omega(L_j) \equiv \phi^{\beta}\left(\frac{F_X(L_j)}{F_D(L_j)}\right)^{1-\beta} \in [0,1]$ is an index of trade freeness. Using (10) and the cut-off conditions gives the cut-off marginal costs:

$$a_{Dj}^{\theta} = \frac{(\beta-1)\,F_E}{F_D(L_j)}\left(\frac{(1-\Omega(L_k))}{1-\Omega(L_j)\Omega(L_k)}\right), \tag{11}$$

[5] This assumption is consistent with the empirical findings by e.g. Axtell (2001).

$$a_{Xjk}^{\theta} = \frac{(\beta-1)\,\Omega(L_k)F_E}{F_X(L_k)}\left(\frac{(1-\Omega(L_j))}{1-\Omega(L_j)\Omega(L_k)}\right). \tag{12}$$

From these, it is seen that, contrary to the standard model by Melitz (2003), the market size will affect the cut-off marginal costs. We will assume that $\frac{F_X(L_k)}{\Omega(L_k)} > F_D(L_j)$ for all $j, k$.[6] This assumption implies that $a_{Xjk} < a_{Dj}\ \forall j, k$.

The price indices may be written as

$$P_j^{1-\sigma} = \frac{\beta}{\beta-1}\left(n_j a_{Dj}^{1-\sigma} + n_k \phi_{kj} a_{Dk}^{(1-\sigma)}\left(\frac{a_{Xkj}}{a_{Dk}}\right)^{\theta+1-\sigma}\right), \tag{13}$$

and the mass of firms in each country can be calculated using (10), (11), and (12) together with the fact that $B_j = \frac{\mu L_j}{P_j^{1-\sigma}}$ :

$$n_j = \frac{\mu(\beta-1)}{F_D(L_j)\beta}\frac{L_j(1-\Omega(L_j)) - L_k\Omega(L_j)(1-\Omega(L_k))}{(1-\Omega(L_j)\Omega(L_k))(1-\Omega(L_j))}. \tag{14}$$

Welfare may be measured by indirect utility, which is proportional to the real wage $\frac{w_j}{p_{Aj}^{1-\mu}P_j^u}$. Since $p_{Aj} = w_j = 1\ \ \forall j$, it suffices to examine $P_j$. Using (11), (12), (13), and (14), we have

$$P_j = \left(\mu^{-\beta}L_j^{-\beta}F_D^{\beta-1}(L_j)F_E(\beta-1)\cdot\frac{1-\Omega(L_k)}{1-\Omega(L_j)\Omega(L_k)}\right)^{\frac{1}{\beta(\sigma-1)}}. \tag{15}$$

This expression shows that, as in the Melitz (2003) model, welfare always increases ($P$ decreases) with trade liberalization; that is, with a higher $\phi_{jk}$ or a lower $\frac{F_X(L_k)}{F_D(L_j)}$.

### 2.3 Parametrisation of the beachhead cost

In the following, we parametrise the beachhead costs as:

$$\widetilde{F}_D(L_j) \equiv f_D + (L_j)^{\gamma}, \quad \widetilde{F}_X(L_k) \equiv f_X + (L_k)^{\gamma}, \quad \gamma > 0. \tag{16}$$

The variable component of the beachhead cost increases in market size, while the constant term picks up costs that are independent of market size. It is quite natural that the beachhead cost would have one fixed and one variable component. The constant $f$ represents the fixed cost of standardizing a product for a particular market or the cost of producing an advertisement tailored to a particular market with its culture and language. The variable cost term $L_j^{\gamma}$ represents the fact that the cost of spreading an advertising message increases with the number of consumers in a market. For instance, the number of free product samples or advertising posters increases with the size of the population. Likewise, the cost of television advertising increases with the number of viewers. We do not put any restriction on the shape of the variable cost term except $\gamma > 0$.[7]

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[6] The corresponding condition in Melitz (2003) is that $\frac{F_X}{\phi} > F_D$.

[7] A simpler alternative would be a multiplicative formulation: $\widetilde{F}_X(L^j) = f_X \cdot (L^j)^{\gamma}$. Some of our results could be derived with this specification but, contrary to our specification, it would e.g. imply that trade shares are

# 3 Results

A large number of comparative static results may be derived. Here, we focus on the more novel aspects of our model, which are related to the effects of market size. From now on, the simplified notation $F_{Dj} \equiv F_D(L_j)$, $F_{Xj} \equiv F_X(L_j)$, and $\Omega_j \equiv \Omega(L_j)$ is adopted.

## 3.1 Productivity

The first set of results concerns the productivity of exporters and non-exporters in the two countries. From (6) and (7)

$$a_{Dj}^{\sigma-1} = \frac{B_j}{F_{Dj}}, \quad a_{Xjk}^{\sigma-1} = \frac{\phi B_k}{F_{Xk}}. \tag{17}$$

A larger market, measured by $L_j$, affects the cutoffs via two channels: First, it increases the demand facing each firm (via $B_j$ and $B_k$, respectively) and, second, it increases the market size dependent beachhead costs. However, note that the demand increase experienced by a firm is dampened by the entry of new firms.

We first turn to the influence of domestic market size. The effect of a larger home market on non-exporters is

$$\frac{\partial a_{Dj}}{\partial L_j} < 0 \quad \text{for} \quad \phi < 1, \tag{18}$$

as shown in appendix 6.3. The negative signs imply that the higher beachhead cost due to a larger market dominates the effect of higher demand, so that the marginal firm must be more productive in a larger market.

Next, from (17)

$$\frac{\partial a_{Xjk}}{\partial L_j} < 0, \tag{19}$$

since $\frac{\partial B_k}{\partial L_j} < 0$. A larger mass of domestic exporters implies stronger competition in the foreign market and the marginal exporter must consequently be more productive. The effects of domestic market size on the productivity of exporters and non-exporters are summarized in Result 1a.

*Result 1a: The average productivity of exporters as well as non-exporters increases in the size of the domestic market as long as $\phi < 1$.*

The aggregate productivity can be expressed as[8]:

$$\overline{\varphi}_j = \left( s_{Dj} \int_0^{a_{Dj}} a^{1-\sigma} dG(a\,|a_{Dj}) + s_{Xjk} \int_0^{a_{Xjk}} a^{1-\sigma} dG(a\,|a_{Dj}) \right)^{\frac{1}{\sigma-1}}, \tag{20}$$

invariant to country size.

[8] See Melitz (2003).

where $s_{Dj}$ is the share of home producers that sells domestically only and $s_{Xjk}$ is the share that exports to country $k$. Using that the ratio of exporters to non-exporters is $\left(\frac{a_{Xjk}}{a_{Dj}}\right)^{\theta}$, $s_{Dj} = \frac{1}{1+\left(\frac{a_{Xjk}}{a_{Dj}}\right)^{\theta}}$, and $s_{Xjk} = \frac{\left(\frac{a_{Xjk}}{a_{Dj}}\right)^{\theta}}{1+\left(\frac{a_{Xjk}}{a_{Dj}}\right)^{\theta}}$, we get:

$$\overline{\varphi}_j = \frac{1}{a_{Dj}} \left(\frac{\theta}{\theta - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \left(\frac{1 + \left(\frac{a_{Xjk}}{a_{Dj}}\right)^{2\theta+1-\sigma}}{1 + \left(\frac{a_{Xjk}}{a_{Dj}}\right)^{\theta}}\right)^{\frac{1}{\sigma-1}}. \tag{21}$$

From (21), it is seen that average productivity increases in $L_j$ since from (18) $\frac{\partial a_{Dj}}{\partial L} < 0$, from (25) $\frac{\partial\left(\frac{a_{Xjk}}{a_{Dj}}\right)}{\partial L_j} > 0$, and $\theta - \sigma + 1 > 0$. It is therefore also the case that aggregate productivity in manufacturing increases in country size.

*Result 1b: Aggregate productivity in manufacturing increases in country size.*

Next, the effect of the foreign market size on non-exporters is

$$\frac{\partial a_{Dj}}{\partial L_k} < 0, \tag{22}$$

from (17), since $\frac{\partial B_j}{\partial L_k} < 0$ by inspection of (10). The intuition is that a larger foreign market implies a larger mass of foreign firms competing in the home market, which decreases the market shares of domestic non-exporters.

The effect of foreign market size on the productivity of the marginal exporter is generally U-shaped, as shown in appendix 6.4:

$$\begin{aligned} \frac{\partial a_{X_{jk}}}{\partial L_k} &\geqslant 0 \quad \text{for} \quad \psi_j^{\beta-1}\psi_k^{(\beta-1)}\left(\beta - (\beta-1)\psi_k\right) \geqslant \phi^{2\beta} \\ \frac{\partial a_{X_{jk}}}{\partial L_k} &< 0 \quad \text{for} \quad \psi_j^{\beta-1}\psi_k^{(\beta-1)}\left(\beta - (\beta-1)\psi_k\right) < \phi^{2\beta}, \end{aligned} \tag{23}$$

where $\psi_k \equiv \frac{F_X(L_k)}{F_D(L_k)} \geqslant 1$ measures the relative market access (the relative beachhead cost) of foreign versus domestic firms. The left-hand side of the inequality, determining the sign of the derivative, increases in $\psi_k$ as is easily shown. Note also that $\lim_{L_k \longrightarrow \infty} \psi_k = 1$, since the fixed cost of the market entry cost becomes irrelevant in the limit. $a_{Xjk}$ therefore decreases in the foreign market size for large enough $L_k$,, simply because firms need to be more productive to overcome the higher (variable) entry cost in a larger market. In the opposite case, when the foreign market is quite small, the fixed market entry cost $f_X$ becomes important. Increasing the foreign market size implies that this cost can be spread over a larger number of units so that the cut-off productivity for entering the foreign market deceases in the market size. The two cases are illustrated in Figure 1, which uses (12) to plot $a_{Xjk}$ against $L_k$ for some standard parameter values ($\phi = 0.8, \theta = 3, \gamma = 1, fx = 2, f_D = 1, F_E = 1, L_j = 2$). The figure also illustrates how a large foreign market is particularly valuable to the exporter when the mark-up is high, which implies that $a_{Xjk}$ tends to increase in the foreign market size when $\sigma$ is low.

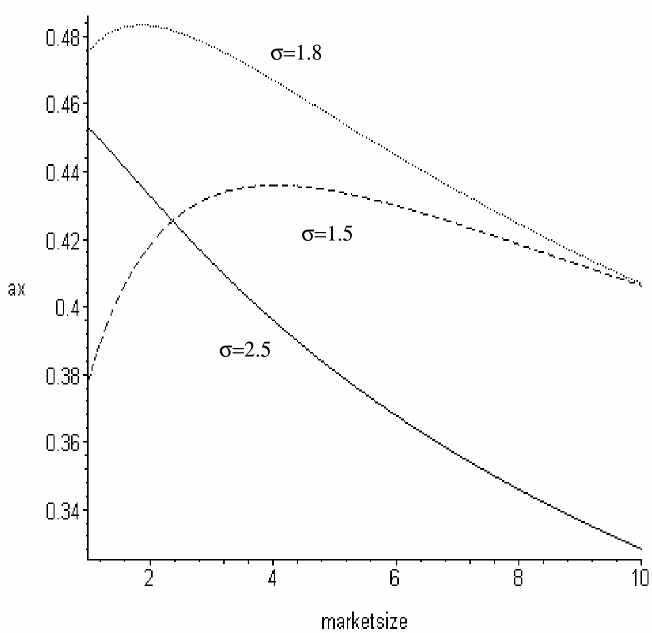


Figure 1: Exporter cut-off (inverse productivity) in relation to foreign market size ($\phi = 0.8, \theta = 3, \gamma = 1, fx = 2, f_D = 1, F_E = 1, L_j = 2$).

Thus, the model encompasses both the case when the marginal exporter needs to be less productive in larger markets as in Arkolakis (2006), and the case in Melitz and Ottaviano (2005) where the opposite holds. Result 2 summarizes the effects of foreign market size.

*Result 2: The average productivity of non-exporters increases in the size of the foreign market. The average productivity of exporters is generally U-shaped in the foreign market size.*

Next using (11) and (12), the relative cut-off productivity for non-exporters and exporters in the home country is

$$\left(\frac{a_{Dj}}{a_{Xjk}}\right)^{\theta} = \frac{F_{Xk}}{F_{Dj}\Omega_k}\left(\frac{1-\Omega_k}{1-\Omega_j}\right) > 1, \quad for \quad \frac{F_{Xj}}{\Omega_j} > F_{Dk}\,\forall\, j,k, \ and \quad \Omega_j, \Omega_k < 1. \tag{24}$$

There is strong empirical support for exporters being more productive than domestic firms, and as mentioned above, we follow Melitz (2003) by making parameter assumptions for this to hold: $\frac{F_{Xj}}{\Omega_j} > F_{Dk}$.

Finally, market size is also of importance for the relative productivity of exporters as compared to non-exporters:

$$\frac{\partial\left(\frac{a_{Dj}}{a_{Xjk}}\right)}{\partial L_j} < 0 \quad for \quad \Omega_j, \Omega_k < 1, \tag{25}$$

as shown in appendix 6.6. The larger is the home country, the less productive are exporters as compared to non-exporters. Essentially, the higher fixed cost associated with the larger home market will push up the relative productivity of domestic firms, which makes exporters look less productive in comparison.

*Result 3: Exporters are more productive than producers for the domestic market. However, this effect decreases in the size of the home country.*

## 3.2 Trade shares

The next set of results concerns the relationship between country size and manufacturing export share. A home exporting firm with the marginal cost $a$, sells $a^{1-\sigma}\phi_{jk}B_k$ in the foreign market. Using (7), the total export volume from home is

$$V_{Xjk} = \int_0^{a_{Xjk}} a^{1-\sigma} dG(a \mid a_{Dj}) \cdot \frac{F_{Xk}}{a_{Xjk}^{1-\sigma}} = \left(\frac{a_{Xjk}}{a_{Dj}}\right)^{\theta} \frac{\beta}{\beta-1} F_{Xk} n_j. \tag{26}$$

Similarly, the total production volume for the home market is

$$V_{Dj} = \int_0^{a_{Dj}} a^{1-\sigma} dG(a \mid a_{Dj}) \cdot \frac{F_{Dj}}{a_{Dj}^{1-\sigma}} = \frac{\beta}{\beta-1} F_{Dj} n_j. \tag{27}$$

The export share may now be written as

$$S_{Xjk} = \frac{V_{Xjk}}{V_{Xjk} + V_{Dj}} = \frac{\Omega_k(1-\Omega_j)}{1-\Omega_j\Omega_k}. \tag{28}$$

Differentiating with respect to country size gives

$$\frac{\partial S_{Xjk}}{\partial L_j} = \frac{\Omega_k(\Omega_k - 1)}{(1-\Omega_j\Omega_k)^2} \frac{\partial \Omega_j}{\partial L_j} < 0, \tag{29}$$

$$\frac{\partial S_{Xjk}}{\partial L_k} = \frac{1-\Omega_j}{(1-\Omega_j\Omega_k)^2} \frac{\partial \Omega_k}{\partial L_k} > 0, \tag{30}$$

which implies that a smaller country has a higher manufacturing export share than a larger one.

*Result 4a: The manufacturing export share of a country decreases in its own size, and increases in the trade partner's size.*

Next, note that for $f_X = f_D$ and the symmetric iceberg trade cost ($\phi_{jk} = \phi_{kj}$), $\Omega_j = \Omega_k = \phi^{\beta}$. This means, from (28), that $S_{Xjk} = S_{Xkj}$; i.e., manufacturing export shares converge as $f_X$ approaches $f_D$. Moreover, since a falling $f_X$ makes exporting easier, export shares converge upwards.

*Result 4b: Falling relative beachhead costs ($f_X$ converging to $f_D$) imply (upwards) converging manufacturing export shares.*

The intuition for *Result 4a* and *Result 4b* derives from the fact that, when selling their product, firms have to pay two different sunk costs: a standardization cost that is independent of market size and a marketing cost which depends on the size of the market reflecting the higher cost of reaching more consumers. Also the standardization for a particular market is more costly for an exporter than for a domestic producer. Since the cost of standardization is independent of market size, it becomes relatively less important as compared to the marketing cost when the market is large. The difference in fixed costs between foreign and domestic firms is therefore relatively smaller in a large market.

For example, suppose that Sweden and the United States have similar levels of regulation but different tastes in the design of labels, packages and instructions. Then, the cost of standardization is similar for an American firm targeting the Swedish market and for a Swedish firm targeting the American market. However, the market size dependent marketing cost is much higher for firms selling in the US as compared to those selling in Sweden. The difference in fixed costs for Swedish exporters and American domestic producers, both serving the same market, is therefore smaller in relative terms than the difference between American exporters and Swedish domestic producers. Consequently, the smaller country, i.e. Sweden, has a larger share of manufacturing exports in its production.

Second, since Swedish exporters are more concerned with the larger marketing costs than the standardization costs, as compared to American exporters to Sweden, it must be the case that the decrease in standardization costs for foreign markets ($f_X$ approaching $f_D$) affects American firms more than Swedish firms. This means that American firms will increase their exports at a greater pace than Swedish firms and therefore, they will start catching up with their Swedish counterparts. In the aggregate, the American export share of manufacturing production will then approach the (larger) Swedish export share and export shares converge across countries. In the extreme, when the cost of standardization is the same for the domestic and the foreign market ($f_X = f_D$), export shares converge completely across countries.

It may be useful to compare our results to the standard models. Here, we use the Melitz (2003) model with a homogenous good and freely traded A-sector à la Helpman et al. (2004). It is easily shown that the manufacturing export shares are independent of country size in this model without our assumption of a market size dependent beachhead cost. However, our result may also be compared to the standard Dixit-Stiglitz trade model without a homogenous good A-sector (see e.g. Helpman (1987)). Like our model, trade shares are negatively related to market size in that model. However, in contrast to our model, manufacturing trade shares diverge as trade costs fall: trade shares increase from zero in autarky to the share of the foreign market in total demand at free trade.[9] As argued below, we believe our prediction of converging manufacturing export shares to be supported by empirical evidence.

### 3.3 The gravity equation

Our modification of the Melitz model has implications for the gravity equation. Consider a setting without the A-sector and with $m$ countries indexed by $j$ and $k$. The factory gate price of each firm is now $p_{ij} = \frac{\sigma}{\sigma-1} a_i w_j$, where $w_j$ is the factor price in country $j$. The export from $j$ to $k$ is

$$E_{jk} = n_j \left( \frac{\sigma}{\sigma - 1} \frac{\tau_{jk} w_j}{P_k} \right)^{1-\sigma} V_{jk} Y_k, \tag{31}$$

where $V_{jk} \equiv \int_0^{a_{Xjk}} a_i^{1-\sigma} dG(a \mid a_{Dj})$. The GDP of country $j$ consists of the sum of sales in all markets including itself:

$$Y_j = \sum_{k=1}^{m} E_{jk} = \sum_{k=1}^{m} n_j \left( \frac{\sigma}{\sigma - 1} \frac{\tau_{jk} w_j}{P_k} \right)^{1-\sigma} V_{jk} Y_k. \tag{32}$$

Following the methodology of Anderson and van Wincoop (2003), a gravity equation of trade may be derived by solving (32) for $n_j \left( \frac{\sigma}{\sigma-1} w_j \right)^{1-\sigma}$ and substituting the solution into (31). This gives:

$$E_{jk} = \frac{Y_j Y_k}{Y^w} \frac{\tau_{jk}^{1-\sigma} V_{jk}}{P_k^{1-\sigma} \sum_{j=1}^{m} \left( \frac{\tau_{jk}^{1-\sigma}}{P_k^{1-\sigma}} \right) s_k V_{jk}}, \tag{33}$$

where $s_k \equiv \frac{Y_k}{Y^w}$, and $Y^w \equiv \sum_{j=1}^{m} Y_j$.

Solving the integral for $V_{jk}$ gives

$$V_{jk} = \frac{\theta}{\theta - \sigma + 1} \left( \frac{a_{Xjk}}{a_{Dj}} \right)^{\theta} a_{Xjk}^{1-\sigma}. \tag{34}$$

Using this and the fact that $a_{Xjk}^{1-\sigma} = \frac{F_{Xk} P_k^{1-\sigma}}{\tau_{jk}^{1-\sigma} Y_k}$ from the cut-off conditions gives the following gravity equation

$$E_{jk} = \frac{Y_j Y_k}{Y^w} \frac{\tau_{jk}^{-\theta} \left( \frac{Y_k}{F_{Xk}} \right)^{\frac{\theta}{\sigma-1} - 1}}{P_k^{-\theta} \sum_{k=1}^{m} \left( \frac{\tau_{jk}}{P_k} \right)^{-\theta} s_k \left( \frac{Y_k}{F_{Xk}} \right)^{\frac{\theta}{\sigma-1} - 1}}. \tag{35}$$

The expression may be compared to the standard Anderson and van Wincoop gravity formulation: $E_{jk} = \frac{Y_j Y_k}{Y^w} \frac{\tau_{jk}^{1-\sigma}}{P_k^{-\theta} \sum_{k=1}^{m} \left( \frac{\tau_{jk}}{P_k} \right)^{1-\sigma} s_k}$. The difference is essentially the term $\left( \frac{Y_j}{F_{Xj}} \right)^{\frac{\theta+1-\sigma}{\sigma-1}}$, which

[9] Naturally, in this model there is no beachhead cost that can be affected by trade liberalization.

multiplies the frictional trade cost.[10] This term measures the importance of the exporters' market entry cost in relation to the size of the export market, that is, trade resistance related to the market entry cost. Note also that a higher $\sigma$ decreases the negative impact of $F_{Xj}$ on trade, as pointed out by Chaney (2008).

Finally, substituting (16) into (35) gives

$$E_{jk} = \frac{Y_j Y_k}{Y^w} \frac{\tau_{jk}^{-\theta} \left( \frac{Y_k}{f_X + L_k^\gamma} \right)^{\frac{\theta}{\sigma-1}-1}}{P_k^{-\theta} \sum_{k=1}^m \left( \frac{\tau_{jk}}{P_k} \right)^{-\theta} s_k \left( \frac{Y_k}{f_X + L_k^\gamma} \right)^{\frac{\theta}{\sigma-1}-1}}. \tag{36}$$

This shows that our specification gives a justification for the common usage of GDP per capita as explanatory variable in the gravity regression. GDP per capita enters because a higher purchasing power per head makes it worth more to firms to pay the entry cost, which increases by heads. However, since the entry cost increases non-linearly in the size of the population, it is $\frac{Y_k}{f_X + L_k^\gamma}$ rather than exactly GDP per capita that should enter the gravity regression.

# 4 Empirical Section

We have derived a set of new results. We will here explicitly test only a few of them. For the other results we will discuss to what extent they seem consistent with existing empirical evidence. One obvious limitation is that several of our results ideally should be tested using a cross country firm level data set, and such a data set is not yet available.

Our first result that firms on average are more productive in large markets is consistent with several empirical studies showing that workers and firms on average are more productive in larger markets (Head and Mayer 2004, Redding and Venables 2004, Syverson 2004, 2006, and Amiti and Cameron 2007), and we do not here perform any independent test of this. Likewise, Result 2, that the productivity of exporters is U-shaped in the size of the export market is consistent with both Arkolakis (2006) indicating that the productivity of exporters decreases in the export market size, and with the opposite result in Melitz and Ottaviano (2005). Concerning Result 3, that country size affects the relative performance of exporters and non-exporters, we turn to stylized evidence in the following section. Results 4a,b are explicitly tested in section 4.2. Finally our result that GDP/capita enters the gravity equation is consistent with a number of empirical studies.

[10] A further difference as compared to the standard gravity equation is that our expression for multilateral resistance, namely $\sum_{k=1}^{m} \left( \frac{\tau_{jk}}{P_k} \right)^{-k} s_k \left( \frac{Y_k}{F_{Xk}} \right)^{\frac{\kappa+1-\sigma}{\sigma-1}}$, does not equal $P_j$ in our set-up. The reason for this is that the exporters' market entry cost $F_{Xk}$ depends on population size and is therefore not symmetric when there is trade between two countries of different size.

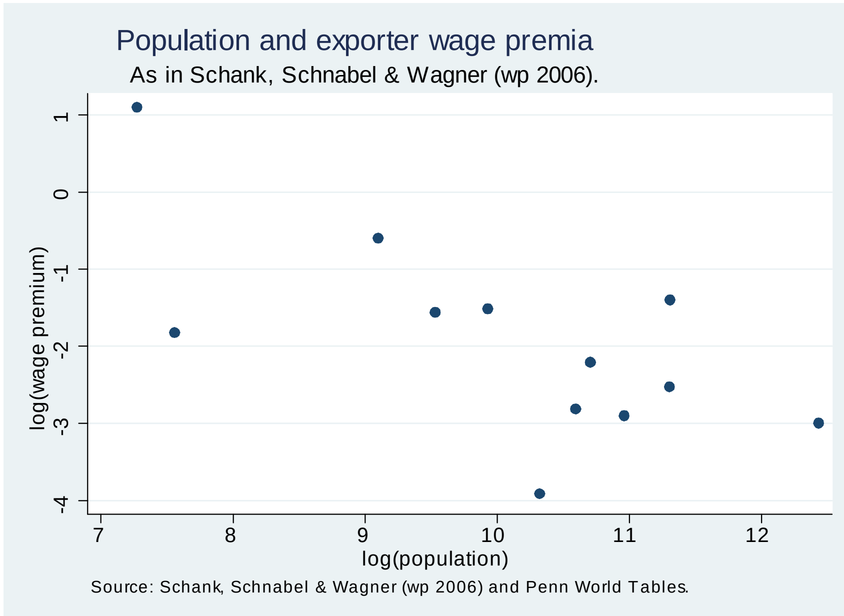


Figure 2: Export premiums decrease in country size.

### 4.1 Empirical evidence of Result 3

We turn to stylized evidence to evaluate Result 3 that country size affects the relative performance of exporters and non-exporters. A study by Schank et al. (2007) offers a literature overview where they measure the wage premium of exporting firms as compared to non-exporting firms. Typically, a regression is run on firm level data with some measure of wages as the dependent variable, and a dummy variable indicating whether the firm is an exporter. The estimated coefficient for this dummy variable is the exporter wage premium as compared to that of non-exporters. We interpret this wage premium to indicate productivity differences between exporters and non-exporters.[11] Figure 2 plots the exporter wage premium versus market size (population) of countries in the studies surveyed in the appendix of Schank et al. (2007).[12] We have also added an observation for Sweden using data provided by Statistics Sweden. Naturally, it must be acknowledged that all regressions are not done with exactly the same methodology or fully comparable data. Nevertheless, Figure 2 shows a negative correlation between export premium and population size. Running a regression on this data gives a slope of $-0.58$ with a t-value of $-2.93$.

[11] This interpretation is consistent with e.g. learning effects as in Malchow-Møller et al. (2007) or by a non-competitive wage setting à la Shapiro and Stiglitz (1984).

[12] We use population to measure market size since it most closely corresponds to our model specification. However, the pattern of exporter wage premia to market size is the same, if GDP is instead used to measure market size.

## 4.2 Empirical evidence of Result 4a,b

In this section, we empirically test predictions of our model related to the effects of market size on trade shares. First, we check that our dataset has the well known property that manufacturing export shares are negatively correlated with country size, as predicted by Result 4a. Even though this is well known theoretical result, it does not typically apply in this type of model. We employ sector level data within the OECD using the STAN database with yearly observations from 1980 to 2003, and run the simple regression

$$s_{ist} = \beta_0 + \beta_1 l_{it} + \varepsilon_{ist}, \tag{37}$$

where $s_{ist} \equiv \log\left(\frac{X}{Y}_{ist}\right)$, $l_{it} \equiv \log L_{it}$. The regression is run at the sectorial level. Table 1 shows the regression of export shares over GDP on a sectorial level in 2001. The regression includes fixed effects for sectors. The coefficient for population, which can be interpreted as a standard elasticity, is highly significant and of the expected sign.

| Year 2001 | |
|---|---|
| Dependent variable | $\frac{\mathrm{X}}{\mathrm{Y}}$ |
| | (1) |
| $l_{it}$ | -0.15*** |
| | (0.024) |
| Sector dummies | Yes |
| Observations | 595 |
| R squared | 0.43 |

The table reports the estimates of the regression of export shares on country size controlling for sector-specific fixed effects. The export share is at the sectoral level and population at the country level. Note: Standard errors in parentheses. * significant at 10%, ** significant at 5%, *** significant at 1%.

Table 1: Export Shares and Country Size

Next Result 4b states that the export share of the manufacturing sector across countries converges as the fixed component of the exporting beachhead cost, $f_X$, approaches the value for the fixed component of the domestic beachhead cost, $f_D$. Given that this has been happening over time, we should observe converging manufacturing trade shares over time. The assumption that the relative access cost to foreign markets, as compared to that of the domestic market, has been falling over time is very much in line with the often cited effect of globalization making the world more alike. A concrete example supporting this assumption is the process of product standardization and removal of non-tariff barriers to trade within the European Union during the last 20-30 years. GATT and WTO negotiations have also aimed at reducing nontariff barriers to trade during this period. Finally, the rapid improvement of telecommunications, including the internet, simplifies business contacts and information gathering about foreign markets, which may be interpreted as a fall in $f_X$.

We look at the evolution of manufacturing export shares over time, at a sectorial level within the OECD using the STAN database with yearly observations from 1980 to 2003.[13] Accepting the assumption that the process of falling access costs to foreign markets has occurred gradually over time during the period investigated, we should observe converging manufacturing export shares. First, we graphically explore the data on manufacturing aggregated to the country level. We want a balanced panel so that we only include country and sector pairs that have nonmissing data throughout the period 1980 to 2000, before we aggregate to the country level. For this period, there are data on most sectors for most countries throughout. The appendix contains a list of the 18 countries that are included. First, Figure 3 plots the evolution of the coefficient of variation of the distribution of trade shares (exports divided by output) for the sample of countries. We use the coefficient of variation since it is neutral to scale. The graph gives an indication that the average dispersion of trade shares in manufacturing across countries decreases throughout the period. This result is driven by the fact that the mean grows more rapidly over time than the standard deviation. Figure 4 plots histograms of country level trade shares for five equally spaced years in the period. It can be seen that the mean increases while a change in the absolute level of dispersion is more difficult to detect.

Next, we proceed to use sectorial data to analyse trade share convergence using a regression framework from the standard empirical growth literature (see e.g. Barro and Sala-i-Martin (1991), Barro and Sala-i-Martin (1992), Bernard and Jones (1996) and Mankiw et al. (1992)). We use the initial value of the manufacturing export share for which we have data and regress the average growth rate in export shares, $\widetilde{\Delta s}_{is}$, on the initial level of trade shares, where the average growth rates are computed as the coefficient on the trend dummy in a regression of logged values on a constant and linear trend, see e.g. Bernard and Jones (1996):

$$\widetilde{\Delta s}_{is} = \beta_0 + \beta_1 s_{is0} + \beta_2 D_s + \varepsilon_{is}.$$

The errors are clustered at the country level and sector dummies are included. The model predicts that $\beta_1$ should be negative since the higher was the initial level, the lower would be the average change over time if convergence held. In Table 2, it is shown that the growth rate

[13] We include all manufacturing sectors except those related to the extraction of raw materials since we do not believe these to be affected by the dynamics described in this paper.

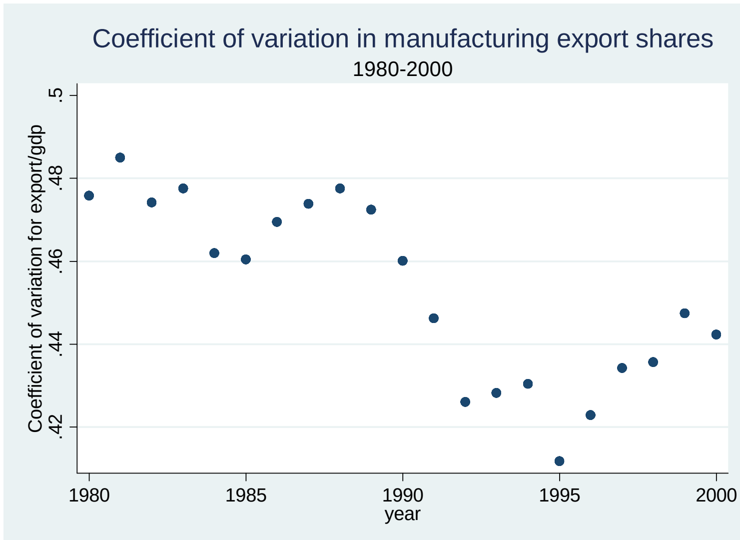


Figure 3: The coefficient of variation for country level export shares. Source: OECD STAN.

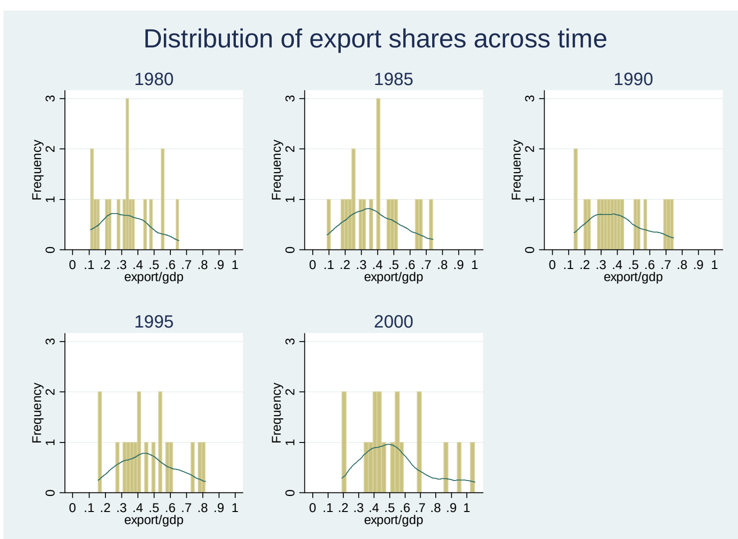


Figure 4: The mean country level export share. Source: OECD STAN Industrial Database.

of export shares depends negatively on the initial level in 1980, thereby suggesting convergence within the OECD at the sectorial level.

| Years 1980 to 2002 | |
|---|---|
| Dependent variable | $\widetilde{\Delta s_{ist}}$ |
| $s_{i,1980}$ | -0.020*** |
| | (0.002) |
| Sector dummies | Yes |
| Observations | 406 |
| R squared | 0.37 |

The table reports the estimates from the convergence regression of the average change of trade shares at the sectoral level on the initial levels of trade shares. Sector-specific effects are controlled for by sectoral fixed effets.

Note: Standard errors in parentheses. Errors are clustered at the country level. * significant at 10% ** significant at 5% *** significant at 1%.

Table 2: Convergence (Initial Values)

## 5 Conclusion

This paper has explicitly modelled a market size dependent market access or beachhead cost in the heterogenous firms and trade model of Melitz (2003). We model this cost as having one variable component that increases in market size, and one fixed component. The fixed component could e.g. be interpreted as the cost of standardizing a product for a particular market, while the variable cost term e.g. represents the fact that the advertising cost of introducing a new product increases with the size of the market (the number of consumers).

In essence we make a relatively small change to the framework of Helpman et al. (2004), which preserves the analytical solvability of the model. Nevertheless it leads to a number of new results. The productivity of non-exporting as well as exporting firms will depend on market size, as will manufacturing export shares. In particular, we show that firms based in a large market are more productive than firms in a smaller market, due to the higher fixed costs related to establish a brand in these markets. The relationship between exporter productivity and export market size is U-shaped, however. This is because a larger market means that the fixed entry cost (e.g. standardization) can be spread over more units, while the variable entry cost (cost of establishing a new brand) increases in the market size. Therefore, for sufficiently small export markets, the productivity of exporters decreases in the export market size, which is consistent with the feature of the firm level data in Eaton et al. (2005) that the number of exporting firms increases in the size of the export market. For sufficiently large exports markets, on the contrary, exporter productivity increases in the export market size in accordance with Melitz and Ottaviano (2005). Furthermore we find that, as in the standard model, exporters are more productive than non-exporters, but this productivity premium decreases in the size of the home country, which is consistent with the stylized facts presented in the paper. Fourth, we show that the manufacturing export share of a country decreases in its own size, and increases in the size of the trade partner. This effect decreases as markets are integrated (in the sense that the fixed beachhead cost of foreign markets declines). Accepting that market access costs into foreign markets have been falling over time as a result of globalization, the model predicts converging manufacturing export shares over time. This result is supported by the empirical section of the paper, where this hypothesis is tested using sector level OECD data from 1980 to 2003. Finally, we derive a gravity equation from our set-up. This equation contains a measure of GDP per capita, which implies that our set-up gives a theoretical rationale for the common practise of including this variable in gravity regressions of trade flows.

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# 6 Appendix

## 6.1 The n-country case

The set of equations, using (6), (7) and (8) for $n$ countries, that produce the solutions for the vector $B \equiv (B_1, ....., B_n)$ for a model with $n$ countries can be generalized into

$$\begin{aligned} B_i^{\beta} F_D(L_i)^{1-\beta} + \sum_{j \neq i} \phi_{ij}^{\beta} B_j^{\beta} F_X(L_j)^{1-\beta} &= (\beta - 1) F_E \\ \sum_{j=1}^{n} \Omega_{ij} B_j^{\beta} F_D(L_j)^{1-\beta} &= (\beta - 1) F_E \qquad \forall i \in \{1, ....., n\}, \end{aligned} \tag{38}$$

where $\Omega_{ij} = \phi_{ij}^{\beta} \frac{F_X(L_j)}{F_D(L_j)}$ and $\Omega_{ii} = 1$. If we define $\gamma_i \equiv B_i^{\beta} F_D(L_i)^{1-\beta}$, the equations generalize to:

$$\sum_{j=1}^{n} \Omega_{ij} \gamma_j = (\beta - 1) F_E \qquad \forall i \in \{1, ....., n\}, \tag{39}$$

or, in a matrix form:

$$\begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & ... & ... & \Omega_{1,n-1} & \Omega_{1,n} \\ \Omega_{2,1} & . & & & & \Omega_{2,n} \\ . & & . & & & . \\ . & & & . & & . \\ \Omega_{n-1,1} & & & & . & \Omega_{n-1,n} \\ \Omega_{n,1} & \Omega_{n,2} & ... & ... & \Omega_{n,n-1} & \Omega_{n,n} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} = \iota (\beta - 1) F_{E}, \tag{40}$$

or

$$\Omega_{n \times n} \gamma_{n \times 1} = \iota_{n \times 1} (\beta - 1) F_E \tag{41}$$

where $\Omega$ consists of entries where the element on the $i$th row and the $j$th column is $\Omega_{ji}$, $\gamma$ is a vector $\left(\gamma_1, \gamma_2, \text{ ..... }, \gamma_{n-1}, \gamma_n\right)'$ and $\iota$ is an $n$ dimensional vector of ones. Inverting the $\Omega_{n \times n}$ matrix yields the $B_j$ which, in turn, determines the cut-off productivity levels.

The case with three countries would then look as follows

$$\begin{bmatrix} 1 & \Omega_{12} & \Omega_{13} \\ \Omega_{21} & 1 & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & 1 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} = \iota (\beta - 1) F_E. \tag{42}$$

The determinant of $\Omega$ is

$$|\Omega| = 1 + \Omega_{12}\Omega_{23}\Omega_{31} + \Omega_{13}\Omega_{21}\Omega_{32} - (\Upsilon_{12} + \Upsilon_{13} + \Upsilon_{23}), \tag{43}$$

where $\Upsilon_{ij} \equiv \Omega_{ij}\Omega_{ji}$ is the product of the bilateral trade costs between $i$ and $j$. Solving the system for $\gamma$ gives:

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} = \frac{1}{|\Omega|} \begin{bmatrix} 1 - \Upsilon_{23} & \Omega_{13}\Omega_{32} - \Omega_{12} & \Omega_{12}\Omega_{23} - \Omega_{13} \\ \Omega_{23}\Omega_{31} - \Omega_{21} & 1 - \Upsilon_{13} & \Omega_{13}\Omega_{21} - \Omega_{23} \\ \Omega_{21}\Omega_{32} - \Omega_{31} & \Omega_{12}\Omega_{31} - \Omega_{32} & 1 - \Upsilon_{12} \end{bmatrix} \iota (\beta - 1) F_E, \tag{44}$$

which gives the following solutions for market size per firm, $B_i$:

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \frac{(\beta-1)\,F_E}{1+\Omega_{12}\Omega_{23}\Omega_{31}+\Omega_{13}\Omega_{21}\Omega_{32}-(\Upsilon_{12}+\Upsilon_{13}+\Upsilon_{23})} \tag{45}$$

$$* \begin{bmatrix} 1-\Upsilon_{23}+\Omega_{13}(\Omega_{32}-1)+\Omega_{12}(\Omega_{23}-1) \\ 1-\Upsilon_{13}+\Omega_{23}(\Omega_{31}-1)+\Omega_{21}(\Omega_{13}-1) \\ 1-\Upsilon_{12}+\Omega_{32}(\Omega_{21}-1)+\Omega_{31}(\Omega_{12}-1) \end{bmatrix} \begin{bmatrix} F_D (L_1)^{\beta-1} \\ F_D (L_2)^{\beta-1} \\ F_D (L_3)^{\beta-1} \end{bmatrix}^T .$$

## 6.2 $\frac{\partial}{\partial L_k}\left(\frac{a_{Dj}}{a_{Dk}}\right)^{\theta} > 0$ for $\Omega_j, \Omega_k < 1$

Proof:

From (**??**)

$$\left(\frac{a_{Dj}}{a_{Dk}}\right)^{\theta} = \frac{F_{Dk}}{F_{Dj}}\left(\frac{1-\Omega_k}{1-\Omega_j}\right).$$

Differentiating w.r.t. $L_k$ gives:

$$\frac{\partial}{\partial L_k}\left(\frac{F_{Dk}}{F_{Dj}}\left(\frac{1-\Omega_k}{1-\Omega_j}\right)\right) = \frac{\gamma L_k^{\gamma-1}}{F_{Dj}(1-\Omega_j)}\left(1-\Omega_k-(\beta-1)\,\Omega_k\left(1-\Omega_k^{\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}\right)\right). \tag{46}$$

The sign of the derivative depends on the sign of the term:

$$\left(1-\Omega_k-(\beta-1)\,\Omega_k\left(1-\Omega_k^{\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}\right)\right). \tag{47}$$

The first- and second-order conditions for a minimum of this term w.r.t. $\Omega(L_k)$ are:

$$\begin{aligned} \frac{\partial}{\partial \Omega_k}\left(\Omega_k\left(1+(\beta-1)\left(1-\Omega_k^{\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}\right)\right)\right) &= \beta\left(\Omega_k^{\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}-1\right)=0 \\ \frac{\partial^2}{\partial^2 \Omega_k}\left(\Omega_k\left(1+(\beta-1)\left(1-\Omega_k^{\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}\right)\right)\right) &= \frac{\beta}{\beta-1}\Omega_k^{\frac{1}{\beta-1}-1}\phi^{\frac{\beta}{1-\beta}} > 0. \end{aligned} \tag{48}$$

The minimum is, thus, given by $\Omega_k = 1$ (since $\Omega_k = 1 \iff \phi = 1$). Substituting $\Omega_k = 1$ into (46) gives $\frac{\partial}{\partial L_k}\left(\frac{a_{Dj}}{a_{Dk}}\right)^{\theta} = 0$. Consequently, it must be the case that $\frac{\partial}{\partial L_k}\left(\frac{a_{Dj}}{a_{Dk}}\right)^{\theta} > 0$ for $\Omega_j, \Omega_k < 1$.

## 6.3 $\frac{\partial a_{Dj}}{\partial L_j} < 0$

From (6.2), we have that

$$\frac{\partial\left(\frac{a_{Dj}}{a_{Dk}}\right)}{\partial L_k} = \frac{\partial a_{Dj}}{\partial L_k}\frac{1}{a_{Dk}} - \frac{a_{Dj}}{(a_{Dk})^2}\frac{\partial a_{Dk}}{\partial L_k} > 0. \tag{49}$$

Since from (11) $\frac{\partial a_{Dj}}{\partial L_k} < 0$, (49) holds iff $\frac{\partial a_{Dk}}{\partial L_k} < 0$.

## 6.4 $\frac{\partial a_{Xjk}}{\partial L_k}$

From (12)

$$a^{\theta}_{Xjk} = \frac{(\beta-1)\,\Omega_k F_E}{F_{Xk}}\left(\frac{(1-\Omega_j)}{1-\Omega_j\Omega_k}\right) = (\beta-1)\,F_E(1-\Omega_j)\frac{1}{F_{Xk}(\frac{1}{\Omega_k}-\Omega_j)}. \tag{50}$$

The sign of $\frac{\partial a_{Xjk}}{\partial L_k}$ is therefore determined by the sign of

$$\begin{aligned} & \frac{\partial}{\partial L_k}[F_{Xk}(\frac{1}{\Omega_k}-\Omega_j)] && (51)\\ = \;& \frac{\partial}{\partial L_k}[F^{\beta}_{Xk}F^{1-\beta}_{Dk}\phi^{-\beta}-F_{Xk}\Omega_j]. && (52)\end{aligned}$$

Now

$$\begin{aligned} \frac{\partial}{\partial L_k}[F^{\beta}_{Xk}F^{1-\beta}_{Dk}\phi^{-\beta}-F_{Xk}\Omega_j] &\lesseqgtr 0\\ &\Leftrightarrow\\ \left(\frac{F_{Xk}}{F_{Dk}}\right)^{\beta}\left(\beta\frac{F_{Dk}}{F_{Xk}}-(\beta-1)\right) &\lesseqgtr \Omega_j\phi^{\beta}\\ &\Leftrightarrow\\ \beta-\frac{F_{Xk}}{F_{Dk}}(\beta-1) &\lesseqgtr \Omega_j\Omega_k. \end{aligned}$$

## 6.5 $\left(\frac{a_{Dj}}{a_{Dk}}\right)^{\theta} > 1$ iff $L_k > L_j$ for $\Omega_j, \Omega_k < 1$

Proof:

First

$$L_j = L_k \Longleftrightarrow \left(\frac{a_{Dj}}{a_{Dk}}\right)^{\theta} = \frac{F_{Dk}}{F_{Dj}}\left(\frac{1-\Omega_k}{1-\Omega_j}\right) = 1.$$

That $L_j = L_k \Longleftrightarrow \left(\frac{a_{Dj}}{a_{Dk}}\right)^{\theta} > 1$ for $\Omega_j, \Omega_k < 1$ now follows from $\frac{\partial}{\partial L_k}\left(\frac{a_{Dj}}{a_{Dk}}\right)^{\theta} > 0$ for $\Omega_j, \Omega_k < 1$.

## 6.6 $\frac{\partial\left(\frac{a_{Dj}}{a_{Xjk}}\right)}{\partial L_j} < 0 \quad for \quad \Omega_j < 1$.

Proof:

$$\frac{\partial}{\partial L_j}\left(\frac{a_{Dj}}{a_{Xjk}}\right)^{\theta} = \gamma L_j^{\gamma-1}\frac{F_{Xk}}{F^2_{Dj}}\frac{(1-\Omega_k)}{\Omega_k(1-\Omega_j)}\left((\beta-1)\frac{\Omega_j\left(1-\frac{F_{Dj}}{F_{Xk}}\right)}{(1-\Omega_j)}-1\right). \tag{53}$$

The sign of (53) will depend on the sign of the term:

$$\Theta_j \equiv \left( \frac{(\beta - 1)\,\Omega_j \left(1 - \frac{F_{Dj}}{F_{Xj}}\right)}{(1 - \Omega_j)} - 1 \right). \tag{54}$$

The F.O.C. when maximising $\Theta_j$ w.r.t. $\phi$ is:

$$\frac{(\beta - 1)\,\beta\Omega_j \left(1 - \frac{F_{Dj}}{F_{Xj}}\right)}{\phi\,(1 - \Omega_j)} - \frac{(\beta - 1)\,\beta\Omega_j^2 \left(1 - \frac{F_{Dj}}{F_{Xj}}\right)}{\phi\,(1 - \Omega_j)^2} = 0 \Longleftrightarrow 1 - \frac{\Omega_j}{(1 - \Omega_j)} = 0. \tag{55}$$

So the only stationary point is $\Omega_j = 1$. Furthermore, $\Theta_j(\Omega_j = 0) = -1$ and $\lim_{\Omega(L_j)\to 1} \Theta_j = 0$. Therefore, it follows that for $\Omega_j \in [0, 1)$:

$$\frac{d}{dL_j} \left( \frac{a_{Dj}}{a_{Xjk}} \right)^{\theta} < 0.$$

### 6.7 Countries included in Figure 4.

The following countries are included in Figure 4. This is a subset of the full STAN sample but it is the only set of countries for which there is data for the full length of 1970 until 2002.

Austria
Belgium
Canada
Denmark
Finland
France
Iceland
Ireland
Italy
Japan
Netherlands
New Zealand
Norway
Portugal
Spain
Sweden
United Kingdom
United States